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DISCUSSION PAPER

Upward Pricing Pressure as a
Predictor of Merger Price Effects

by

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Abstract

We use Monte Carlo experiments to evaluate whether “upward pricing pressure” (UPP) accurately predicts the price effects of mergers, motivated by the observation that UPP is a restricted form of the first order approximation derived in Jaffe and Weyl (2013). Results indicate that UPP is quite accurate with standard log-concave demand systems, but understates price effects if demand exhibits greater convexity. Prediction error does not systematically exceed that of misspecified simulation models, nor is it much greater than that of correctly-specified models simulated with imprecise demand elasticities. The results also support that both UPP and the HHI change provide accurate screens for anticompetitive mergers.

1 Introduction

In a number of recent antitrust enforcement actions, the U.S. Department of Justice (DOJ) and the Federal Trade Commission (FTC) have alleged that mergers between producers of competing differentiated products would adversely affect unilateral pricing incentives.¹ This follows a decades-long trend that has both spurred on and been informed by academic research on how mergers affect prices (e.g., Davidson and Deneckere (1985); Berry and Pakes (1993); Hausman, Leonard and Zona (1994); Werden and Froeb (1994); Nevo (2000); Jaffe and Weyl (2013); Carlton and Keating (2015)). Continuing this evolution, the DOJ and the FTC updated their Horizontal Merger Guidelines in 2010, in part motivated by a desire to better align the document with economic theory and antitrust practice as they relate to markets with differentiated products (Shapiro (2010)).

One point of emphasis in the 2010 Horizontal Merger Guidelines is that mergers between competitors create opportunity costs, which in turn place upward pricing pressure (or “UPP”) on the combining firms. This principle is easily derived from basic economic models, and the magnitude of the opportunity costs often can be quantified with information from only the merging parties. This combination of theoretical and practical simplicity make UPP a useful diagnostic tool. Referring to UPP as the value of diverted sales, the Guidelines state that “[t]he Agencies rely more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated products.”² The FTC has employed UPP calculations to support arguments in court (FTC v. Sysco Corporation, et al.) and to justify enforcement decisions (Family Dollar/Dollar Tree).³

Although UPP has a clear relationship to firms’ pricing incentives, antitrust economists have been wary about using it as a prediction of price effects. UPP does not incorporate the manner in which the pass-through of costs to prices depends on the higher-order properties of the underlying demand system. Nor does it account for the possibility that non-merging competitors change their prices as the market shifts to a new equilibrium. Two of the principal authors of the 2010 Horizontal Merger Guidelines, Joseph Farrell and Carl Shapiro, emphasize in their academic work that “UPP does not predict post-merger prices, but only predicts the sign of changes in price” (Farrell and Shapiro (2010)).⁴ Furthermore, Jaffe and

¹E.g., U.S. v. H& R Block Inc., et al.; U.S. v. AT&T Inc., et al.; U.S. v. Bazaarvoice, Inc.; FTC v. Sysco Corporation, et al.; U.S. v. AB Electrolux, et al.

²See Section 6.1.

³The FTC Press Release is available online: https://www.ftc.gov/system/files/documents/public_statements/681781/150713dollartree-jdwstmt.pdf.

⁴See also Shapiro (2010), who writes that:

The value of diverted sales, taken alone, does not purport to quantify the magnitude of any

Weyl (2013) show that UPP must be scaled by an appropriate measure of pass-through to provide a first order approximation to price effects. Obtaining estimates of the requisite pass-through information can be difficult even in advantageous empirical settings (e.g., Miller, Osborne and Sheu (2015)).

In this paper, we revisit whether UPP can be applied to predict the magnitude of price effects. We begin with a theoretical discussion in which we develop that, in some standard settings, using an identity matrix to proxy for the relevant pass-through matrix may introduce only limited misspecification error. By implication, UPP itself may provide a reasonable approximation to the true price effects. We then explore this possibility using a large-scale Monte Carlo experiment that simulates mergers in markets with differentiated products. Results indicate that UPP is quite accurate with standard log-concave demand systems, but understates price effects if demand exhibits greater convexity. The prediction error that arises with UPP does not systematically exceed the prediction error that occurs due to functional form misspecification in simulation models, nor is it much greater than the prediction error that arises in correctly specified simulation models that rely on imprecise econometric estimates of their structural parameters.

The Monte Carlo experiments follow the data generating process developed in Miller, Remer, Ryan and Sheu (2016). We repeatedly draw randomized market shares and a single price-cost margin, and use these data to calibrate the parameters of a logit demand system. We then calibrate the less restrictive linear, almost ideal, and log-linear demand systems to match the elasticities of the logit model, sometimes incorporating a degree of measurement error in the elasticities. The analysis thus features two log-concave demand systems (linear and logit demand) alongside two demand systems that exhibit greater convexity (e.g., almost ideal and log-linear demand). These four demand systems are commonly employed in antitrust analyses (Werden, Froeb and Scheffman (2004); Werden and Froeb (2008)), and also have been used in academic studies that examine the effect of demand curvature on the precision of counterfactual simulations (e.g., Crooke, Froeb, Tschantz and Werden (1999); Huang, Rojas and Bass (2008)).⁵

The analysis is subject to a number of caveats and limitations. The most serious pertain

post-merger price increase.... The value of diverted sales is a measure of the extra (opportunity) cost the merged firm bears in selling units of Product 1. Higher costs give the merged firm an incentive to raise the price of Product 1. But further analysis is needed to determine how that cost increase translated into a price increase.

⁵Our research also is related to Cheung (2013), who compares the performance of predictions arising from a structurally estimated merger simulation model with those from UPP in the context of the airline industry.

to external validity. First, we focus exclusively on pricing in differentiated products markets, although the UPP calculation itself can be generalized to other settings (e.g., Jaffe and Weyl (2013)). Second, we impose the Nash-Bertrand equilibrium concept throughout the data generating process in order to focus the analysis on the “unilateral effects” of mergers. UPP is unlikely to perform as well for mergers that create coordinated effects. Lastly, we make a number specific assumptions about the demand systems and marginal cost functions that are necessitated by the Monte Carlo approach. We do not seek to provide the most general results available. Instead, we seek to establish certain relationships that advance the dialog on UPP and motivate future research.

The remainder of the paper proceeds as follows. Section 2 details the theoretical connection between UPP, first order approximation, and the price effects of mergers. Section 3 describes the Monte Carlo experimental design and provides summary statistics. Section 4 presents the results. There we plot the raw data, compare the prediction error that arises from UPP with that from merger simulation, and also discuss the use of UPP and HHI-based measures as early-stage screens. Section 5 concludes with a summary and a discussion of the appropriate scope of application for UPP.

2 Theoretical Framework

2.1 Merger price effects and UPP

We examine the connection between different methods of merger price prediction within the context of Nash-Bertrand price competition between multi-product firms. Assume that each firm, i , produces a subset of products available to consumers, faces a twice-differentiable demand function, and maximizes the following profit function:

$$\pi_i = P_i^T Q_i(P) - C_i(Q_i(P))$$

where P_i is a vector of firm i 's prices, Q_i is a vector of firm i 's unit sales, P is a vector containing the prices of every product, and C_i is the cost function. The superscript T denotes the vector/matrix transpose. Profit-maximizing prices are characterized by first-order conditions:

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)}{\partial P_i} \right]^T Q_i(P) - (P_i - MC_i) = 0, \quad (1)$$

where $MC_i = \partial C_i / \partial Q_i$ is a vector of firm i 's marginal costs. Now consider a merger between two firms j and k that, for simplicity, does not affect the cost functions. The post-merger first-order conditions are given by

$$h_i(P) \equiv f_i(P) + g_i(P) = 0 \quad \forall i \in I \quad (2)$$

where

$$g_j(P) = - \underbrace{\left(\frac{\partial Q_j(P)^T}{\partial P_j} \right)^{-1} \left(\frac{\partial Q_k(P)^T}{\partial P_j} \right)}_{\text{Matrix of Diversion from } j \text{ to } k} \underbrace{(P_k - MC_k)}_{\text{Markup of } k} \quad (3)$$

and $g_k(P)$ is defined analogously, while $g_i(P) = 0$ for all $i \neq j, k$. Prices that satisfy the post-merger first order conditions can be computed given sufficient information on the demand system and marginal costs. Directly computing post-merger prices using this information is referred to as merger simulation and has been a main focus of research spanning more than two decades; numerous literature reviews summarize the topic (e.g., Werden and Froeb (2008); Budzinski and Ruhmer (2010); Baker and Reitman (2013)).

The merger can be interpreted as creating an opportunity cost within the joined firm. Aggressive pricing from one merging partner creates forgone profits that otherwise would be earned by the other. The magnitude of these opportunity costs – given by the $g(P)$ function – depends multiplicatively on the customer diversion rates between the merging firms and their markups. Reinforcing this interpretation is the fact that both marginal costs and $g_i(P)$ are additively separable in the post-merger first order conditions. Farrell and Shapiro (2010) refer to these opportunity costs as the UPP due to the merger. They propose UPP as an initial screen in merger investigations, on the basis that higher marginal costs tend to be associated with higher prices.

2.2 UPP as a price predictor

The equilibrium post-merger price effects in this model depend upon how pricing pressure is passed through to consumers. In merger simulation models, pass-through behavior is determined by the demand system, and there is existing research that explores how functional form restrictions on demand affect the accuracy of simulation (e.g., Crooke, Froeb, Tschantz and Werden (1999); Miller, Remer, Ryan and Sheu (2016)). A somewhat more general solution to calculating merger price effects is provided by Jaffe and Weyl (2013). If pass-through is observed (or can be estimated from data) then a first order approximation to the

price change can be obtained as

$$\Delta P = - \left(\frac{\partial h(P)}{\partial P} \right)^{-1} \Bigg|_{P=P^0} g(P^0) \quad (4)$$

where P^0 is the vector of pre-merger prices. The first order approximation equals UPP pre-multiplied by the opposite inverse Jacobian of $h(P)$, which Jaffe and Weyl refer to as the merger pass-through matrix. By inspection, merger pass-through depends on the first and second derivatives of demand, while omitting higher order terms. Miller, Remer, Ryan and Sheu (2016) provide Monte Carlo evidence that the first order approximation is an accurate predictor of true price effects.

With this foundation in place, it follows that UPP itself may provide a useful prediction of the price effect, insofar as the identity matrix can accurately proxy for the merger pass-through matrix. We provide a simple numerical example to fix ideas. Consider three firms, each of which has a margin of 0.50 and a 30% market share (the outside good has a 10% share). Consumer behavior is given by the logit demand system. With a merger between the first two firms, equation (4) becomes

$$\begin{bmatrix} 0.204 \\ 0.204 \\ 0.052 \end{bmatrix} = \begin{bmatrix} 0.771 & 0.180 & 0.297 \\ 0.180 & 0.771 & 0.297 \\ 0.122 & 0.122 & 0.776 \end{bmatrix} \begin{bmatrix} .214 \\ .214 \\ 0 \end{bmatrix} \quad (5)$$

Here the value of UPP (0.214) nearly equals the first order approximation (0.204) for the merging firms. This happens because the diagonal elements of the merger pass-through matrix are somewhat below one, while the off-diagonal elements are positive. Thus, using an identity matrix to proxy merger pass-through overstates some effects and understates others; the balance is a prediction close to the first order approximation. Further, again in this example, it is worth noting that both UPP and the first order approximation are close to the true price effects (0.190 for the merging firms).

This idea extends beyond the simple example provided. Countervailing biases arise provided that (i) the diagonal elements of the merger pass-through matrix are below unity, and (ii) prices are strategic complements so the off-diagonal elements are positive. In such settings, UPP may provide a reasonable approximation to the true price effects. The two demand systems we consider that exhibit log-concavity (linear and logit) always satisfy both conditions.⁶ If instead demand exhibits greater convexity, then the diagonal elements of the

⁶Log-concavity is sufficient to ensure incomplete *cost* pass-through (e.g., Bulow and Pfleiderer (1983)),

merger pass-through matrix can exceed one, and UPP should understate price effects. As we develop below, this typically is the case with almost ideal and log-linear demand.

The limited informational requirements of UPP could make it a valuable tool for antitrust authorities. UPP can be calculated with diversion and markups for only the merging parties, and such information often becomes available during the course of merger investigations. By contrast, merger simulation models typically require a full set of demand elasticities, encompassing consumer responses to the prices of all firms in the model. FOA requires these demand elasticities along with pass-through. To the extent that UPP provides accurate price predictions, the importance of obtaining elasticities and pass-through would be diminished. This motivates the Monte Carlo experiments developed below.

2.3 Market shares and HHI

The economic theory outlined above demonstrates that competition between merging firms, as characterized by diversion ratios and markups, is directly related to unilateral price effects in markets with differentiated products. Except in a special case, no such direct theoretical connection exists between unilateral price effects and market concentration. Nevertheless, the 2010 Guidelines maintain that the HHI is useful in informing competitive effects, at least in the broad sense of “identify[ing] some mergers unlikely to raise competitive concerns and some others for which it is particularly important to examine whether other competitive factors confirm, reinforce, or counteract the potentially harmful effects of increased concentration.”⁷ We calculate HHI as the sum of squared market shares:

$$HHI = \sum_i s_i^2 \tag{6}$$

where s_i is between 0 and 100 and represents the market share of firm i . The change in HHI due to a merger between firms j and k requires only the merging parties’ market shares:

$$\Delta HHI = 2s_j s_k \tag{7}$$

The Monte Carlo experiments allow us to evaluate the accuracy of the HHI statistics as a screening device. Because HHI and ΔHHI are merger-specific statistics (unlike UPP which is firm-specific), we compare them to the average price change of the merging firms.

The direct theoretical connection between unilateral price effects and market concen-

but cost pass-through differs somewhat from merger pass-through. See Jaffe and Weyl (2013) for details.

⁷See the 2010 Horizontal Merger Guidelines, Section 5.3.

tration arises if consumer diversion is proportional to market share. Then diversion from product j to product k equals $s_k/(1-s_j)$, and can be approximated by $s_k(1+s_j)$ for small s_j . Diversion from k to j is analogous, meaning that the sum of the approximate diversion ratios is $s_j + s_k + 2s_j s_k$ or, equivalently, $s_j + s_k + \Delta\text{HHI}$.⁸ This provides a theoretical foundation for the many empirical studies that relate merger price effects to the predicted change in HHI (e.g., Dafny, Duggan and Ramanarayanan (2012); Ashenfelter, Hosken and Weinberg (2015)). Even in this special case, the correlation between ΔHHI and the price effects may be weak because pricing pressure is determined through the interaction of diversion and markups. Lastly, we note that the *level* of HHI often is influenced greatly by the shares of non-merging firms. While the strategic reactions of rivals affect post-merger equilibrium, typically they are of secondary importance.⁹

3 Design of the Monte Carlo Experiments

3.1 Data generation

We generate data that are consistent with the theoretical model outlined in the previous section. Each draw of simulated data is independent and characterizes the pre-merger equilibrium conditions of a single market. Together, the data cover a wide range of competitive conditions. The markets contain six firms that produce differentiated products at constant marginal cost. Firms compete in prices and equilibrium is Nash-Bertrand. All pre-merger prices are normalized to one, which results in price effects that are identical in levels and percentages. The specific data generating process is as follows:

1. Randomly draw (i) market shares for six firms and an outside good, and (ii) the first firm's margin based on a uniform distribution bounded between 0.20 and 0.80.
2. Calibrate the parameters of a logit demand system based on the margin and market shares, and calculate the demand elasticities that arise in the pre-merger equilibrium. This entails selecting demand parameters that rationalize the random data. The parameters are exactly identified given market shares, prices, and a single margin.
3. Calibrate linear, almost ideal, and log-linear demand systems based on the logit demand elasticities. The parameters of these systems are exactly identified given market

⁸We first encountered these mathematics in Shapiro (2010).

⁹The market shares of competitors is more relevant for the likelihood and repercussions of post-merger coordinated effects. We confine the analysis to unilateral effects by maintaining the Nash-Bertrand equilibrium concept throughout the paper.

shares, prices, and the logit elasticities.¹⁰

4. Calculate UPP for a merger of the first and second firms in each market, based on the elasticities and margins for each draw of data. UPP is invariant to the demand system. Simulate the effect of the merger under each of the demand systems.
5. Repeat steps (1) - (4) until 4,500 draws of data are obtained.

The algorithm generates 18,000 mergers to be examined, each defined by a draw of data and a demand system. See Appendix A for mathematical details on the calibration process.

The data generating process allows us to assess the accuracy of UPP both in absolute terms and relative to merger simulation conducted with a functional form misspecification or with imprecise demand elasticities. To develop the first comparison, we note that the elasticities of each demand system are identical in the pre-merger equilibrium, for a given draw of data, so that differences in price effects arise solely due to functional form. Suppose the true demand system is logit. The prediction error of UPP can be compared against simulation results using almost ideal, linear, and log-linear demand.

For the second comparison, we incorporate imprecision into the observed demand elasticities, and evaluate how the predictive accuracy of UPP and simulation degrade. Specifically, we add a uniformly distributed error to each product’s own-price elasticity of demand. To do so in a manner that preserves the property that own-price elasticities are less than negative one, we define the observed own-price elasticities to be

$$\tilde{\epsilon}_{kk} = \epsilon_{kk} + \nu \quad \text{where} \quad \nu \sim U(-t(\epsilon_{kk} + 1), t(\epsilon_{kk} + 1)) \quad (8)$$

The support of the error is element-specific and depends on $t \in [0, 1]$. We examine three levels of error: $t = (0.2, 0.5, 0.8)$. We then scale each product’s cross-price elasticity according to the percent error of that products’ own-price elasticity, i.e. $\tilde{\epsilon}_{jk} = \epsilon_{jk} \frac{\tilde{\epsilon}_{kk}}{\epsilon_{kk}}$. This restriction eliminates economically unlikely scenarios in which a substitution away from a given product (in response to a price increase) is exceeded by substitution to other products. When we generate data without such a restriction, even modest amounts of error often result in negative price predictions. We focus the second comparison on the linear, almost-ideal, and log-linear demand systems because these can accommodate changes in the elasticity matrix

¹⁰In the pre-merger equilibrium, consumer substitution between products is proportional to market share because all the systems are calibrated based on logit elasticities. This reduces the dimensionality of the random data that must be drawn. The diversion-by-share property is retained away from the pre-merger equilibrium only for logit demand.

with a fixed set of market shares.

3.2 Summary statistics

Table 1 summarizes the empirical distributions of the data. The distribution of firm 1's share is centered around 14 percent, which reflects that shares are allocated among six products and the outside good. The margin distribution is determined by the uniform draws with support over $(0.20, 0.80)$. The own-price elasticity of demand, which equals the inverse margin, has a distribution centered around two. 90 percent of the own-price elasticities fall between 1.31 and 4.37. The diversion ratios have a distribution centered at .17, and 90 percent of diversion ratios fall between .02 and .32.

Market concentration is based on pre-merger market shares, following equations (6) and (7). The median pre- and post-merger HHI are 1562 and 1931, respectively, and the median change is 317. The simulated merger results in an HHI increase of greater than 200 in 65 percent of markets, an increase of between 100 and 200 in 15 percent of markets, and an increase of less than 100 in 19 percent of markets. The median UPP is 0.07, and 90 percent of UPP values fall between 0.01 and 0.22. All of the above statistics are invariant to the posited demand system, because the demand systems are calibrated to produce the same first-order characteristics in the pre-merger equilibria.¹¹

The median merger price effects are 0.06, 0.11, 0.05, and 0.18 for the logit, almost ideal, linear, and log-linear demand systems, respectively. Because pre-merger prices are normalized to one, these statistics reflect both the median level change and median percentage change. Dispersion across demand systems reflects the specific pass-through properties of the systems, with greater own pass-through associated with larger price effects. This relationship, first observed in Froeb, Tschantz and Werden (2005), is explained by the theoretical results of Jaffe and Weyl (2013). Dispersion within demand systems mainly reflects the range of market conditions that arise from the data generating process, which is demonstrated in part by the range of UPP.

¹¹In calculating HHI, we use the shares of the six firms that strategically react to the merger, and ignore the share of the outside good. This is equivalent to an assumption that the outside good is sold by an infinite number of atomistic firms, and leads to a conservative estimate of the HHI level.

Table 1: Order Statistics

| | Median | 5%. | 10% | 25% | 75% | 90% | 95% |
|---|--------|------|------|------|------|------|------|
| <i>Market Conditions</i> | | | | | | | |
| Market share | 0.15 | 0.02 | 0.04 | 0.08 | 0.20 | 0.25 | 0.28 |
| Margin | 0.49 | 0.23 | 0.26 | 0.35 | 0.64 | 0.73 | 0.76 |
| Elasticity | 2.03 | 1.31 | 1.37 | 1.56 | 2.89 | 3.87 | 4.37 |
| Diversions | 0.17 | 0.02 | 0.04 | 0.09 | 0.23 | 0.29 | 0.32 |
| <i>Herfindahl–Hirschman Index (HHI)</i> | | | | | | | |
| Pre-Merger | 1562 | 1163 | 1231 | 1361 | 1816 | 2073 | 2255 |
| Post-Merger | 1931 | 1346 | 1452 | 1642 | 2277 | 2667 | 2937 |
| Change | 317 | 22 | 48 | 139 | 555 | 833 | 968 |
| <i>Upward Pricing Pressure</i> | | | | | | | |
| UPP | 0.07 | 0.01 | 0.01 | 0.04 | 0.12 | 0.18 | 0.21 |
| <i>Merger Price Effects</i> | | | | | | | |
| Logit | 0.06 | 0.01 | 0.01 | 0.03 | 0.11 | 0.16 | 0.20 |
| AIDS | 0.11 | 0.01 | 0.02 | 0.05 | 0.28 | 0.69 | 1.14 |
| Linear | 0.05 | 0.00 | 0.01 | 0.02 | 0.08 | 0.12 | 0.15 |
| Log-Linear | 0.18 | 0.01 | 0.03 | 0.08 | 0.46 | 1.18 | 2.13 |

Notes: Summary statistics are based on 4,500 randomly-drawn sets of data on the pre-merger equilibria. The market share, margin and elasticity are for the first firm. The cross-diversion is diversion from firm 1 to firm 2 (the two merging firms). Market share and margin are drawn randomly in the data generating process. The elasticity is the own-price elasticity of demand and equals the inverse margin. The merger price effects are the change in firm 1's equilibrium price.

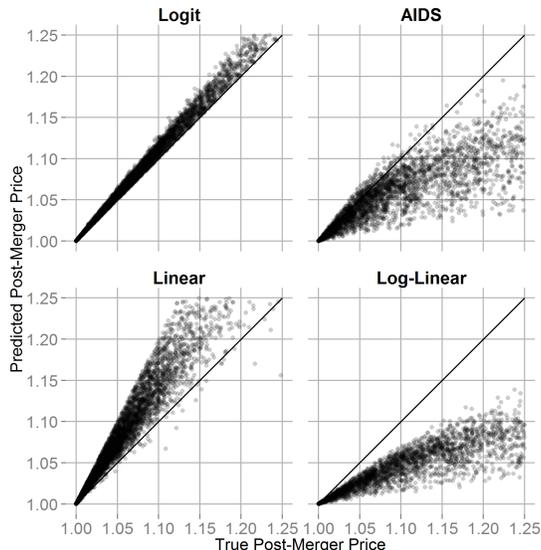


Figure 1: Graphical Illustration of UPP as a Price Predictor

Notes: The scatter plots characterize the accuracy of UPP as a price prediction when the underlying demand system is logit, almost ideal, linear, and log-linear. Each dot represents the first firm’s predicted and actual post-merger prices for a given draw of data.

4 Results

4.1 Graphical analysis

We begin by plotting the data. Figure 1 depicts the accuracy of UPP in predicting post-merger price increases under each of the demand systems considered. Each dot represents the predicted and true changes in firm 1’s price for a given draw of data; its vertical position is the prediction of simulation and its horizontal position is the true price effect. Dots that fall along the 45-degree line represent exact predictions while dots that fall above (below) the line represent over (under) predictions.

UPP appears to be quite accurate, albeit somewhat larger in magnitude than the true price effect, if the underlying demand is logit or linear. These systems are log-concave so the two biases developed in Section 2 are countervailing. UPP exceeds the actual price increases because using ones to proxy the diagonal pass-through elements (which amplifies predictions with incomplete pass-through) has a somewhat larger affect on results than suppressing the cross terms (which damps predictions with strategic complements). UPP understates price increases with almost ideal and log-linear demand, both of which exhibit greater convexity. Because the diagonal elements of merger pass-through with these systems typically exceed unity, the biases caused by using an identity matrix as a proxy are not countervailing and

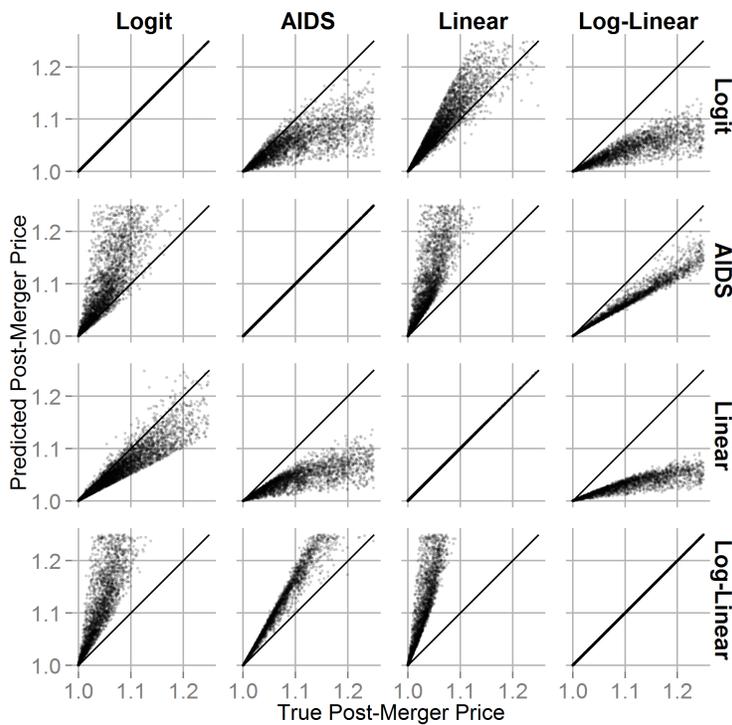


Figure 2: Prediction Error from Standard Merger Simulations

Notes: The scatter plots characterize the accuracy of merger simulations when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3), and log-linear (column 4). Merger simulations are conducted assuming demand is logit (row 1), almost ideal (row 2), linear (row 3), and log-linear (row 4). Each dot represents the first firm’s predicted and actual post-merger prices for a given draw of data.

instead tend to lower UPP relative to the actual price increases.

To put this in context, Figure 2 depicts the accuracy of merger simulation conducted with incorrect functional form assumptions. The scatter plots show data sorted by the underlying demand system: logit (column 1), almost ideal (column 2), linear (column 3), and log-linear (column 4), and by the merger simulation model: logit (row 1), almost ideal (row 2), linear (row 3), and log-linear (row 4). In many instances, the prediction error that arise due to functional form misspecification visibly exceeds the prediction error of UPP. This sensitivity of merger simulation to functional form assumptions is well known (e.g., Crooke, Froeb, Tschantz and Werden (1999); Miller, Remer, Ryan and Sheu (2016)) and, in antitrust settings, it is standard practice to generate predictions under multiple assumptions as a way to evaluate the scope of potential price changes.

For a second comparison, Figure 3 depicts the accuracy of merger simulation conducted with imprecisely measured demand elasticities. We plot only part of the generated data, for

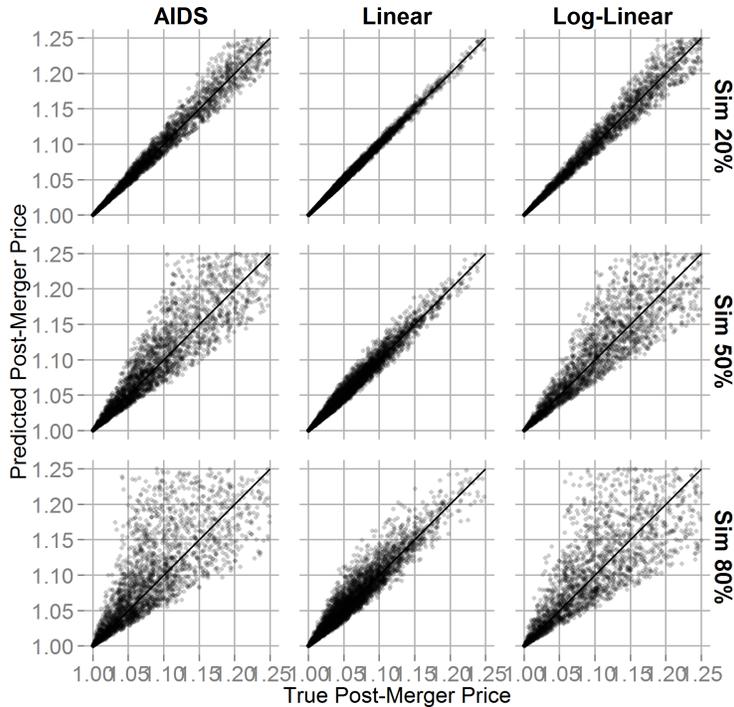


Figure 3: Prediction Error from Standard Merger Simulations

Notes: The scatter plots characterize the accuracy of merger simulations when there is error in the observed elasticities of demand. Merger simulations are conducted assuming the true demand system is known: almost ideal (column 1), linear (column 2), and log-linear (column 3). Each dot represents the first firm’s predicted and actual post-merger prices for a given draw of data.

the sake of brevity, showing the cases in which the elasticities of demand are observed with 20% error (row 1), 50% error (row 2), 80% error (row 3). As one might expect, predictions are centered around the true effects but prediction error increases as elasticities lose precision. Interestingly, predictions are relatively robust to imprecision in the elasticities with linear demand; we suspect this is because pass-through with linear demand does not change much with the elasticities.¹² We explore the relationship between these predictors and UPP in greater detail next.

4.2 Numerical analysis

Table 2 presents the median absolute prediction error (“MAPE”) of UPP when the true underlying demand system is logit, almost ideal, linear, or log-linear. UPP is quite accurate if demand is logit; the MAPE is roughly 10% of the median price effect. UPP loses

¹²See the mathematics in Miller, Remer and Sheu (2013).

Table 2: Median Absolute Prediction Error

| | Underlying Demand System: | | | |
|-----------------------|---------------------------|-------|--------|------------|
| | Logit | AIDS | Linear | Log-Linear |
| UPP | 0.006 | 0.042 | 0.022 | 0.110 |
| Logit Simulation | 0.000 | 0.049 | 0.014 | 0.117 |
| AIDS Simulation | 0.050 | 0.000 | 0.068 | 0.065 |
| Linear Simulation | 0.014 | 0.066 | 0.000 | 0.132 |
| Log-Linear Simulation | 0.123 | 0.065 | 0.139 | 0.000 |

Notes: The table provides the median absolute prediction error of UPP and standard simulations when the true underlying demand system is logit, almost ideal, linear, and log-linear.

Table 3: Frequency with Which UPP Improves Accuracy

| | Underlying Demand System: | | | |
|--------------------|---------------------------|-------|--------|------------|
| | Logit | AIDS | Linear | Log-Linear |
| Logit Simulation | . | 92.2% | 3.2% | 100% |
| AIDS Simulation | 95.1% | . | 90.8% | 10.6% |
| Linear Simulation | 69.0% | 98.5% | . | 99.0% |
| Log-Lin Simulation | 100% | 74.6% | 100% | . |

Notes: This table shows the fraction of mergers for which UPP has a smaller absolute prediction error than standard merger simulations in predicting the price change.

some accuracy if demand is linear or almost ideal, but prediction error remains small once benchmarked against median price effects. With log-linear demand, the MAPE of UPP is more than 50% of the median price effect. The table also shows the MAPEs that arise with misspecified merger simulations. Looking across the four demand systems, the MAPE of UPP is always smaller than at least two of the three misspecified simulations.

Table 3 shows the frequency with which UPP provides a more accurate prediction than misspecified simulation. Among the twelve comparison groups the data generating process allows, UPP dominates misspecified in all but two instances (the exceptions being logit simulation with linear demand and almost ideal simulation with log-linear demand). We interpret the results summarized in Tables 2 and 3 as suggestive that the inaccuracies associated with using UPP as a price predictor often are somewhat smaller than the inaccuracies that can arise due to functional form misspecification in merger simulation models.

Table 4: MAPE with Imprecise Demand Elasticities

| Panel A: Simulation | | | |
|---------------------|----------------------------------|--------|------------|
| | <u>Underlying Demand System:</u> | | |
| | AIDS | Linear | Log-Linear |
| 20% Error | 0.007 | 0.001 | 0.012 |
| 50% Error | 0.019 | 0.004 | 0.030 |
| 80% Error | 0.031 | 0.006 | 0.049 |

| Panel B: Upward Pricing Pressure | | | |
|----------------------------------|----------------------------------|--------|------------|
| | <u>Underlying Demand System:</u> | | |
| | AIDS | Linear | Log-Linear |
| 20% Error | 0.042 | 0.022 | 0.110 |
| 50% Error | 0.041 | 0.022 | 0.108 |
| 80% Error | 0.045 | 0.023 | 0.104 |

Notes: Panel A shows the median absolute prediction error of simulation when elasticities are observed with 20%, 50%, and 80% error and the true demand system is known. Panel B shows the median absolute prediction error of UPP when elasticities are observed with 20%, 50%, and 80% error.

We next consider merger simulation conducted with the correct demand system but imprecise demand elasticities. In this exercise, we recalculate UPP based on the imprecise elasticities in order to help facilitate an “apples-to-apples” comparison. Panel A of Table 4 summarizes the prediction errors that arise with merger simulation. The MAPEs increase significantly with the amount of imprecisions, but remain small relative to the median price prediction for each underlying demand system. Panel B shows that the MAPEs of UPP, by contrast, do not increase much with imprecision. We suspect the reason is that effects of imprecision are partially “canceled out” when the own and cross elasticities are converted into diversion. Despite this robustness, the MAPEs of UPP exceed those of merger simulation even with a substantial amount of measurement error. Consistent with these results, UPP is more accurate than simulation in a minority of the mergers, at each level of imprecision considered.¹³

¹³The analog to Table 3 is available upon request.

Table 5: UPP as a Screen

| | Underlying Demand System: | | | |
|---------------------------------|---------------------------|-------|--------|------------|
| | Logit | AIDS | Linear | Log-Linear |
| False Positives (Type I Error) | 5.0% | 0.2% | 18.4% | 0.0% |
| False Negatives (Type II Error) | 0.0% | 22.4% | 0.0% | 36.6% |

Notes: Panel A shows the fraction of mergers for which the price change is less than 10% but UPP exceeds 10%. Panel B the fraction of mergers for which the price change exceeds 10% but UPP is less than 10%.

4.3 Preliminary screens in merger analysis

4.3.1 Upward Pricing Pressure

The Monte Carlo experiments also allow us to assess the properties of UPP as a preliminary screen in merger analysis. The evidence shown thus far – most visibly in Figure 1 – demonstrates that UPP is strongly correlated with price effects, a property that is highly desirable in a screen. Indeed, UPP is almost perfectly correlated with price changes under logit and linear demand, and highly correlated with price changes under almost ideal and log-linear demand. The correlation coefficients are 0.996, 0.955, 0.857 and 0.895, respectively.

Consider the following thought experiment: the objective of the antitrust authority is to block mergers that increase price more than 10%, and employs a screen in which it investigates if and only if UPP exceeds 10%. How well would the antitrust authority sort mergers?¹⁴ Table 5 provides the frequency of the two possible errors: “false positives” (benign mergers that are investigated) and “false negatives” (anticompetitive mergers that are not investigated). Results are broken out by the true underlying demand system. If demand is log-concave (i.e., linear or logit) then false positives are much more likely than false negatives. From a policy standpoint, this probably is desirable to the extent that false positives can be identified and cleared in the subsequent investigation, while no such correction is available with false negatives. If demand instead exhibits considerable convexity then false negatives far exceed false positives. This is problematic from a policy standpoint, by the same logic. Taken together, the results support the notion that it is prudent to employ an inclusive UPP screen, at least to the extent that substantial convexity in demand is deemed realistic.

¹⁴We select 10% solely based on the empirical distribution of price changes in the data. We have examined other thresholds, and the qualitative results are unaffected.

4.3.2 Herfindahl-Hirschman Index

The 2010 Horizontal Merger Guidelines define a set of HHI levels and changes to help stratify mergers into those that are unlikely to pose a problem and those that warrant a close investigation. Five categories are defined as follows:

- (i) Post-merger $HHI > 2500$ and $\Delta HHI > 200$.
- (ii) Post-merger $HHI > 2500$ and $\Delta HHI \in (100, 200]$.
- (iii) Post-merger $HHI \in (1500, 2500]$ and $\Delta HHI > 100$.
- (iv) Post-merger $HHI \leq 1500$.
- (v) $\Delta HHI < 100$.

The Guidelines state that categories (i)-(iii) are likely to raise competitive concern and lead to further investigation, while mergers in categories (iv) and (v) are unlikely to create competitive problems. Many in the antitrust community view the latter categories as providing safe harbors, although this is not specifically endorsed in the Guidelines. Our Monte Carlo experiments allow us to evaluate these HHI thresholds. Because the theoretical connection between ΔHHI and unilateral effects is strongest with proportional consumer substitution, the results can be interpreted from a best-case scenario for the concentration measures.

Table 6 shows the fraction of mergers that result in price increases of at least 5% (Panel A) and 10% (Panel B), sorted by HHI category. Mergers in categories (i) and (iii) frequently produce substantial price increases. This is especially true of mergers in category (i), which generate price elevations above 5% in around 90% of the mergers with logit and linear demand, and in more than 95% of the mergers with almost ideal and log-linear demand. Perhaps more surprising, mergers in category (ii) appear relatively benign, and never produce a 5% price increase with log-concave demand. The results for category (iv) indicate that a nontrivial minority of mergers in markets that are not deemed “moderately concentrated” nonetheless result in price increases of 5% or higher. By contrast, virtually no mergers in category (v) result in such price increases if demand is log-concave.

These results reinforce that ΔHHI is more directly connected to unilateral effects theory than the post-merger HHI (e.g., see Section 2). Accordingly, we investigate whether the ΔHHI could be used effectively as a screen. The data indicate a strong correlation between ΔHHI and the price change. The correlation coefficients range between 0.519 and 0.846 depending on the demand system. Table 7 shows the fraction of mergers that result in

Table 6: HHI Category Screens

| Panel A: Frequency of 5% Price Increase | | | | |
|---|-------|-------|--------|------------|
| Underlying Demand System: | | | | |
| | Logit | AIDS | Linear | Log-Linear |
| Category (i) | 90.9% | 95.7% | 86.7% | 98.8% |
| Category (ii) | 0.0% | 33.3% | 0.0% | 61.9% |
| Category (iii) | 63.8% | 79.8% | 45.1% | 92.4% |
| Category (iv) | 19.3% | 46.6% | 6.0% | 61.5% |
| Category (v) | 0.2% | 20.7% | 0.0% | 30.2% |

| Panel B: Frequency of 10% Price Increase | | | | |
|--|-------|-------|--------|------------|
| Underlying Demand System: | | | | |
| | Logit | AIDS | Linear | Log-Linear |
| Category (i) | 53.2% | 80.2% | 49.5% | 93.6% |
| Category (ii) | 0.0% | 9.5% | 0.0% | 33.3% |
| Category (iii) | 17.7% | 57.1% | 8.0% | 73.9% |
| Category (iv) | 0.7% | 27.0% | 0.0% | 36.7% |
| Category (v) | 0.0% | 5.8% | 0.0% | 9.9% |

Notes: Panel A shows the fraction of mergers in each HHI category for which the weighted-average change in the merging firms' prices is greater than 5%. Panel B shows the the same statistic for a price increase greater than 10%. Category (i): Post-merger $HHI > 2500$ and $\Delta HHI > 200$. Category (ii): Post-merger $HHI > 2500$ and $\Delta HHI \in (100, 200]$. Category (iii): Post-merger $HHI \in (1500, 2500]$ and $\Delta HHI > 100$. Category (iv): Post-merger $HHI \leq 1500$. Category (v): $\Delta HHI < 100$.

Table 7: Screens Based on ΔHHI

| Panel A: Frequency of 5% Price Increase | | | | |
|---|-------|-------|--------|------------|
| Underlying Demand System: | | | | |
| | Logit | AIDS | Linear | Log-Linear |
| $\Delta\text{HHI} > 200$ | 76.1% | 87.0% | 60.3% | 96.9% |
| $\Delta\text{HHI} \in (100, 200)$ | 20.2% | 53.4% | 0.3% | 71.7% |
| $\Delta\text{HHI} < 100$ | 0.2% | 20.7% | 0.0% | 30.2% |

| Panel B: Frequency of 10% Price Increase | | | | |
|--|-------|-------|--------|------------|
| Underlying Demand System: | | | | |
| | Logit | AIDS | Linear | Log-Linear |
| $\Delta\text{HHI} > 200$ | 27.4% | 65.8% | 17.6% | 81.9% |
| $\Delta\text{HHI} \in (100, 200)$ | 0.0% | 30.9% | 0.0% | 45.5% |
| $\Delta\text{HHI} < 100$ | 0.0% | 5.8% | 0.0% | 9.9% |

Notes: Panel A shows the fraction of mergers in each ΔHHI category for which the weighted-average change in the merging firms' prices is greater than 5%. Panel B shows the same statistic for a price increase greater than 10%.

prices increases of at least 5% (Panel A) and 10% (Panel B), this time sorted by ΔHHI . Results are provided for (i) change in HHI greater than 200, (ii) change in HHI between 100 and 200 and (iii) change in HHI less than 100. Most mergers with $\Delta\text{HHI} > 200$ produce substantial price increases, as do a nontrivial minority of mergers with $\Delta\text{HHI} \in (100, 200)$. Mergers with $\Delta\text{HHI} < 100$ also cause substantial price increases – notice that this corresponds with category (v) above. Virtually no mergers with $\Delta\text{HHI} < 100$ result in substantial price increases if demand is log-concave (this corresponds to category (v) above).

5 Conclusion

This research evaluates the accuracy of UPP in predicting post-merger price changes, using a large-scale Monte Carlo experiment. The results are supportive overall: we find that UPP is quite accurate with standard log-concave demand systems, but understates price effects if demand exhibits greater convexity. Prediction error does not systematically exceed that of misspecified simulation models, nor is it much greater than that of correctly-specified models simulated with imprecise demand elasticities. We provide a theoretical basis for these results

by observing that UPP is a restricted version of the first order approximation derived in Jaffe and Weyl (2013). We conclude that UPP has greater utility than currently is recognized.

That UPP often outperforms simulation models in our Monte Carlo experiments raises a question about the appropriate scope of application. In our view, the value of UPP as a price predictor is greatest in merger investigations and similar policy endeavors, due to its expediency and simplicity. The academic literature provides an array of methodologies that are capable of both limiting functional-form misspecification in simulation models and reducing standard errors in structural estimation. These methodologies (which we do not examine in the Monte Carlo experiments) may well allow simulation to produce more robust and accurate predictions than are available from UPP. Thus, we are skeptical that our results have significant bearing on empirical industrial economics. By contrast, because state-of-art academic methodologies often may be too time-consuming to be used within the constraints of merger investigations, our results are immediately relevant for antitrust practice.

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Appendix

A Mathematical Details of the Calibration Process

We provide mathematical details on the calibration process in this appendix. To distinguish the notation from that of Section 2, we move to lower cases and let, for example, s_i and p_i be the market share and price of firm i 's product, respectively.¹⁵ Recall that in the data generating process we randomly assign market shares among the four single-product firms and the outside good, draw the price-cost margin of the first firm's product from a uniform distribution with support over $(0.2, 0.8)$, and normalize all prices to unity. The calibration process then obtains parameters for the logit, almost ideal, linear and log-linear demand systems that reproduce these draws of data.

Calibration starts with multinomial logit demand, the basic workhorse model of the discrete choice literature. The system is defined by the share equation

$$s_i = \frac{e^{(\delta_i - \alpha p_i)}}{1 + \sum_{j=1}^N e^{(\delta_j - \alpha p_j)}}. \quad (\text{A.1})$$

The parameters to be calibrated include the price coefficient α and the product-specific quality terms δ_i . We recover the price coefficient by combining the data with the first order conditions of the first firm. Under the assumption of Nash-Bertrand competition this yields:

$$\alpha = \frac{1}{m_1 p_1 (1 - s_1)} \quad (\text{A.2})$$

where m_1 is the price-cost margin of firm 1. We then identify the quality terms that reproduce the market shares:

$$\delta_i = \log(s_i) - \log(s_0) + \alpha p_i, \quad (\text{A.3})$$

for $i = 1 \dots N$. We follow convention with the normalization $\delta_0 = 0$. Occasionally, a set of randomly-drawn data cannot be rationalized with logit demand and we replace it with a set that can be rationalized. This tends to occur when the first firm has both an unusually small market share and an unusually high price-cost margin.

The logit demand system sometimes is criticized for its inflexible demand elasticities. Here, the restrictions on substitution are advantageous and allow us to obtain a full matrix of elasticities with a tractable amount of randomly drawn data. The derivatives of demand

¹⁵We define market share $s_i = q_i / \sum_{j=1}^N q_j$, where q_i represents unit sales.

with respect to prices, as is well known, take the form

$$\frac{\partial q_i}{\partial p_j} = \begin{cases} \alpha s_i(1 - s_i) & \text{if } i = j \\ -\alpha s_i s_j & \text{if } i \neq j \end{cases} \quad (\text{A.4})$$

We use the logit derivatives to calibrate the more flexible almost ideal, linear and log-linear demand systems. This ensures that each demand system has the same first order properties in the pre-merger equilibrium, for a given draw of data.

The AIDS is written in terms of expenditure shares instead of quantity shares Deaton and Muellbauer (1980). The expenditure share of product i takes the form

$$w_i = \alpha_i + \sum_{j=0}^N \gamma_{ij} \log p_j + \beta_i \log(x/P) \quad (\text{A.5})$$

where x is total expenditure and P is a price index. We incorporate the outside good as product $i = 0$ and normalize its price to one; this reduces to N^2 the number of price coefficients in the system that must be identified (i.e., γ_{ij} for $i, j \neq 0$). We further set $\beta_i = 0$ for all i , a restriction that imposes in income elasticity of unity. Under this restriction, total expenditures are given by

$$\log(x) = (\tilde{\alpha} + u\tilde{\beta}) + \sum_{k=1}^N \alpha_k \log(p_k) + \frac{1}{2} \sum_{k=1}^N \sum_{j=1}^N \gamma_{kj} \log(p_k) \log(p_j) \quad (\text{A.6})$$

for some utility u . We identify the sum $\tilde{\alpha} + u\tilde{\beta}$ rather than $\tilde{\alpha}$, u and $\tilde{\beta}$ individually.¹⁶ Given this structure, product i 's unit sales are given by $q_i = xw_i/p_i$ and the first derivatives of demand take the form

$$\frac{\partial q_i}{\partial p_j} = \begin{cases} \frac{x}{p_i^2}(\gamma_{ii} - w_i + w_i^2) & \text{if } i = j \\ \frac{x}{p_i p_j}(\gamma_{ij} + w_i w_j) & \text{if } i \neq j \end{cases} \quad (\text{A.7})$$

The calibration process for the AIDS then takes the following four steps:

1. Calculate x and w_i from the randomly drawn data on market shares, using a market size of one to translate market shares into quantities.
2. Recover the price coefficients γ_{ij} for $i, j \neq 0$ that equate the AIDS derivatives given in

¹⁶The price index P is defined implicitly by equation (A.6) as the combination of prices that obtains utility u given expenditure x . A formulation is provided in Deaton and Muellbauer (1980).

equation (A.7) and the logit derivatives given in equation (A.4). Symmetry is satisfied because consumer substitution is proportional to share in the logit model. The outside good price coefficients, γ_{i0} and γ_{0i} for all i , are not identified and do not affect outcomes under the normalization the $p_0 = 1$. Nonetheless, they can be conceptualized as taking values such that the adding up restrictions $\sum_{i=0}^N \gamma_{ij} = 0$ hold for all j .

3. Recover the expenditure share intercepts α_i from equation (A.5), leveraging the normalization that $\beta_i = 0$. The outside good intercept α_0 is not identified and does not affect outcomes, but can be conceptualized as taking a value such that the adding up restriction $\sum_{i=0}^N \alpha_i = 1$ holds.

4. Recover the composite term $(\tilde{\alpha} + u\tilde{\beta})$ from equation (A.6).

This process creates an AIDS that, for any given set of data, has quantities and elasticities that are identical in the pre-merger equilibrium to those that arise under logit demand. The system possess all the desirable properties defined in Deaton and Muellbauer (1980). Our approach to calibration differs from Epstein and Rubinfeld (2001), which does not model the price index as a function of the parameters, and from Crooke, Froeb, Tschantz and Werden (1999), which assumes total expenditures are fixed.

We turn now to the linear and log-linear demand systems. Linear demand takes the form

$$q_i = \alpha_i + \sum_j \beta_{ij} p_j \quad (\text{A.8})$$

The parameters to be calibrated include the firm specific intercepts α_i and the price coefficients β_{ij} . We recover the price coefficients directly from the logit derivatives in equation (A.4). We then recover the intercepts to equate the implied quantities in equation (A.8) with the randomly drawn market shares, again using a market size of one. Of similar form is the log-linear demand system:

$$\log(q_i) = \gamma_i + \sum_j \epsilon_{ij} \log p_j \quad (\text{A.9})$$

where the parameters to be calibrated are the intercepts γ_i and the price coefficients ϵ_{ij} . Again we recover the price coefficients from the logit derivatives (converting first the derivatives into elasticities). We then recover the intercepts to equate the implied quantities with the market share data. This process creates linear and log-linear demand systems that, for any given set of data, has quantities and elasticities that are identical to those of the calibrated logit and almost ideal demand systems, in the pre-merger equilibrium.