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Bundling with Resale

By
Drew Vollmer[1]

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[1] Antitrust Division, U.S. Department of Justice. Email: Andrew.Vollmer@usdoj.gov.

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Drew Vollmer*

U.S. Department of Justice

Abstract

How does resale affect multiproduct bundling? I investigate using a model of monopoly bundling with costly resale. Consumers purchase in the primary market while anticipating resale, then participate in a resale market with market-clearing prices. Resale forces the monopolist to balance the additional profit from a discounted bundle against the opportunity for consumer arbitrage. In equilibrium, the monopolist may still offer a discounted bundle, but resale reduces the returns to bundling and has an ambiguous effect on consumer and total welfare. When consumers have heterogeneous costs of resale, it is possible for consumers to resell in equilibrium.

1 Introduction

Although economists have devoted significant attention to multiproduct bundling, the literature has not considered a critical feature of many bundles: that they can be split apart and resold. Consider season tickets for sports and theater, which consumers separate into tickets they use and tickets they resell on StubHub. Or think of books sold in box sets, which consumers can list on eBay and Amazon. Other examples include cookware sets, golf club sets, and trading cards. More generally, bundles can be split and resold whenever their component goods are not physically attached.[1]

Studying resale matters because resale affects the incentives to bundle. Discounted bundles encourage consumers to buy more goods, but resale undermines the strategy by allowing them to purchase the bundle, resell some goods, and keep the discount. The effects of resale on the buyer's

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[1] Some bundles, such as cable TV packages, cannot be separated for resale. Others can be separated but may be impractical to resell, like fast food combo meals.

purchase decisions and the seller's incentives to discount the bundle have not yet been explored. Consequently, our understanding of markets that combine bundling and resale is incomplete.

The purpose of this paper is to examine the effects of resale on bundling. I study a model of monopoly bundling with costly resale to determine the effects of resale on profit, welfare, and the monopolist's problem. Resale introduces a new tradeoff to the monopolist's problem, reduces profit, and has an ambiguous effect on consumer and total welfare.

Resale complicates the monopolist's problem by creating a tradeoff between the profitability of bundling and consumers' gains from resale. The essence of bundling is that larger bundle discounts raise profit, but the gains are tempered when large discounts encourage consumers to buy the bundle and break it apart to resell. Resale is harmful for the monopolist because it only receives the discounted bundle price instead of the higher prices for the constituent goods.

I find that resale limits, but does not eliminate, the returns to bundling in equilibrium. The monopolist would earn more without resale, but it still discounts the bundle because resale is costly for consumers. More surprisingly, the ability to resell may actually harm consumers and society. For a fixed allocation, resale should boost welfare. However, the monopolist responds to resale by changing its prices and hence the initial allocation of goods, making the effects of resale on consumer and total welfare ambiguous.

The analysis contributes to our understanding of both multiproduct bundling and resale. Although a small literature considers resale when a monopolist sells many units of one good (e.g. Alger (1999)), no prior study has considered how resale affects multiproduct bundling. The combination of bundling and resale matters because resale affects welfare and the returns to bundling—a traditional focus of the bundling literature. Moreover, bundling and resale coexist in economically significant markets: revenue from sports tickets, sold in mixed bundles, exceeded \$10bn in 2019 (Statista (2020)) and revenue from just one trading card game is estimated to exceed \$500m per year (Deaux (2019)). The welfare results also have implications for resale policy. U.S. law guarantees a right to resell most goods[2] and some US states have expanded resale rights further (Pender (2017)). I show that consumers and society do not necessarily benefit from resale when the seller bundles.

The basis for the analysis is a model in which a monopolist seller practices mixed bundling and atomistic consumers have access to a resale market with transaction costs. The monopolist starts by setting primary market prices for two goods and a bundle containing both goods. After

[2] The first-sale doctrine prevents copyright holders from restricting the buyer's ability to resell in 17 U.S.C. §109.

observing prices, consumers purchase goods in the primary market while anticipating the ability to engage in costly resale. Finally, consumers participate in a resale market where resale prices are determined by market clearing.

Consumers resell in equilibrium when the cost of purchasing the bundle and participating in the resale market is lower than the total cost of buying each component good in the primary market. In Section 3, I study the case where consumers have a homogeneous cost of resale and show there is a threshold level of the bundle discount where consumers become willing to resell. In equilibrium, resale causes the monopolist to shrink its discount until no consumer is willing to resell. The result is a simple characterization of the monopolist's problem: it maximizes profit without resale subject to a constraint on the bundle discount. The model is also tractable enough to clarify the forces affecting welfare. I show that when the cost of resale changes, the change in consumer welfare depends entirely on inframarginal consumers, but for total welfare it depends entirely on marginal consumers.

The results with a single cost of resale are appealingly simple, but do not match the motivating examples that involve both bundle discounts and observed resale. In Section 4, I study consumers with heterogeneous costs of resale and show that the equilibrium can feature both bundle discounts and resale transactions. Unlike the case with homogeneous costs, heterogeneous costs of resale allow the number of willing resellers to rise gradually with the bundle discount. The result is a more complex pricing problem: the monopolist must choose a discount balancing the number of consumers who resell with the profit earned from consumers who do not. In equilibrium, the monopolist might tolerate resale among low-cost types if a large discount earns more profit from high-cost types.

A contribution of the model is its use of an endogenous resale market where resale prices are determined by market clearing. The approach is novel because earlier studies of price discrimination with arbitrage rely on simplifying assumptions to limit aftermarket outcomes. An example is Alger (1999), who assumes that consumers evenly share multi-unit packages at no cost—for instance, paying half of the price for half of the quantity. As a result, aftermarket prices are directly determined by the monopolist's menu. In contrast, resale prices in this paper are an equilibrium outcome determined by competitive forces and reservation values in the resale market. In addition to providing a richer equilibrium, the assumption of market power in the primary market and market clearing in the resale market is more realistic: in observed markets, sellers control the prices of differentiated products and small resellers interact in online resale markets.

The paper proceeds as follows. I start by reviewing the relevant literature. In Section 2, I introduce the model, and in Section 3 I study a benchmark case with homogeneous costs of resale. I consider heterogeneous costs in Section 4. In Section 5, I conclude.

Related Literature. A primary focus of the bundling literature to date is to determine when monopoly bundling is profitable. Studies have considered how the value of bundling depends on the distribution of consumer values, as in McAfee et al. (1989) and Chen and Riordan (2013), the number of goods sold, as in Bakos and Brynjolfsson (1999), and uncertainty over future states of the world, as in Alexandrov and Bedre-Defolie (2014). This paper contributes by demonstrating that resale is a determinant of the profitability of bundling.

Researchers have also considered bundling outside the traditional setting with a monopolist seller. Chen and Li (2018) consider the effect of bundling when a buyer must procure several products. Zhou (2017) and Zhou (2021) study the effects of bundling in a competitive environment. Pagnozzi (2009) considers bundling in auctions when consumers can resell after the auction, but the implications of resale in his study are specific to auctions rather than monopoly bundling.

Bundling has also been studied empirically. Gandal et al. (2018) use data on computer software to determine the effect of correlations in consumer values on the profitability of bundling. Chu et al. (2011) use data on theater ticket sales to compare the performance of theoretically optimal bundle pricing to simpler rules. Crawford and Yurukoglu (2012) study bundling in a competitive setting, cable television, and consider its effect on upstream bargaining.

A separate literature considers the effects of resale markets. The effect of resale on sellers of durable goods has been widely studied, for instance in Chen et al. (2013). Sellers of durable goods can benefit from resale because it allocates past vintages to the consumers who value them most, but resale forces sellers to compete against past vintages. This paper features a similar tradeoff, with resale increasing consumers' willingness to pay for the bundle but introducing competition for individual goods. I find that the harms of resale always outweigh the benefits to the seller. Sellers of perishable goods can also benefit from reallocation when there is limited capacity and consumers receive preference shocks, as in Cui et al. (2014).

Chen et al. (2021) consider how to price loot boxes, random prizes that can be thought of as bundles of award probabilities. Their analysis of salvage—letting consumers return an unwanted item for a partial refund—is similar to allowing resale, but it differs in that returned goods do not compete with the seller's offerings.

Finally, this paper is related to the literature on price discrimination with resale, which focuses on multi-unit sales of one product. The most notable paper in this literature is Alger (1999), who considers pricing when consumers can make costless joint purchases. In the equilibrium of that analysis, the seller sets prices that prevent all resale. Several other papers, such as Gans and King (2007) and McManus (2001), consider settings where the seller benefits from or practices price discrimination when consumers can resell.

2 Model

A monopolist with zero fixed and marginal costs and no capacity constraints sells two goods, called 1 and 2. The monopolist sets primary market prices $P = (P_1, P_2, P_B)$, where P_1 and P_2 are the prices of goods 1 and 2 and P_B is the price of a bundle containing both goods. The bundle price satisfies $P_B \leq P_1 + P_2$ because consumers can buy each good separately.

The market includes a mass of consumers normalized to one. Each consumer has type (v_1, v_2), where v_1 and v_2 denote the consumer's value for each good and $v_1 + v_2$ is the consumer's value for the bundle. Values are drawn from the joint distribution $F(v_1, v_2)$. I assume that $F(v_1, v_2)$ has support on $[0, 1]^2$ with a strictly positive, atomless density.

The game proceeds as follows. First, the monopolist sets primary market prices P. Next, consumers purchase goods in the primary market. Finally, consumers participate in a resale market with a vector of endogenously determined clearing prices $P^s = (P_1^s, P_2^s)$. The bundle is not offered in the resale market because it does not affect market outcomes.[3] Consumers and the monopolist know the distribution of values $F(v_1, v_2)$ throughout the game.

Participating in the resale market, either as a buyer or reseller, is costly. Consumer k must pay a cost c_k for each good resold or bought from a reseller. For example, when a single good is resold the buyer and reseller each pay the cost, and if a consumer resells both goods he incurs the cost $2c_k$. Costs are independent of values, $c_k \perp (v_1, v_2)$, and follow the distribution $G(c)$, which has density $g(c)$ and satisfies $G(\underline{c}) = 0$ and $G(\bar{c}) = 1$ for some real numbers $0 \leq \underline{c} < \bar{c}$. The distribution of costs of resale is also common knowledge. The cost of resale can be interpreted as the time and effort needed to participate.[4]

[3] Buyers would not pay more than P_B and resellers could not acquire the bundle for less than P_B. The bundle could be resold at $P_B^s = P_B$ if resale were frictionless, but profit and surplus would be the same if consumers purchased in the primary market.

[4] An alternative assumption would be that part of the cost is paid to the resale market operator, like the fees on sites like eBay and StubHub. Under this assumption, the cost would count towards total surplus. This would have no effect on the results in Section 3, but it would increase total welfare in Section 4.

I consider subgame perfect Nash equilibria of the model, which require consumers to have correct expectations for resale prices P^s when they make purchase decisions in the primary market. Equilibrium is a pair (P^*, P^{s*}) of the monopolist's primary market prices P and resale market prices P^s such that (i) primary market prices P^* maximize profit given consumer demand, (ii) consumers make optimal primary market purchases under the expectation that the resale market will clear at prices P^{s*}, and (iii) the prices P^{s*} clear the resale market when consumers make optimal primary market purchases anticipating P^{s*}.

Consumers choose whether to act as buyers or resellers in the resale market. For instance, suppose consumer k wants to acquire good 1. She can earn surplus $v_1 - (P_1^s + c_k)$ by buying it in the resale market, but she can also earn $v_1 - (P_B - P_2^s + c_k)$ by purchasing the bundle, reselling good 2, and keeping good 1. Each consumer chooses the option maximizing surplus.

Surplus-maximizing choices define a map from types (v_1, v_2) to purchase decisions for each cost c_k. Figure 1 depicts the purchase regions for the case without resale. The diagonal line, $v_1 + v_2 = P_B$, separates consumers with positive and negative surplus from the bundle. The horizontal and vertical lines $v_1 = P_1$ and $v_2 = P_2$ do the same for the individual goods. Some consumers have positive surplus for several choices, so the regions denote the surplus-maximizing option.

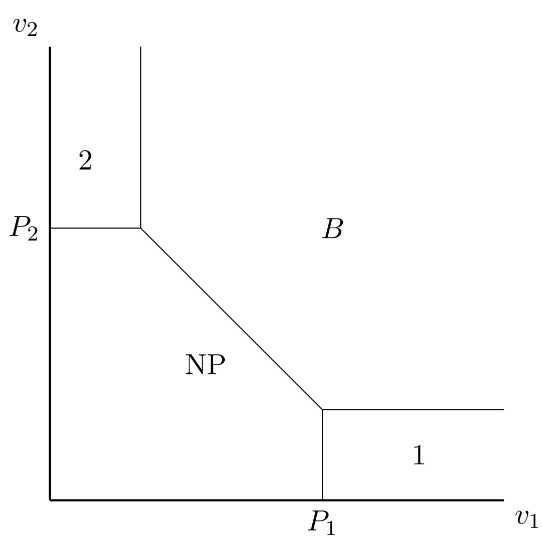


Figure 1: Allocations when there is no resale. Consumers in the B regions purchase the bundle, those in the 1 and 2 regions purchase only good 1 or good 2, and those in the NP region make no purchase.

3 A Homogeneous Cost of Resale

I begin the analysis by considering the case where all consumers share the same cost of participating in the resale market, $c_k = c$ for all consumers k.

3.1 Resale Equilibrium

Suppose that the monopolist has announced its price vector P and consider the subgame in which consumers make primary and then resale market purchase decisions. An equilibrium of the subgame is a vector of secondary market prices $P^{s*}(P)$ such that consumers make their optimal purchase decisions in the primary market believing that resale prices will be $P^{s*}(P)$ and the vector of resale prices $P^{s*}(P)$ clears the resale market after optimal purchases in the primary market. The goal of this subsection is to characterize equilibrium resale prices and the conditions necessary for resale in equilibrium.

The characterization of equilibrium relies on two supporting results. The first establishes a condition for resale prices.

Lemma 1. In any equilibrium with resale market transactions, $P_1^{s*} + P_2^{s*} = P_B$.

Lemma 1 follows from the need for a buyer and seller in each resale transaction. If the sum of resale prices were any higher, there would be no resale buyers, and if it were any lower, there will be no resellers. I focus on equilibria in which $P_1^{s*} + P_2^{s*} = P_B$ because the class includes all equilibria with resale.

The result is useful because it narrows the search for equilibrium resale prices and clarifies that resale supply comes from consumers who purchase the bundle to resell one good and keep the other. No consumer purchases an individual good for resale or tries to resell both goods in the bundle when $c > 0$ because it results in a loss in equilibrium. Consumers may be willing to do so when resale is frictionless, but doing so does not affect market outcomes in the equilibrium of the full model.

An additional implication is that there is no meaningful distinction between consumers who purchase good 1 in the resale market and consumers who buy the bundle to resell good 2: both hold good 1 at the end of the market and earn the same surplus.[5] Consequently, I refer to consumers as wanting to buy good 1 or 2 through resale and do not specify whether they buy or resell in the resale market.

[5] Observe that $v_1 - (P_1^s + c) = v_1 - (P_B - P_2^s + c)$.

The second supporting result establishes that, when the monopolist only sells a bundle, there is a unique vector of equilibrium resale prices when the cost of resale is small enough. The requirement for the cost of resale is given in Assumption 1.

Assumption 1. The cost c satisfies $c < \frac{P_B}{2}$ when $P_B < 1$ and $c < \frac{2-P_B}{2}$ otherwise.

The assumption rules out cases where consumers do not resell because the costs are high relative to the price of the good, either because the price is too low to justify resale ($P_B < 1$) or because the price is so high that it is close to the maximum possible valuation ($P_B \geq 1$). If values for each good were unbounded, the case where $P_B \geq 1$ would be unnecessary. I assume that Assumption 1 is satisfied for the rest of the analysis.

Lemma 2. When the monopolist only sells the bundle ($P_1 = P_2 > 1$), there exists an equilibrium vector of resale market prices $\hat{P}^{s*}$. Under Assumption 1, the vector of resale prices is unique.

The prices $\hat{P}^{s*}$ are called the pure-bundling resale prices and would prevail in the resale market if no consumer bought an individual good in the primary market. They are useful in finding resale prices in the mixed bundling equilibrium.

To see why an equilibrium exists, consider the case of pure bundling with resale depicted in Figure 2. Consumers know that the bundle price is P_B, which defines the diagonal line as $v_1 + v_2 = P_B$, and believe that resale prices will be P^s. Types in region B find it optimal to buy the bundle; those in regions 1 and 2 find it optimal to acquire one good through resale. The key difference from the case with no resale is that the individual good prices (P_1, P_2) have been replaced with cost-inclusive resale prices $(P_1^s + c, P_2^s + c)$. The dashed lines from the (cost-exclusive) resale prices meet at the diagonal by Lemma 1.

The primary market runs before the resale market. Consumers in the B region purchase the bundle and keep it. Half of consumers who want only one good purchase the bundle in order to resell.[6] Finally, consumers participate in the resale market. Consumers who bought the bundle but only want one good resell to consumers who only want one good and did not buy the bundle. The resale market only clears at the expected prices P^s if the mass of consumers in regions 1 and 2 are equal.

Equilibrium therefore requires a vector of resale prices equating the mass of the two regions. Such a price vector exists under mild conditions. By Lemma 1, any increase in P_1^s must cause P_2^s to fall (and vice versa). The increase in P_1^s causes fewer consumers to want good 1 and more to want

[6] Each consumer could flip a coin, for example.

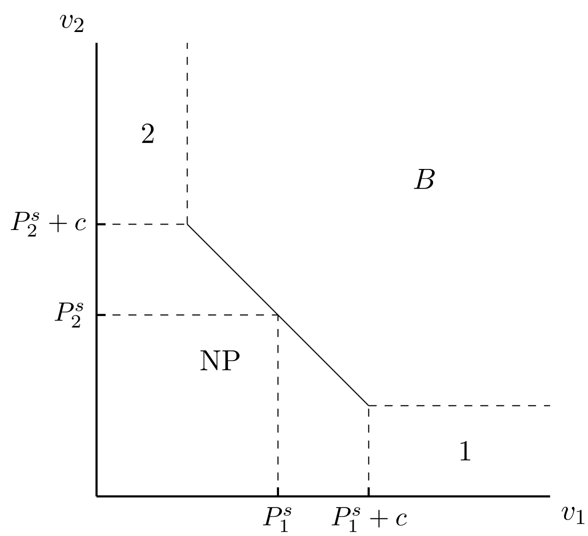


Figure 2: Pure bundling allocations when the resale price is P^s.

good 2. In Figure 2, the intersection of the dashed lines from P_1^s and P_2^s slides southeast along the diagonal, shrinking the 1 region and enlarging the 2 region. With a strictly positive, atomless density, there is a vector of resale prices making the mass of consumers in the two regions equal.

The resale price vector is unique if there are prices such that consumers want to purchase both goods in the resale market. Graphically, the regions 1 and 2 cannot be empty at the same time. Assumption 1 guarantees that resale prices are unique by ruling out cases where both regions are empty because resale is prohibitively expensive.

Lemmas 1 and 2 provide the tools needed to characterize the full resale equilibrium. I add one additional assumption to simplify the conclusion: when all consumers are indifferent between the primary and resale markets, they choose to purchase in the primary market.

Assumption 2. When $P_1 = P_1^{s*} + c$ and $P_2 = P_2^{s*} + c$, no consumers participate in resale.

Theorem 1. Let $P = (P_1, P_2, P_B)$ be a vector of primary market prices. There is resale in equilibrium if and only if $P_1 + P_2 > P_B + 2c$. Resale prices are unique if there are resale transactions.

Theorem 1 shows that the existence of resale in equilibrium reduces to a simple condition: whether $P_1 + P_2 > P_B + 2c$. The condition has a natural interpretation. Resale involves sharing the bundle at price P_B, but incurs the extra cost $2c$. Consumers only share the bundle through resale when it is strictly cheaper than buying each individual good from the seller.

The result also connects resale to the monopolist's ability to discriminate. Bundling is profitable because it lets the monopolist discount the bundle relative to buying each good individually, $P_B <$

$P_1 + P_2$. Doing so lets it boost sales among consumers with high average valuations, like those in the triangle bounded by $v_1 + v_2 = P_B$, $v_1 < P_1$, and $v_2 < P_2$. But discounting the bundle lets consumers share the discount through the resale market, leading to resale whenever the discount exceeds $2c$.

Assumption 2 plays a limited role in the proof of Theorem 1. It allows resale to imply that $P_B + 2c < P_1 + P_2$ by ruling out equilibria of the subgame where consumers resell even though they earn the same surplus in the primary market. The assumption has no substantive consequences for the rest of the analysis. The subgame equilibria it excludes cannot occur in the equilibrium of the full model when $c > 0$ and have no effect on outcomes when $c = 0$.[7]

3.2 The Monopolist's Problem

The monopolist's problem is to set its profit-maximizing price vector P^* given the resale equilibrium described in Theorem 1. The key insight is that it is never profitable for the monopolist to allow resale. To see why, suppose that some consumers resell at prices P^{s*}. Consumers would make the same choices if primary market prices were $(P_1^{s*} + c, P_2^{s*} + c, P_B)$, but they would purchase in the primary market, letting the monopolist earn an additional $2c$ on each transaction that used to involve resale.

The conclusion that it is never optimal to allow resale, coupled with the conditions for resale in Theorem 1, allow a complete description of the monopolist's problem.

Theorem 2. Let $\pi_N(P_1, P_2, P_B)$ be the monopolist's profit when there is no resale. There are no resale transactions in equilibrium. The monopolist's problem is

$$\max_P \pi_N(P_1, P_2, P_B) \text{ subject to } P_1 + P_2 \leq P_B + 2c. \tag{1}$$

Theorem 2 establishes that the effect of resale is to limit the seller's bundle discount, and hence its ability to use price discrimination. Without resale, the monopolist is free to choose any bundle discount it likes, but with resale, it is limited to discounts smaller than $2c$. When resale is frictionless, the monopolist offers no discount at all.

The implication that resale is harmful for sellers rings true because sellers of bundles have attempted to prevent resale. The NFL set a price floor in the resale market before the New York Attorney General's office intervened (Belson (2016)). The Denver Broncos NFL team went so far as

[7] For an explanation of why the subgame equilibrium cannot occur in the full game, see the proof of Theorem 2. The proof does not invoke the assumption for the case where $c > 0$.

to revoke season tickets for consumers who resell too frequently (Thomas (2017)). Preventing the resale of durable goods is more difficult because consumers have a legal right to resell, but sellers can make resale more difficult, for example by printing several books in a single volume.

The condition for resale in equilibrium and the constraint on prices in the monopolist's problem are appealingly simple and demonstrate the effect of resale, but they rely on the assumption that all consumers have the same cost of resale c. With a homogeneous cost, the monopolist can prevent all resale by setting a discount less than $2c$. But the instant the discount exceeds $2c$, consumers flock to the resale market. Later, I show that consumers can resell in equilibrium when there are heterogeneous costs of resale. The prediction that there is no resale in equilibrium is shared in other studies of price discrimination with arbitrage, such as Alger (1999).

3.3 Comparative Statics

To fully describe the effects of resale on bundling, I present comparative statics for profit, the bundle discount, and welfare as the cost of resale changes.

Corollary 1. The monopolist's profit and bundle discount are weakly increasing in the cost of resale c.

The conclusion of Corollary 1 follows from the monopolist's constraint in Theorem 2. It confirms that resale is harmful to monopoly bundling and that the ease of resale matters for the returns to bundling. It also establishes that, when resale becomes more costly, the monopolist weakly increases the bundle discount.

The changes in consumer and total welfare are less clear-cut. When the initial allocation of goods is fixed, consumers benefit from resale by engaging in welfare-enhancing trade. But consumers may not benefit because the monopolist revises its prices in response to resale, changing the initial allocation. As for price discrimination more generally, the effect of resale on consumer welfare is ambiguous.

To make the analysis tractable, I impose the following assumption.

Assumption 3. When there is a homogeneous cost of resale c, the monopolist's optimal prices $P^*(c)$ are differentiable in c and satisfy $P_B^{*'}(c) \leq 0$, $P_1^{*'}(c) \geq 0$, and $P_2^{*'}(c) \geq 0$.

The assumption requires the distribution of values $F(v_1, v_2)$ to be sufficiently regular that prices are continuous in c. The requirement for the derivatives matches the idea that the monopolist weakly increases its bundle discount as c increases, lowering the price of the bundle and raising the prices of the individual goods. Assumption 3 holds when $F(v_1, v_2)$ is uniform.

Under Assumption 3, an increase in c affects consumer welfare by increasing surplus for consumers buying the bundle and lowering it for those who buy the individual goods. Because marginal buyers have surplus zero, the change in consumer welfare depends only on inframarginal consumers.

Theorem 3. Let $\mu_1(c)$, $\mu_2(c)$, and $\mu_B(c)$ denote the masses of consumers buying good 1, good 2, and the bundle when the cost of resale is c. Consumer welfare weakly increases in c if and only if

$$\frac{\partial P_1^*(c)}{\partial c}\mu_1(c) + \frac{\partial P_2^*(c)}{\partial c}\mu_2(c) \leq -\frac{\partial P_B^*(c)}{\partial c}\mu_B(c). \tag{2}$$

The direction of the change in consumer welfare in Theorem 3 is necessarily ambiguous: bundling has an ambiguous effect, and the integral of the change in consumer welfare over c must equal the difference between component pricing and mixed bundling.

The change does not have to be monotone in the cost of resale. The sign of the change depends on how many consumers buy the bundle relative to the number buying the individual goods, and the share in each group varies with the cost of resale. The example in Section 3.4 illustrates the non-monotonicity.

Unlike the change in consumer welfare, the change in total welfare only depends on marginal consumers. Inframarginal consumers make the same purchases when the cost changes and thus do not contribute to the welfare change.

Theorem 4. Total welfare weakly increases in c if and only if

$$\begin{aligned}0 \leq\ & -\frac{\partial P_1^*(c)}{\partial c}\int_0^{P_B^*(c)-P_1^*(c)} P_1^*(c) f(P_1^*(c), v_2)\, dv_2 - \\
& \frac{\partial P_2^*(c)}{\partial c}\int_0^{P_B^*(c)-P_2^*(c)} P_2^*(c) f(v_1, P_2^*(c))\, dv_1 - \\
& \left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_1^*(c)}{\partial c}\right)\int_{P_1^*(c)}^1 \left(P_B^*(c) - P_1^*(c)\right) f(v_1, P_B^*(c) - P_1^*(c))\, dv_1 - \\
& \left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_2^*(c)}{\partial c}\right)\int_{P_2^*(c)}^1 \left(P_B^*(c) - P_2^*(c)\right) f(P_B^*(c) - P_2^*(c), v_2)\, dv_2 - \\
& \frac{\partial P_B^*(c)}{\partial c}\int_{P_B^*(c)-P_2^*(c)}^{P_1^*(c)} P_B^*(c) f(v_1, P_B^*(c) - v_1)\, dv_1.\end{aligned} \tag{3}$$

Each term in Theorem 4 captures the effect of changing c for a line segment of marginal buyers separating the purchase regions in Figure 1. The first two terms describe the change due to consumers who switch to buying nothing when the prices of individual goods rise. The next two are the changes from consumers who switch from buying one good to the bundle. The last term comes from consumers who switch from buying nothing to buying the bundle.

As with consumer welfare, the change in total welfare when the cost c changes is ambiguous because the total effect of bundling on total welfare—an integral over all values of c—is also ambiguous.

3.4 An Example

I conclude the discussion of homogeneous costs with an example in which values are distributed uniformly on the unit square, $f(v_1, v_2) = 1$. If there were no resale, the optimal prices would be $P^* = (\frac{2}{3}, \frac{2}{3}, \frac{4-\sqrt{2}}{3})$, constraining the monopolist whenever $c < \frac{\sqrt{2}}{6}$.[8]

Results are shown in Figure 3. Profit is increasing in the cost c, in line with Theorem 1. Further, the bundle price is decreasing in c and the prices of the individual goods are increasing in c, as assumed in the welfare discussion in the last subsection.

Total welfare is monotone in the cost of resale in this example, but the change in consumer welfare is not. It is decreasing at low values of c and reaches its minimum around $c = .08$. As c increases, consumer welfare starts to increase and reaches its highest level at $c = \frac{\sqrt{2}}{6}$—the equilibrium without resale.

The non-monotonicity is consistent with Theorem 3. The change in consumer welfare is driven by inframarginal consumers, with an increase in c benefiting consumers who buy the bundle and harming consumers who buy individual goods. As c varies, the bundle price falls and the prices of the individual goods rise. At low values of c, the number of consumers buying individual goods is high relative to the number buying the bundle, so the left side of equation (2) is large and consumer welfare falls. But as c increases and more consumers purchase the bundle, the change in consumer welfare turns positive. The change in the set of consumers purchasing each good as c increases is illustrated for $c = .05$ and $c = .2$ in Figure 4.

4 Heterogeneous Costs of Resale

In this section, I show that the monopolist may allow resale in equilibrium when consumers have heterogeneous costs of resale. Resale still reduces profit and has an ambiguous effect on total and consumer welfare, but the monopolist may optimally reduce its bundle discount when the

[8] See the *Journal's* editorial web site for closed-form expressions for prices, profit, and welfare as a function of c.

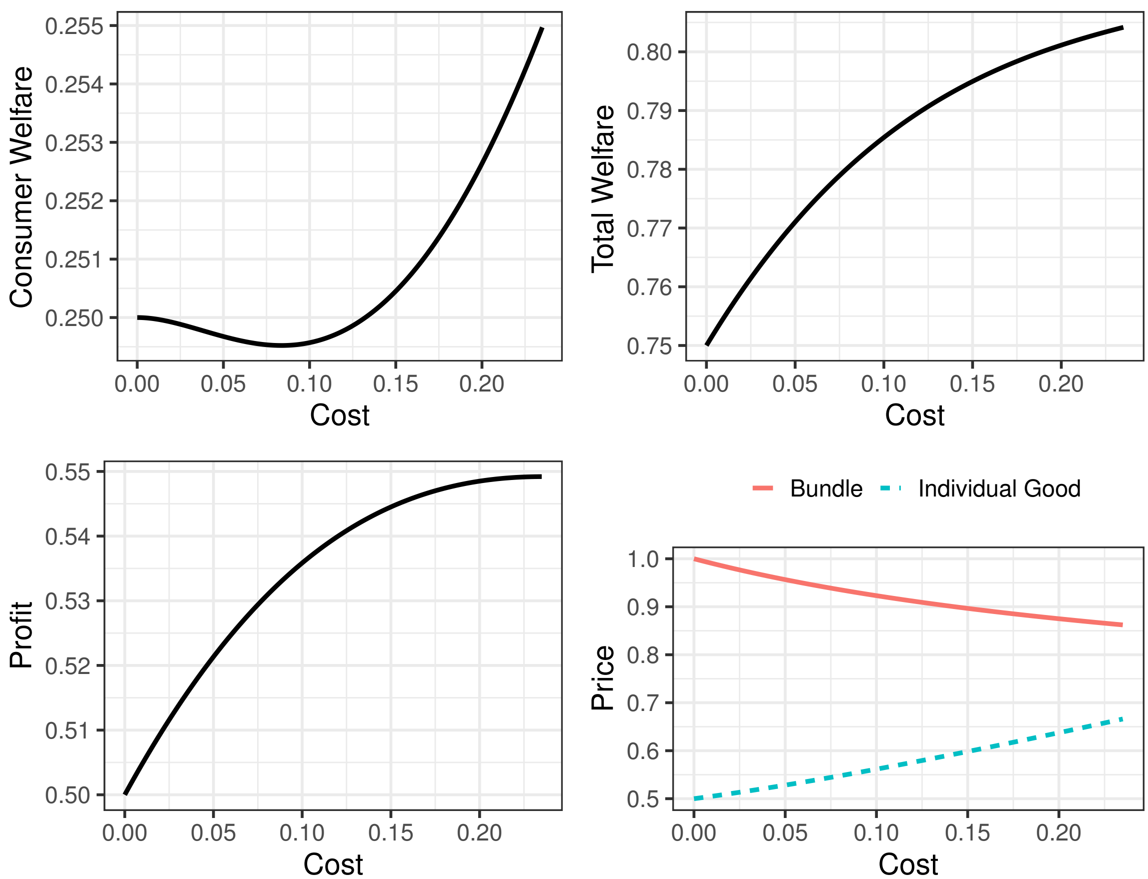
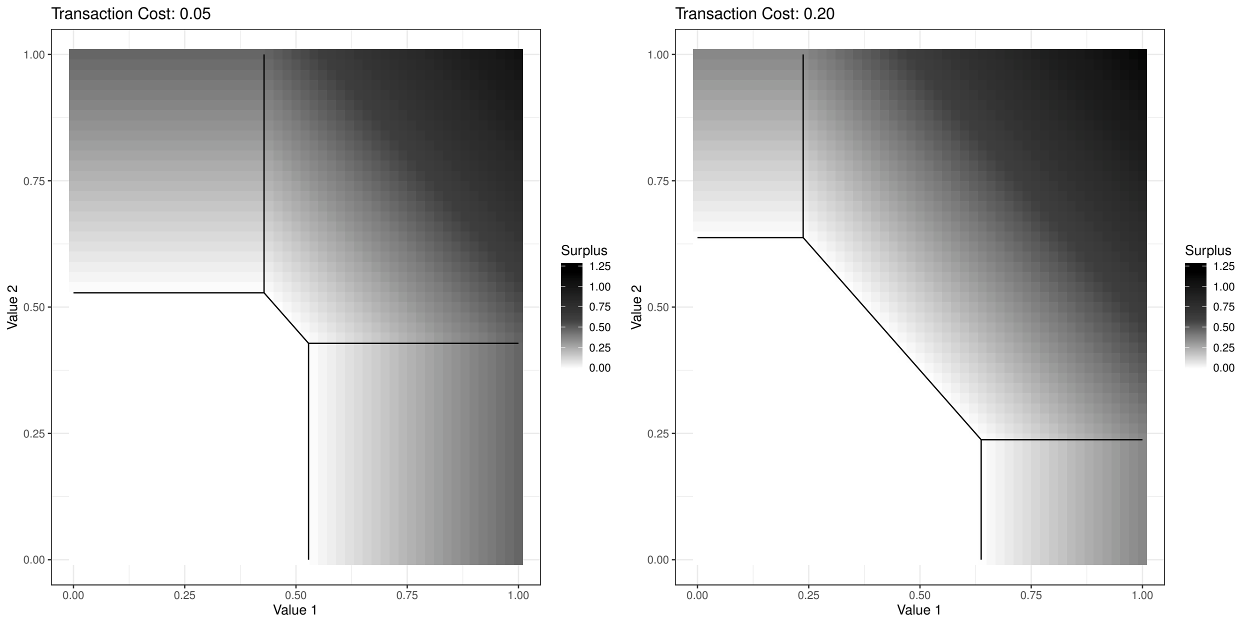


Figure 3: Consumer welfare, total welfare, profit, and prices when F is uniform.

distribution of costs of resale shifts upward.

The monopolist allows resale in equilibrium because preventing low-cost types from reselling reduces the profit earned from high-cost types. Some consumers may have very low costs, forcing the monopolist to set a negligible bundle discount if it wants to prevent resale. Instead, it may prefer a higher discount that earns more from other consumers but allows low-cost consumers to resell. The tradeoff is presented in Example 1, followed by a general analysis.

Example 1. Suppose that values are distributed uniformly, half of consumers have $c = .2$, and the other half have $c = 0$. Recall that values and costs are independent. The monopolist has two choices: it can prevent all resale by using component pricing ($P_1 = P_2 = .5$, $P_B = 1$) and earn profit .5, or it can offer a larger bundle discount to those with $c = .2$ and allow consumers with $c = 0$ to resell.



(a) $c = .05$ (b) $c = .2$

Figure 4: Consumer welfare for $c = .05$ (left panel) and $c = .2$ (right panel) when F is uniform. The solid lines separate the regions purchasing each good.

The monopolist finds it optimal to practice mixed bundling, setting $P_B^* = .9231$ and $P_1^* = P_2^* = .6615$. It earns $\pi^* = .5218$—higher than the .5 earned without bundling—and allows consumers with $c = 0$ to resell.

To prevent resale, the monopolist in Example 1 would have to abandon bundling altogether. Instead, it prefers to allow half of consumers to resell because bundling is profitable for the other half. The example also demonstrates that the monopolist's optimal prices do not maximize profit earned from consumers who do not resell. The optimal prices if all consumers have $c = .2$ involve the lower bundle price $P_B = .875$; the monopolist adjusts its prices to earn more from the consumers who resell.

Heterogeneous costs are a plausible explanation for observed resale. Some consumers are keen to pocket a few dollars by purchasing resold goods while others do not bother. Sellers are almost certainly aware that consumers can resell parts of the bundle, demonstrated by efforts to prevent resale, but they may continue to bundle as long as there is relatively little resale.

4.1 Equilibrium

To simplify the analysis, I impose Assumption 4.

Assumption 4. $F(v_1, v_2)$ is symmetric.

Assumption 4 simplifies the results by implying that the monopolist's optimal prices satisfy $P_1^* = P_2^*$. Because they are equal in equilibrium, I use P_i to refer to P_1 and P_2.

Define the critical cost c^* as $c^* \equiv P_i - \frac{1}{2}P_B$ and let

$$\mu_B(P_B, c) = \int_{\frac{1}{2}P_B - c}^{1} \int_{\frac{1}{2}P_B - c}^{1} f(v_1, v_2)\, dv_1\, dv_2 - \int_{\frac{1}{2}P_B - c}^{\frac{1}{2}P_B + c} \int_{\frac{1}{2}P_B - c}^{P_B - v_1} f(v_1, v_2)\, dv_2\, dv_1, \tag{4}$$

$$\mu_R(P_B, c) = \int_{\frac{1}{2}P_B + c}^{1} \int_{0}^{\frac{1}{2}P_B - c} f(v_1, v_2)\, dv_2\, dv_1, \tag{5}$$

$$\pi_R(P_B, G, c^*) = \int_{\underline{c}}^{c^*} P_B(\mu_B(P_B, c) + \mu_R(P_B, c))\, dG(c). \tag{6}$$

The expressions describe purchase decisions for consumers with cost c who are willing to resell at bundle price P_B. Consumers are willing to resell if c is below the critical cost c^*, defined from $2P_i = P_B + 2c^*$, and find resale too costly if $c \geq c^*$. The function μ_B gives the fraction of consumers with cost $c < c^*$ who purchase the bundle and do not resell when the bundle price is P_B. Similarly, μ_R gives the fraction with $c < c^*$ who purchase the bundle to resell. An additional fraction μ_R purchases in the resale market. The function π_R gives profit earned from consumers with types $c < c^*$ when c is distributed according to $G(c)$.

Theorem 5. The monopolist's problem is

$$\max_{P} (1 - G(P_i - \frac{1}{2}P_B))\pi_N(P_i, P_B) + \pi_R(P_B, G, P_i - \frac{1}{2}P_B). \tag{7}$$

Equilibrium resale prices are $P^{s*} = (\frac{P_B}{2}, \frac{P_B}{2})$. Consumers with $c < c^*$ resell and those with $c \geq c^*$ do not.

The monopolist now must consider the profit earned from consumers who find resale too costly (the first term), consumers with costs low enough to resell (the second), and the number of consumers in the two groups (which depends on c^*). The result is a tradeoff: the monopolist can generally earn more from consumers who do not resell by increasing its bundle discount, but doing so increases the number of consumers who engage in resale and contribute less to profit.

The tradeoff substantially complicates the monopolist's problem. In Section 3, the effect of resale was to cap its bundle discount. Now, resale forces the monopolist to strike a balance between the profitability of its prices for consumers without resale and the number of consumers who engage in arbitrage at those prices. The tradeoff is essential to explain why the monopolist might allow resale in equilibrium.

4.2 Resale in Equilibrium

Example 1 suggests that resale is possible in equilibrium. I provide a sufficient condition for resale in equilibrium in Theorem 6.

Theorem 6. Assume that the prices $\dot{P}$ maximizing $\pi_N(P_i, P_B)$ subject to $P_i \leq \frac{1}{2}P_B + \underline{c}$ have $\dot{P}_i = \frac{1}{2}\dot{P}_B + \underline{c}$. The monopolist allows the lowest-cost type $\underline{c}$ to resell in equilibrium if $2\underline{c}\mu_R(\dot{P}_B, \underline{c})g(\underline{c}) < \partial\pi_N(\dot{P}_i, \dot{P}_B)/\partial P_i$.

Theorem 6 formalizes the intuition from Example 1. By preventing the lowest-cost type from reselling, the monopolist can gain $2\underline{c}$ from each transaction that used to involve resale. But doing so might not be worthwhile because the monopolist must change its prices and earn less in profit from consumers who did not resell.

Note that the resale condition in Theorem 6 is sufficient but not necessary. It only applies to the lowest-cost type and there could be a global optimum where resale is tolerated even if there are no local improvements.[9] Nonetheless, the intuition explains why the monopolist might allow many types to resell. Table 1 presents examples in which more than a third of consumers have costs low enough to resell at the monopolist's optimal prices.

Theorem 6 requires a mild assumption, that the constraint $P_i \leq \frac{1}{2}P_B + \underline{c}$ binds. The assumption is needed to use the derivative at $\underline{c}$ and holds whenever the monopolist can increase profit by using a larger bundle discount.

4.3 Comparative Statics

Next I consider how pricing, profit, and welfare change when the distribution of costs of resale changes. Corollary 1 offered a general conclusion for pricing in the benchmark model: the monopolist's bundle discount is weakly increasing in the cost of resale, implying more intense price discrimination when resale becomes more costly. Surprisingly, the conclusion is no longer true with a distribution of costs.

[9] The complexity of the seller's problem prevents me from obtaining necessary conditions.

Why might the monopolist reduce its discount when there are stronger barriers to resale? Unlike in Section 3, the monopolist's prices do not necessarily maximize profit from consumers who do not resell for a given bundle discount—they also affect sales to consumers who resell. It is therefore possible that, after resale becomes more costly, the seller might choose a smaller discount that earns more from consumers who do not resell.

Rows 3 and 4 of Table 1 provide an example. The distribution in row 4 first-order stochastically dominates the one in row 3, yet the monopolist sets a smaller discount when costs of resale are higher. The example demonstrates that simple models without resale in equilibrium do not capture the seller's incentives for pricing. However, it remains true that profit increases when resale becomes more difficult.

Theorem 7. Let $\tilde{G}(c)$ be a distribution such that $\tilde{g}(c) \leq g(c)$ for all $c \leq c^*$. Then the firm's profit is higher at $\tilde{G}(c)$ than $G(c)$.

Theorem 7 is directly analogous to Corollary 1 in the original model. The assumption on the distributional shift is necessary because profit earned from resellers is not necessarily monotone in c.[10] The distributional shifts between each pair of rows in Table 1 satisfy the assumptions of Theorem 7. As expected, profit increases in each case.

Changes in consumer and total welfare remain ambiguous, but the expressions are far more complex than in Section 3. In the more stylized model, it is possible to consider the derivative of prices with respect to the cost, reducing welfare changes to the effect of price changes on marginal and inframarginal buyers. Such conclusions are not possible with a distribution of costs because a shift in the distribution does not lead to a smooth change in optimal prices.

Consider consumer welfare, which is now the integral over c of the welfare earned by consumers with each cost of resale c. Let $CW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Consumers with $c \geq c^*$ earn $CW_N(P_i, P_B)$ and all others earn $CW_N(\frac{P_B}{2} + c, P_B)$, making overall consumer welfare with distribution of costs $G(c)$

$$CW(G, P) = (1 - G(c^*))CW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} CW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \tag{8}$$

If the change in optimal prices were continuous, the change in consumer welfare for consumers with each cost c would depend only on inframarginal consumers, as in Theorem 3. Consumers with

[10] With non-monotonicity, it is possible to have some $\tilde{G}(c) \geq_{FOSD} G(c)$ at which P is less profitable, for instance if profit from resellers is decreasing in c on some interval and $\tilde{G}(c)$ shifts mass upwards only on that interval.

$c < c^*$ would only be affected by the change in P_B when the monopolist changes prices.

If prices jump when G shifts to G', however, some consumers will switch purchases and earn surplus. The overall change in consumer welfare includes both the inframarginal consumers and the switchers, adding the changes in surplus for each cost c weighted by the change in the number of consumers with that cost across the distributions. As before, the change is ambiguous: Table 1 contains examples in which consumer welfare moves in each direction as the distribution of costs shifts upwards.

The story is similar for total welfare. Let $TW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Total welfare $TW(G, P)$ for the distribution G is

$$TW(G,P) = (1 - G(c^*))TW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} TW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \qquad (9)$$

Without a continuous change in prices when G shifts to G', the change in total welfare cannot be distilled to the values of marginal buyers. But the main insight of Theorem 4 applies because, for consumers with cost c, the change in total welfare is driven by consumers who change their purchase decisions at the new prices. The aggregate change is the sum of those changes, weighted by the change in the number of consumers with type c between the distributions. The change remains ambiguous, as demonstrated by the comparisons in Table 1.

4.4 Examples

To illustrate equilibrium and the comparative statics, I simulate the market for various distributions of costs of resale when values are distributed uniformly. The examples are paired, with resale becoming more costly from the first to the second distribution. The second distribution always dominates the first in the sense of first-order stochastic dominance; the changes also satisfy the stronger criterion in Theorem 7.

The examples use three types of distributions of costs: normal distributions, uniform distributions on $[0, .23]$ (ending at approximately the optimal bundle discount without resale), and a split uniform distribution. The split uniform distribution $SplitUnif(x, y)$ spreads probability y evenly on the interval $[0, x]$ and probability $1 - y$ evenly on $[x, .23]$. The split uniform is used in the third, fifth, and sixth rows. For example, in the third row, it spreads .3 of the probability between 0 and .02 and the rest between .02 and .23.

In each shift, profit increases after resale becomes more difficult. In the shift from row 3 to

	P_i^*	P_B^*	c^*	π	π_N	CW	TW
$N(.1, .05)$	0.5476	0.9431	0.0761	0.5210	0.5297	0.3354	0.8564
$N(.14, .05)$	0.5615	0.9271	0.0980	0.5282	0.5354	0.3072	0.8354
$\mathit{SplitUnif}(.02, .3)$	0.5696	0.9481	0.0956	0.5156	0.5345	0.4129	0.9285
$\mathit{Unif}(0, .23)$	0.5639	0.9374	0.0952	0.5206	0.5346	0.3675	0.8881
$\mathit{SplitUnif}(.1, .5)$	0.5569	0.9433	0.0853	0.5187	0.5321	0.3723	0.8910
$\mathit{SplitUnif}(.15, .5)$	0.5785	0.9258	0.1156	0.5245	0.5390	0.3569	0.8814
$N(.08, .001)$	0.5459	0.9372	0.0773	0.5300	0.5300	0.2504	0.7804
$N(.2, .001)$	0.6347	0.8764	0.1965	0.5483	0.5483	0.2525	0.8008

Table 1: Examples of equilibrium with a distribution of costs of resale on $[0, .23]$ when values are uniformly distributed. The distribution SplitUnif has mass uniformly distributed on either side of a tilt point. The tilt point is the first argument and the amount of mass to the left is the second.

row 4, the optimal bundle discount falls. Consumer and total welfare decline in the first three comparisons but increase in the last.

5 Conclusion

The goal of this paper has been to determine the effects of resale on bundling by analyzing a model of monopoly bundling with costly resale. I showed that resale adds a new dimension to the monopolist's problem, reduces profit, and has an ambiguous effect on consumer and total welfare.

The research adds to the bundling literature because resale has a significant effect on the profitability of bundling. It also matters for policy regarding resale, demonstrating that consumers and society may not benefit from resale when the seller bundles.

An additional contribution is the model's use of market clearing to determine resale prices. In the model, resale prices emerge in the equilibrium of a subgame where atomistic consumers interact in a resale market. The approach matches observed markets, where many small sellers with varied reservation prices trade, but has not been widely adopted in other studies of price discrimination with resale.

Proofs

3.1 *Proof of Lemma 1*

Proof. Assume that P^{s*} is an equilibrium price vector and that good 2 is resold in the secondary market. Buyers must prefer purchasing good 2 in the resale market at price $P_2^{s*} + c$ to purchasing the bundle and reselling good 1, which costs $P_B - (P_1^{s*} - c)$. Hence $P_1^{s*} + P_2^{s*} \leq P_B$.

There are three ways for resellers to provide good 2, and each requires $P_B \le P_1^{s*} + P_2^{s*}$. First, consumers could purchase the bundle to keep good 1 and resell good 2, paying $P_B - (P_2^{s*} - c)$ for good 1. To do so, this must be cheaper than the total resale price $P_1^{s*}+c$, implying $P_B \le P_1^{s*}+P_2^{s*}$. Second, they could purchase the bundle to resell both goods, but would only do so if it is profitable, $P_B + 2c \le P_1^{s*} + P_2^{s*}$. Third, they could purchase only good 2 to resell, earning $P_2^{s*} - P_2 - c$. This requires $P_2^{s*} = P_2$ and $c = 0$, and I show that $P_1^{s*} + P_2^{s*} < P_B$ is impossible in this case. It cannot be that $P_1^{s*} < P_1$ because some consumers would demand good 1 and none would resell it: resellers would receive $P_1^{s*} - P_1 < 0$ if they bought 1 to resell, $P_1^{s*} + P_2^{s*} - P_B < 0$ if they bought the bundle to resell both goods, and $v_2 - P_B + P_1^{s*} < v_2 - P_2^{s*}$ if they bought the bundle to resell 2. Then $P_1 \le P_1^{s*}$, implying $P_1 + P_2 \le P_1^{s*} + P_2^{s*} < P_B$, which contradicts $P_B \le P_1 + P_2$. □

Proof of Lemma 2

Proof. Equilibrium requires the masses of consumers demanding each good in the resale market to be equal. When the monopolist only offers the bundle, the masses $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ of consumers who only want good 1 or good 2 are

$$\mu_1(P_B, P_1^s) = \int_{P_1^s+c}^{1} \int_{0}^{P_B - P_1^s - c} f(v_1, v_2)\, dv_1\, dv_2$$
$$\mu_2(P_B, P_1^s) = \int_{0}^{P_1^s - c} \int_{P_B - P_1^s + c}^{1} f(v_1, v_2)\, dv_1\, dv_2.$$

Define $\delta(P_1^s) = \mu_1(P_B, P_1^s) - \mu_2(P_B, P_1^s)$ on $[0, P_B]$. There is some $\hat{P}_1^{s*}$ satisfying $\delta(\hat{P}_1^{s*}) = 0$ because $\delta(\cdot)$ is a continuous (since F is atomless) decreasing function satisfying $\delta(0) \ge 0$ and $\delta(P_B) \le 0$.

To be unique, $\delta(\cdot)$ can only cross zero at a single point and so $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ can never simultaneously be zero. Because F has a strictly positive atomless density, this only requires that the regions of integration not simultaneously be empty. For μ_1, this requires $P_1^s + c < 1$ and $0 < P_B - P_1^s - c$, or $P_1^s < \min\{1 - c, P_B - c\}$. For μ_2, it requires $0 < P_1^s - c$ and $P_B - P_1^s + c < 1$, or $P_1^s > \max\{P_B - 1 + c, c\}$. Such a P_1^s exists when $\max\{c, P_B - 1 + c\} < \min\{1 - c, P_B - c\}$. The condition becomes $P_B - 1 + c < 1 - c$ when $P_B \ge 1$ and $c < P_B - c$ when $P_B < 1$. □

Proof of Theorem 1

Proof. ($\Rightarrow$) Without loss of generality, suppose that good 1 is resold. The reseller pays $P_B + c - P_1^{s*}$ for good 2 and the buyer pays $P_1^{s*} + c$ for good 1. Equilibrium requires $P_B + c - P_1^{s*} \leq P_2$ and $P_1^{s*} + c \leq P_1$. Assumption 2 implies that one inequality must be strict and so $P_B + 2c < P_1 + P_2$.

($\Leftarrow$) Suppose $P_1 + P_2 > P_B + 2c$ and let $(\hat{P}_1^{s*}, \hat{P}_2^{s*})$ be the vector of pure-bundling resale prices corresponding to P_B. There are two cases. First, $\hat{P}_1^{s*} + c < P_1$ and $\hat{P}_2^{s*} + c < P_2$. All consumers who want one good strictly prefer the secondary market and $\hat{P}^{s*}$ is the unique vector of resale prices. Second, suppose $\hat{P}_1^{s*} + c \geq P_1$ and $\hat{P}_2^{s*} + c < P_2$. Buyers of good 2 strictly prefer the resale market, so P_1^s must fall (and P_2^s must rise) until either $P_1 = P_1^s + c$ or $P_2 = P_2^s + c$. Since $P_1 + P_2 > P_B + 2c$, it must be that $\hat{P}_1^{s*} + c - P_1 < P_2 - \hat{P}_2^{s*} - c$, so prices adjust until $P_1^s + c = P_1$ and $P_2^s + c < P_2$. The resulting vector $P^{s*} = (P_1 - c, P_B - P_1 + c)$ is an equilibrium: fewer consumers only want good 2 than good 1 (since $P_1^{s*} < \hat{P}_1^{s*}$), and consumers who want good 1 are indifferent between the two markets, making some willing to resell and clear the resale market. The equilibrium is unique. At any higher P_1^{s*}, there would be no supply of good 1 in the resale market. At any lower P_1^{s*}, there would be excess demand for good 1. □

3.2 *Proof of Theorem 2*

Proof. I start by showing that the optimal prices satisfy $P_1 + P_2 \leq P_B + 2c$. Suppose they do not, so Theorem 1 implies there is resale in equilibrium at price vector P^{s*}. The monopolist could earn weakly more by setting $P_i = P_i^{s*} + c$ for $i = 1, 2$ if all purchases were made in the primary market: consumers would receive the same goods as before, but the monopolist would earn an additional $2c$ on all transactions that formerly involved resale. When $c > 0$, the increase in profit would be strict. Next I show profit still weakly increases if there is resale when $P_i = P_i^{s*} + c$. The profit function without resale is continuous, so the monopolist can slightly lower P_1 and P_2 below $P_i^{s*} + c$ and earn profit arbitrarily close to its level if no consumers resold and $P_i = P_i^{s*} + c$.

Theorem 1 completes the argument by showing that no consumers resell when the monopolist's prices satisfy $P_1 + P_2 \leq P_B + 2c$, but the result relies on Assumption 2. When $c > 0$, Assumption 2 is not needed. Suppose consumers resold in equilibrium when the optimal prices without resale such that $P_1 + P_2 \leq P_B + 2c$ satisfy $P_1 + P_2 = P_B + 2c$. The monopolist would maximize profit by setting the highest prices such that $P_i < P_i^{s*} + c$. No such prices exist, so the only equilibrium involves no resale. When $c = 0$, I rely on Assumption 2 to prevent resale, but in that case resale does not affect profit or welfare. □

3.3 *Proof of Corollary 1*

Proof. The monopolist's optimal prices for cost c are feasible at $c' > c$. $\square$

Proof of Theorem 3

Proof. The regions are defined as

$$\mu_1(c) = \int_{P_1^*(c)}^{1} \int_{0}^{P_B^*(c)-P_1^*(c)} f(v_1, v_2)\, dv_2\, dv_1$$

$$\mu_2(c) = \int_{P_2^*(c)}^{1} \int_{0}^{P_B^*(c)-P_2^*(c)} f(v_1, v_2)\, dv_1\, dv_2$$

$$\mu_B(c) = \int_{P_B^*(c)-P_2^*(c)}^{1} \int_{P_B^*(c)-P_1^*(c)}^{1} f(v_1, v_2)\, dv_2\, dv_1 - \int_{P_B^*(c)-P_2^*(c)}^{P_1^*(c)} \int_{P_B^*(c)-P_1^*(c)}^{P_B^*(c)-v_1} f(v_1, v_2)\, dv_2\, dv_1.$$

Consumer welfare is

$$\begin{aligned} CW(c) =& \Big\{ \int_{P_1^*(c)}^{1} \int_{0}^{P_B^*(c)-P_1^*(c)} (v_1 - P_1^*(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\} + \\ & \Big\{ \int_{P_2^*(c)}^{1} \int_{0}^{P_B^*(c)-P_2^*(c)} (v_2 - P_2^*(c)) f(v_1, v_2)\, dv_1\, dv_2 \Big\} + \\ & \Big\{ \int_{P_B^*(c)-P_2^*(c)}^{1} \int_{P_B^*(c)-P_1^*(c)}^{1} (v_1 + v_2 - P_B^*(c)) f(v_1, v_2)\, dv_2\, dv_1 - \\ & \int_{P_B^*(c)-P_2^*(c)}^{P_1^*(c)} \int_{P_B^*(c)-P_1^*(c)}^{P_B^*(c)-v_1} (v_1 + v_2 - P_B^*(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\}. \end{aligned} \tag{10}$$

Taking the derivative with respect to c and comparing it to zero yields the result. $\square$

Proof of Theorem 4

Proof. Total welfare is

$$\begin{aligned} TW(c) =& \Big\{ \int_{P_1^*(c)}^{1} \int_{0}^{P_B^*(c)-P_1^*(c)} v_1 f(v_1, v_2)\, dv_2\, dv_1 \Big\} + \\ & \Big\{ \int_{P_2^*(c)}^{1} \int_{0}^{P_B^*(c)-P_2^*(c)} v_2 f(v_1, v_2)\, dv_1\, dv_2 \Big\} + \\ & \Big\{ \int_{P_B^*(c)-P_2^*(c)}^{1} \int_{P_B^*(c)-P_1^*(c)}^{1} (v_1 + v_2) f(v_1, v_2)\, dv_2\, dv_1 - \\ & \int_{P_B^*(c)-P_2^*(c)}^{P_1^*(c)} \int_{P_B^*(c)-P_1^*(c)}^{P_B^*(c)-v_1} (v_1 + v_2) f(v_1, v_2)\, dv_2\, dv_1 \Big\}. \end{aligned} \tag{11}$$

And so the change in total welfare with respect to c is

$$
\begin{aligned}
\frac{\partial TW(c)}{\partial c} = & -\frac{\partial P_1^*(c)}{\partial c}\int_0^{P_B^*(c)-P_1^*(c)} P_1^*(c) f(P_1^*(c), v_2)\, dv_2 + \\
& \left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_1^*(c)}{\partial c}\right)\int_{P_1^*(c)}^1 v_1 f(v_1, P_B^*(c) - P_1^*(c))\, dv_1 + \\
& -\frac{\partial P_2^*(c)}{\partial c}\int_0^{P_B^*(c)-P_2^*(c)} P_2^*(c) f(v_1, P_2^*(c))\, dv_1 + \\
& \left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_2^*(c)}{\partial c}\right)\int_{P_2^*(c)}^1 v_2 f(P_B^*(c) - P_2^*(c), v_2)\, dv_2 + \\
& -\left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_1^*(c)}{\partial c}\right)\int_{P_1^*(c)}^1 (v_1 + P_B^*(c) - P_1^*(c))\, f(v_1, P_B^*(c) - P_1^*(c))\, dv_1 + \\
& -\left(\frac{\partial P_B^*(c)}{\partial c} - \frac{\partial P_2^*(c)}{\partial c}\right)\int_{P_2^*(c)}^1 (v_2 + P_B^*(c) - P_2^*(c))\, f(P_B^*(c) - P_2^*(c), v_2)\, dv_2 + \\
& -\frac{\partial P_B^*(c)}{\partial c}\int_{P_B^*(c)-P_2^*(c)}^{P_1^*(c)} P_B^*(c) f(v_1, P_B^*(c) - v_1)\, dv_1 .
\end{aligned}
\tag{12}
$$

□

4.1 *Proof of Theorem 5*

Proof. Lemma 1 applies with heterogeneity in c because its proof applies for each type c. By the symmetry of $F(\cdot)$, $P_1 = P_2$ and so equilibrium resale prices are $P^{s*} = (\frac{P_i}{2}, \frac{P_i}{2})$ in any equilibrium with resale transactions.

For prices $P = (P_i, P_B)$, all types with $c \geq P_i - \frac{1}{2}P_B$ find resale too costly because $P_B + 2c \geq 2P_i$. The monopolist therefore earns $\pi_N(P_i, P_B)$ from a fraction $1 - G(P_i - \frac{1}{2}P_B)$ of consumers.

All other consumers are willing to share the bundle through resale. Surplus maximization with prices $(\frac{1}{2}P_B + c_k, P_B)$ implies that for types c_k satisfying $c_k < P_i - \frac{1}{2}P_B$, a fraction $\mu_R(P_B, c_k)$ acquire good 1 through resale and a fraction $\mu_B(P_B, c)$ purchase the bundle. For type c_k, the monopolist sells $\mu_R(P_B, c_k)$ bundles to be shared and $\mu_B(P_B, c_k)$ to consumers who do not resell. Integrating over all such types, it earns $\pi_R(P_B, G, P_i - \frac{1}{2}P_B)$ from types who resell. □

4.2 *Proof of Theorem 6*

Proof. Assume that the monopolist prevents consumers with cost $\underline{c}$ from reselling. By assumption, the monopolist's optimal prices under the constraint have $\dot{P}_i = \frac{1}{2}\dot{P}_B + \underline{c}$. I show that the monopolist can increase profit by increasing P_i when $g(\underline{c})$ is small relative to the change in profit. The derivative of profit with respect to P_i at $\underline{c}$ is

$$
\begin{aligned}
\frac{\partial}{\partial P_i}\pi(\dot{P}) &= g(\underline{c})\left[\dot{P}_B\left(\mu_B(\dot{P}_B,\underline{c})+\mu_R(\dot{P}_B,\underline{c})\right)-\pi_N(\dot{P}_i,\dot{P}_B)\right]+(1-G(\underline{c}))\frac{\partial\pi_N(\dot{P}_i,\dot{P}_B)}{\partial P_i}\\
&= g(\underline{c})\left(-2\underline{c}\mu_R(\dot{P}_B,\underline{c})\right)+(1-G(\underline{c}))\frac{\partial\pi_N(\dot{P}_i,\dot{P}_B)}{\partial P_i}\\
&= -2\underline{c}g(\underline{c})\mu_R(\dot{P}_B,\underline{c})+\frac{\partial\pi_N(\dot{P}_i,\dot{P}_B)}{\partial P_i}.
\end{aligned}
$$

The second step relies on the fact that $\dot{P}_B(\mu_B(\dot{P}_B,\underline{c})+\mu_R(\dot{P}_B,\underline{c}))-\pi_N(\dot{P}_i,\dot{P}_B)=-2\underline{c}\mu_R(\dot{P}_B,\underline{c})$, which is true because the seller earns $2\underline{c}$ on each formerly resold transaction when type $\underline{c}$ moves to the primary market. The conclusion follows by setting the derivative greater than or equal to zero. □

4.3 *Proof of Theorem 7*

Lemma 3. At the monopolist's optimal prices P^*, for all $c \leq c^*$,

$$\pi_N(P^*) \geq P_B^*\left(\mu_B(P_B^*,c)+\mu_R(P_B^*,c)\right).$$

Proof. Let $\tilde{c}=\sup_{c\in[\underline{c},c^*]}\{P_B^*\left(\mu_B(P_B^*,c)+\mu_R(P_B^*,c)\right)\}$. Assume for contradiction that $\pi_N(P^*) < P_B^*\left(\mu_B(P_B^*,\tilde{c})+\mu_R(P_B^*,\tilde{c})\right)$. If the monopolist deviated to set $P=(\frac{1}{2}P_B^*+\tilde{c},P_B^*)$, then it would strictly increase profit from consumers with $c \geq \tilde{c}$ and earn the same amount from consumers with $c < \tilde{c}$. □

Proof. Let P be the optimal prices when costs are distributed under $G(c)$. I show that the monopolist earns higher profit at P under $\tilde{G}$. Let $\Delta G(c)=G(c)-\tilde{G}(c)$, which satisfies $\Delta G(c)\geq 0$ for $c\leq c^*$. Consider profit from resellers at G and use the fact that $\pi_N(P_i,P_B)\geq P_B\left(\mu_B(P_B,c)+\mu_R(P_B,c)\right)$ for $c\leq c^*$ by Lemma 3.

$$
\begin{aligned}
&\int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B,c)+\mu_R(P_B,c)\right)dG(c)\\
&=\int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B,c)+\mu_R(P_B,c)\right)d\Delta G(c)+\int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B,c)+\mu_R(P_B,c)\right)d\tilde{G}(c)
\end{aligned}
$$

$$
\begin{aligned}
&\le \int_{\underline{c}}^{c^*} \pi_N(P_i, P_B)\, d\Delta G(c) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= (G(c^*) - \tilde{G}(c^*))\pi_N(P_i, P_B) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c).
\end{aligned}
$$

Let $\pi^G(P)$ give profit for prices P under the distribution of costs G. Substituting into the profit function at G gives

$$
\begin{aligned}
\pi^G(P) &= (1 - G(c^*))\pi_N(P_i, P_B) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) dG(c) \\
&\le (1 - G(c^*))\pi_N(P_i, P_B) + (G(c^*) - \tilde{G}(c^*))\pi_N(P_i, P_B) + \\
&\quad \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= \pi^{\tilde{G}}(P).
\end{aligned}
$$

$\square$

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