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FORWARD CONTRACTS, MARKET STRUCTURE, AND THE WELFARE EFFECTS OF MERGERS

by

Nathan H. Miller

and

Joseph U. Podwol

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* Miller: Georgetown University, nhm27@georgetown.edu. Podwol: U.S. Department of Justice, joseph.podwol@usdoj.gov

This paper subsumes an earlier working paper titled, "Forward Contracting and the Welfare Effects of Mergers" (2013). We thank Jeff Lien and Jeremy Verlinda for valuable comments and Jan Bouckaert and Louis Kaplow for helpful discussions. The views expressed in this paper are those of the authors and are not purported to reflect the views of the U.S. Department of Justice.

Forward Contracts, Market Structure, and the Welfare Effects of Mergers*

Nathan H. Miller†
Georgetown University

Joseph U. Podwol‡
U.S. Department of Justice

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Abstract

We examine how forward contracts affect economic outcomes under generalized market structures. In the model, forward contracts discipline the exercise of market power by making profit less sensitive to changes in output. This impact is greatest in markets with intermediate levels of concentration. Mergers reduce the use of forward contracts in equilibrium and, in markets that are sufficiently concentrated, this amplifies the adverse effects on consumer surplus. Additional analyses of merger profitability and collusion are provided. Throughout, we illustrate and extend the theoretical results using Monte Carlo simulations. The results have practical relevance for antitrust enforcement.

Keywords: forward contracts; hedging; mergers; antitrust policy
JEL classification: L13; L41; L44

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†Georgetown University, McDonough School of Business, 37th and O Streets NW, Washington, DC 20057. Email: nhm27@georgetown.edu.

‡U.S. Department of Justice, Antitrust Division, Economic Analysis Group, 450 5th St. NW, Washington, DC 20530. Part of this research was conducted while I was the Victor Kramer fellow at Harvard Law School, Cambridge, MA. Email: joseph.podwol@usdoj.gov.

1 Introduction

A long-standing result in the theoretical literature is that forward markets can increase output and lower prices in imperfectly competitive industries (Allaz and Vila (1993)). Underlying the result is that forward sales discipline the exercise of market power in the spot market by making profit less sensitive to the changes in output. Little attention has been played, however, to the role of competition in determining the magnitude of these effect—the existing literature is developed almost exclusively using models of symmetric duopoly. In the present study, we examine the effects of forward markets under generalized market structures, and obtain results that are of practical relevance to antitrust authorities.

Our model features an oligopolistic industry in which firms sell a homogeneous product and compete through their choices of quantities. Competition happens first in one or more contract markets, and later in a spot market. Following Perry and Porter (1985), firms have heterogeneous marginal cost schedules that reflect their respective capacities. The model can incorporate any arbitrary number of firms and any combination of capacities, and thereby facilitates an analysis of market structure. Thus, we bring together two established theoretical literatures: one on strategic forward contracts (e.g., Allaz and Vila (1993)), and the other on the effects of horizontal mergers with homogeneous products (e.g., Perry and Porter (1985); Farrell and Shapiro (1990)).

We establish that the presence of forward markets weakly increases aggregate output in equilibrium, relative to a Cournot benchmark, regardless of market structure. Forward markets allow firms to make strategic commitments, and the ensuing competition for Stackleberg leadership increases output relative to a Cournot baseline. This effect is largest for intermediate levels of market concentration, and converges to zero as market structure approaches the limit cases of monopoly and perfect competition. The non-monotonicity arises because increasing the number of firms intensifies the competition for Stackleberg leadership and thereby pushes the industry toward a perfectly competitive equilibrium faster than would be the case under Cournot competition. However, there are diminishing returns: firms do not sell output for less than their marginal cost, regardless of their forward position. As the number of firms grows large, competitive outcomes are obtained with or without forward markets. A simple Monte Carlo experiment suggests that, with a single period of forward contracting, the increase in consumer surplus is maximized at a Hirshmann-Herfindahl Index (HHI) of around 0.30, corresponding roughly to a three firm oligopoly. The increase in total surplus is maximized at an HHI around 0.40.

These results suggest that the presence of forward markets has nuanced implications for merger analysis. Indeed, we establish that forward contracting exacerbates the loss of consumer surplus caused by mergers if the market is sufficiently concentrated, but mitigates loss otherwise. This can be understood as the combination of two forces. First, forward contracts discipline the exercise of market power, which would be sufficient to mitigate output loss if firms' forward contracting practices were to remain unchanged post-merger. However, mergers also lessen the competition for Stackleberg leadership, thereby softening the constraint on the exercise of market power. The latter effect dominates if the market is sufficiently concentrated. Returning to Monte Carlo experimentation, forward markets tend to amplify consumer surplus loss if the post-merger HHI exceeds 0.40, roughly between a symmetric tripoly and duopoly levels.

While it is difficult to obtain general analytical results on the profitability of mergers in our setting, the Monte Carlo experiments we conduct have the striking feature that every merger considered is privately profitable in the presence of forward markets. To motivate this numerical result, we point out that mergers are not profitable in Cournot models with constant marginal costs except in the case of merger to monopoly (Salant, Switzer and Reynolds (1983)). With increasing marginal cost schedules, some mergers are profitable, but many still are not (Perry and Porter (1985)). Thus our finding is somewhat novel. We demonstrate analytically that it stems from the merging firm's ability to influence the output of its rivals through forward commitments: consolidation damps the incentives for all firms to hedge, and the output expansion by non-merging is mitigated sufficiently to bring about profitability.

Our final set of results pertain to collusion. Liski and Montero (2006) show that the presence of a forward market can reduce the critical discount rate necessary to sustain collusion in the case of symmetric duopoly. We advance the literature by considering how this relationship depends on industry structure, namely changes in the number of firms. We find that, (i) the presence of a forward market decreases the critical discount rate relative to Cournot; and (ii) this effect is more pronounced for small N. This suggests that it is more likely that, in the presence of a forward market, firms will switch from competition to collusion in response to an increase in concentration.

One limitation of our model is that it does not incorporate risk aversion. However, Allaz (1992) shows that risk-hedging and strategic motives can coexist in equilibrium, with each contributing to an expansion of output relative to the Cournot benchmark. The mechanisms that we identify extend to that setting cleanly. Further, we anticipate that many of our results also would extend to models in which forward contracts exist only to hedge risk (e.g., Eldor and Zilcha (1990)); the basis being that if the exercise of market power is relatively more profitable, but for some limiting constraint, then firms have relatively stronger incentives to relax the constraint. Thus, for instance, one might expect firms in less competitive industries to bear somewhat more risk. This principle applies well beyond models of forward contracting; the dynamic price signaling game of Sweeting and Tao (2016) is one recent example that shares a core intuition with our own research.

This study blends the literatures on horizontal mergers and strategic forward contracting. In the former literature, Perry and Porter (1985) introduce the concept of capital stocks to model mergers among Cournot competitors as making the combined firm larger instead of merely reducing the number of firms. McAfee and Williams (1992) solve for the equilibrium strategies under any arbitrary allocation of capital stocks. Farrell and Shapiro (1990) allow for fully general cost functions which incorporate the possibility of merger-specific cost efficiencies, and also develop the usefulness of examining "first-order" impact of mergers. Jaffe and Wyle (2013) apply the first-order approach to study merger effects under a general model of competition that nests conjectural variations, Cournot, and Bertrand as special cases. The solution techniques that we employ extend the methodologies developed in these articles.

The seminal article on strategic forward contracting is Allaz and Vila (1993). The main result developed is that as the number of contracting stages increases in a model of duopoly, total output approaches the perfectly competitive level. The subsequent literature has gone in a number of directions. Hughes and Kao (1997) and Ferreira (2006) consider the importance of the assumption that contracts are observable to the market. Green (1999) extends the model to markets in which firms submit supply schedules. Mahenc and Salanie (2004) analyze the impact of forward contracting when firms compete via differentiated products Bertrand in the spot market. Ferreira (2003) explores equilibria of the game with infinitely many contracting rounds. Liski and Montero (2006) consider the role of forward contracting in sustaining collusive outcomes. All of these studies suggest that the extent to which our results are applicable in real-world settings will depend on a number of features of the industry in question. Empirical evidence on the importance of forward contracting is presented in Wolak (2000), Bushnell (2007), Bushnell, Mansur and Saravia (2008), Hortacsu and Puller (2008) and Brown and Eckert (2016).

Among the aforementioned studies, the closest to our research are Bushnell (2007) and Brown and Eckert (2016). Bushnell (2007) examines the welfare impact of a forward market for a symmetric N-firm oligopoly with a single round of forward contracting. The model is calibrated to a number of deregulated electricity markets in order to ascertain the impact of forward markets on prices and output. Mergers are not examined. Brown and Eckert allow firms to have heterogeneous capital stocks as we do, but the focus is primarily empirical and as a result, they do not analytically solve for the equilibrium with an arbitrary number of contracting rounds and heterogeneous firms as we do.

The paper proceeds as follows. Section 2 describes the model of multistage quantity competition and solves for equilibrium strategies using backward induction. Section 3 analyzes the welfare impact of forward contracting, showing that the welfare impact of a forward market is non-monotonic in concentration. Section 4 formally models the welfare impacts of mergers highlighting how the results differ from the baseline model of Cournot competition. Section 5 provides an extension to collusion and Section 6 concludes with a discussion of the applicability of our results.

2 Model

2.1 Overview

We consider a modified Cournot model that features T contracting stages. The model is a variant of Allaz and Vila (1993) but we allow for an arbitrary number of producers with heterogeneous production technologies as in McAfee and Williams (1992). In each of T periods prior to production, firms can contract at a set price to buy or sell output to be delivered at time $t = 0$. Denote each of these contracting stages as $T, \ldots, t, \ldots, 1$ such that stage t occurs t periods before production. Following the conclusion of each contracting stage, contracted quantities are observed by all market participants and are taken into account in the subgame that follows. At $t = 0$, production takes place, contracts are settled, and producers compete via Cournot to sell any residual output in the spot market. The solution concept is Subgame Perfect Nash Equilibrium ("SPE").

Formally, let f_i^t denote the quantity contracted by producer $i \in \{1, \ldots, N\}$ in stage t, and let $q_i^t = \sum_{\tau=t+1}^{T} f_i^\tau$ denote the producer's forward position at the beginning of period

t. Forward contracts in stage t are agreed upon taking as given the forward price, P^t, and the vector of forward positions, $\mathbf{q}^t = \{q_1^t, ..., q_N^t\}$, and with knowledge of the corresponding subgame equilibrium that follows. At $t = 0$, each producer sells q_i^s in the spot market taking into account the vector of forward positions $\mathbf{q}^0 = \{q_1^0, ..., q_N^0\}$ and given other producers' output. This determines the producer's output, q_i, as the sum of its contracted and spot sales. Producers are "short" in the spot market if $q_i^0 > 0$. Total output is the sum of all firms' output and is denoted $Q = \sum_i q_i$. Buyers are passive entities and are represented by the linear inverse demand schedule $P(Q) = a - bQ$, for $a, b > 0$.

Each producer i is characterized by its capital stock, k_i, a proxy for its productive capacity. Total costs are $C_i(q_i) = cq_i + eq_i^2/2k_i$, so that marginal costs, $C_i'(q_i) = c + eq_i/k_i$, are increasing in output but decreasing in the capital stock. As a result, firms with greater capital stocks will have higher market shares owing to this cost advantage. We assume $a > c \geq 0$ to ensure that gains to trade exist. The parameter e is binary ($e \in \{0, 1\}$) and allows the model to nest constant marginal costs as a special case.

2.2 Spot market subgame

Solutions are obtained via backward induction: first considering the output decisions of producers in the spot market, given any vector of contracted quantities, and then considering the contract market. The spot price is determined by total output, $Q(\mathbf{q^0})$, which is itself a function of the vector of forward positions, $\mathbf{q^0}$. Producer i chooses its output, q_i (the sum of forward and spot market quantities), taking as given $\mathbf{q^0}$ as well as the vector of other producers' output, $\mathbf{q_{-i}}$, to maximize the profit function,

$$\pi_i^s\left(q_i; \mathbf{q^0}, \mathbf{q_{-i}}\right) = P\left(Q\left(\mathbf{q^0}, \mathbf{q_{-i}}\right)\right)\left(q_i\left(\mathbf{q^0}, \mathbf{q_{-i}}\right) - q_i^0\right) - C_i\left(q_i\left(\mathbf{q^0}, \mathbf{q_{-i}}\right)\right).$$

Suppressing dependence on $\mathbf{q^0}$ and $\mathbf{q_{-i}}$, the first-order condition implies that

$$P(Q) + \left(q_i - q_i^0\right) P'(Q) = C_i'(q_i). \tag{1}$$

If the producer holds a short position (i.e. $q_i^0 > 0$), then the inclusion of q_i^0 in equation (1) says that, relative to Cournot, revenue is less sensitive to output because selling an additional unit has no effect on the price received from forward sales. This amounts to an outward shift in the firm's marginal revenue function, holding fixed the output of other producers.[1] If competing producers increase their output relative to Cournot due to their own forward positions, this will cause i's marginal revenue function to shift back somewhat.

We derive closed-form expressions for equilibrium price and quantities by making use of the following terms:

$$\beta_i = \frac{bk_i}{bk_i + e},\ B = \sum_i \beta_i,\ B_{-i} = \sum_{j \neq i} \beta_j,\ F^0 = \sum_i \beta_i q_i^0,\ F_{-i}^0 = \sum_{j \neq i} \beta_j q_j^0,$$

Proposition 1 *In the spot market subgame with vector of forward positions, $\mathbf{q^0}$, there exists a unique Nash equilibrium in which price, total output and individual firms' output are given by:*

$$\begin{aligned} P\left(\mathbf{q^0}\right) &= c + \frac{a-c}{1+B} - \frac{bF^0}{1+B} \\ Q\left(\mathbf{q^0}\right) &= \left(\frac{a-c}{b}\right)\frac{B}{1+B} + \frac{F^0}{1+B} \\ q_i\left(\mathbf{q^0}\right) &= \left(\frac{a-c}{b}\right)\frac{\beta_i}{1+B} + \frac{\beta_i}{1+B}\left[\left(1+B_{-i}\right)q_i^0 - F_{-i}^0\right] \end{aligned}$$

[1] Anderson and Sundaresan (1984) use this very argument to show that given a short forward position, a monopolist will necessarily increase output relative to Cournot. They rely on risk aversion to explain why a monopolist would hold a short position in the first place.

All proofs are in the Appendix. The above values have been expressed so as to illustrate the differences between the multi-stage model of competition considered here and a baseline model of Cournot competition without forward contracts in which $q_i^0 = F_{-i}^0 = F^0 = 0$. In Cournot, total output is increasing while price is decreasing in B. A larger value of B corresponds to conditions typically associated with a more competitive industry: a larger number of firms, holding fixed capital stock per firm; greater capacity (i.e. capital stock) per firm, holding fixed the number of firms; and a more symmetric distribution of capacity among firms.

If F^0, a weighted average of producers' forward positions, is positive (i.e. producers are short on net) then price is lower and total quantity is higher than under Cournot. This foreshadows the results obtained below. A given producer's quantity may be higher or lower than the Cournot baseline, depending on how its forward position compares to that of other producers. One could imagine a producer would want to contract a large share of its productive capacity to become a Stackelberg leader. However, since other producers are employing the same strategy, each must adjust its output to the contracted quantities of its rivals. We will be able to say more about which of these forces dominates after deriving the equilibrium in the contract market.

2.3 Contract market

The contract market consists of speculators and producers. Speculators serve to take the opposite side of any long or short position of the producers subject to the constraint that the trade cannot be unprofitable ex ante. Producers take the contract price as given and simultaneously choose quantities. Suppose further that there are at least two speculators. With perfect information about the future, the resulting spot price is known as are all prices and quantities in subsequent contracting rounds, conditional on equilibrium (pure) strategies. Perfect foresight along with competition among speculators rules out any price other than the resulting spot price. Therefore, we require that the period-τ contract price, P^{τ}, satisfy, $P^{\tau} = \cdots = P^{1} = P\left(Q\left(\mathbf{q}^{\tau}\right)\right)$, where $Q\left(\mathbf{q}^{\tau}\right)$ is total output conditional on period-τ forward positions, $\mathbf{q}^{\tau}$, given equilibrium behavior in what follows. We refer to this as the "no arbitrage" condition.[2] Finally, we assume no discounting of profits.[3]

Consider then producer i's decision of how much to supply (or demand) in the contract market. Taking as given $\mathbf{q}^{\tau}$ and $\mathbf{q_{-i}}$, producer i chooses f_i^{τ} to maximize its profit function,[4]

$$\begin{aligned}
\pi_i\left(f_i^{\tau}; \mathbf{q}^{\tau}, \mathbf{q_{-i}}\right) &= P^{\tau} f_i^{\tau} + \sum_{t=1}^{\tau-1} P^{t} f_i^{t} + P\left(Q\right)\left(q_i - q_i^{0}\right) - C_i\left(q_i\right) \\
&= P\left(Q\right)\left(q_i - q_i^{\tau}\right) - C_i\left(q_i\right)
\end{aligned}$$

The first line on the right-hand side says that the producer takes into account that transactions in the current period affect prices and quantities in subsequent contracting periods as well as in the spot market. The second line on the right-hand side follows from the no-arbitrage condition. This shows that when the producer believes that all subsequent forward prices will adjust to the rationally anticipated spot price, it need only be concerned with how its decision today affects the spot price.

The first-order condition implies that,

$$P\left(Q\right) + \left(q_i - q_i^{\tau}\right)\left(1 + R_i^{\tau}\right) P'\left(Q\right) = C_i'\left(q_i\right), \tag{2}$$

where $R_i^{\tau} \equiv \sum_{j \neq i} \frac{\partial q_j}{\partial f_i^{\tau}} / \frac{\partial q_i}{\partial f_i^{\tau}}$. The interpretation of R_i^{τ} is as follows: if producer i takes an action in stage τ that increases its output by one unit, R_i^{τ} is the quantity response from all other producers. This term may be thought of as a conjectural variation, albeit one that is derived endogenously from equilibrium play. In a Cournot game with "Nash conjectures"

[2] The issue of commitment arises in that given a fixed number of contracting periods, a firm would always wish to to increase its contracting opportunities so as to disadvantage its rivals. Our results require that contracting frictions limit firms to a finite number of contracting periods.

[3] Including a discount rate changes nothing as shown by Liski and Montero (2006).

[4] We suppress dependence on q^{τ} and q_{-i} for readability.

(McAfee and Williams (1992)), this term is zero. But when competition spills across multiple periods as in the current setting, each producer recognizes that a marginal increase in its own short position, will reduce the amount competing firms produce. This creates an incentive for each firm to expand output beyond its Cournot level.

We derive R_i^τ recursively, relying on equilibrium behavior.

Lemma 1 *The conjectural variation in stage* 1 *with respect to producer* i*'s output as derived from Nash equilibrium behavior in the subgame beginning in stage* 0 *is,*

$$R_i^1 = -\frac{B_{-i}}{1 + B_{-i}}.$$

For any $\tau \geq 1$, *define* $\mu_i^\tau = \frac{\beta_i}{1+\beta_i R_i^\tau}$ *and* $M_{-i}^\tau = \sum_{j \neq i} \mu_j^\tau$. *The conjectural variation in stage* $\tau + 1$ *with respect to producer* i*'s output as derived from SPE behavior in the subgame beginning in stage* τ *is,*

$$R_i^{\tau+1} = -\frac{M_{-i}^\tau}{1 + M_{-i}^\tau}.$$

We can use Lemma 1 to show how the firm's problem is impacted by the presence of a forward market. It is evident that the marginal revenue curve facing firm i in the contract market as expressed in equation (2) is flatter in own output than it would be under Cournot. Since $1 + R_i^\tau < 1$, a marginal increase in firm i's contracted quantity does not reduce the price by as much as it would under Cournot because other firms respond by reducing their own output. Holding all other firms' output fixed at their Cournot levels and assuming no forward position in period τ (i.e., $q_i^\tau = 0$), the inclusion of $1 + R_i^\tau$ in equation (2) pivots firm i's marginal revenue curve up from the vertical axis, which suggests firm i will increase output relative to Cournot. As we saw in the spot market subgame, incorporating a short position shifts the firm's marginal revenue curve outward, thereby reinforcing this effect. However, if the same incentives facing firm i lead other firms to increase their output relative to Cournot, firm i's marginal revenue curve shifts down because quantities are strategic substitutes. This shift curbs firm i's incentive to increase output relative to Cournot and may even decrease it if other firms increase their output by a large enough amount.

We can now derive the equilibrium of the full game. Let $M^\tau = \sum_i \mu_i^\tau$ for any $\tau \geq 1$ and for completeness of notation, let $R_i^t = 0$ for all i when $t = 0$.

Proposition 2 *There exists a unique SPE of the game beginning in period T such that in each period, a producer anticipates producing q_i and sells a strictly positive fraction of its uncommitted anticipated output which rationalizes q_i as an equilibrium. The equilibrium is characterized by a vector of outputs, $\{q_i\}_i$, a sequence of forward sales, $\{f_i^t\}_{i,t}$, total output, Q, and price, P, satisfying:*

$$
\begin{aligned}
q_i &= \left(\frac{a-c}{b}\right)\frac{\mu_i^T}{1+M^T} \\
f_i^\tau &= \frac{R_i^{\tau-1}-R_i^\tau}{1+R_i^{\tau-1}}\left(q_i - \sum_{t=\tau+1}^{T} f_i^t\right) \\
Q &= \left(\frac{a-c}{b}\right)\frac{M^T}{1+M^T} \\
P &= c + \frac{a-c}{1+M^T}
\end{aligned}
$$

Absent a contract market (i.e., $R_i^t = 0 \; \forall \; i,t$), μ_i^T and M^T reduce to β_i and B, respectively, so that the price and quantities in Proposition 2 collapse to their values in the Cournot game of McAfee and Williams (1992). We can assess the impact of a forward market more broadly by analyzing changes in equilibrium outcomes as T increases from zero as in Cournot to positive values. We have that,

Corollary 1 *For any $T \in \{0, 1, \ldots\}$, price is (weakly) lower and total output is (weakly)* *higher in the SPE of the game with $T+1$ contracting rounds than with T. Each inequality is strict outside of the monopoly case. An individual producer's output can nevertheless be lower in the game with $T+1$ contracting rounds relative to T if its capital stock is sufficiently small relative to that of its competitors.*

Allaz and Vila (1993) provide a special case of this result for a symmetric two-firm oligopoly. When firms are symmetric, our model shows that all firms increase their output as T increases, as they do in Allaz and Vila (1993). Corollary 1 shows that this may no longer be the case when firms are asymmetric. This result suggests that the introduction of a forward market may increase concentration as measured by output, even as it improves welfare.

The impact of the forward market on output can be substantial. Consider the special case of constant marginal cost ($e = 0$) and a single contracting stage ($T = 1$). In this case, $\beta_i = 1$ so that $h_i = \frac{N-1}{N}$, $\mu_i^1 = N$, and $M^1 = N^2$. The presence of a forward market increases output by 140 percent when $N = 2$ and by nearly 600 percent when $N = 6$. These increases would be somewhat smaller if marginal costs were instead increasing ($e = 1$) and larger with multiple rounds of contracting ($T > 1$).

3 Market Structure and Welfare

We now examine the role of market structure in evaluating the impact of a forward market on welfare. Whereas Allaz and Vila (1993) showed that welfare can span duopoly-Cournot to perfect competition levels as the number of contracting rounds increases, our focus is on how the welfare impact of a forward market is influenced by market structure. As such, we treat T as fixed, determined by the particulars of the industry.[5]

[5] Bushnell (2007) discusses the institutional details of forward sales within wholesale electricity.

3.1 Market structure and hedge rates

The welfare impact of a forward market is related to the fraction of each firm's output that is contracted in the forward market, i.e. its "hedge rate." The following result aids the understanding of this relationship.

Lemma 2 *Given equilibrium strategies within the SPE of the $(T+1)$-stage game, the hedge rate can be expressed as* $h_i \equiv \frac{q_i^0}{q_i} = |R_i^T| = \frac{M_{-i}^T}{1+M_{-i}^T}$.

The result is fairly general in that the first equality, $h_i = |R_i^T|$, does not rely on the shape of the demand or cost functions. It follows from the fact that a firm, when deciding how much to supply on the contract market, takes into account that a marginal increase in supply will be met by a decrease in its competitors' sales in subsequent periods. Thus, while a marginal increase in contracted supply on its own causes the price to decline, the corresponding decrease in competitors' outputs partially offsets this. The optimum equates marginal revenue across each of $T+1$ stages much in the way that a third-degree price discriminating monopolist equates marginal revenue across customer segments.

A firm's hedge rate depends at a first order on the amount of capital stock controlled by its competitors as well as the distribution of capital stock among them.[6] Competitors with larger capital stocks produce more irrespective of hedging, so their response to firm i's contracted quantity will be larger. At the same time, because larger firms make less efficient use of their capital stocks, firm i's hedge rate is larger when the capital stocks of its competitors are more symmetrically distributed.[7] The upshot is that the structural conditions which lead a firm to sell a larger fraction of its output in the contract market are

[6] In the game with $T = 1$ contracting stages, a firm's hedge rate, $h_i = B_{-i}/(1+B_{-i})$, depends only on the capital stocks of its competitors. But when $T > 1$, the hedge rate depends on μ_{-i}^T, each of which depends on firm i's capital stock through its influence on every other firm's hedge rate. The effect of β_i on h_{-i} is of a second-order magnitude, however.

[7] Each β_i is concave in capital due to increasing marginal costs. Thus, firms with larger capital stocks produce less per unit of capital than smaller firms. Note that if marginal costs are constant ($d = 0$) then $\beta_i = 1\ \forall i$.

the same conditions that lead to greater output in the baseline Cournot model.

As a further illustration, consider the perfectly symmetric case (i.e., $\beta_i = \beta$ for all i). The (common) hedge rate when $T = 1$ is,

$$h^{(1)} = \frac{(N-1)\,\beta}{1 + (N-1)\,\beta}. \tag{3}$$

That $h^{(1)}$ is larger for larger values of N suggests that from a welfare perspective, a forward market is not a perfect substitute for a competitive industry structure because forward contracting is more prevalent when the industry is more competitive. This interpretation continues to hold for larger values of T. To see this, we have from Lemmas 1 and 2 that the hedge rate when $T = 2$ is,

$$h^{(2)} = H\left(h^{(1)}\right) \equiv \frac{(N-1)\,\beta}{1 + \left(N - 1 - h^{(1)}\right)\beta}. \tag{4}$$

Since H is monotonically increasing in $h^{(1)}$ and larger for larger values of N, it follows that iteration $T-1$, $h^{(T)} = H^{T-1}\left(h^{(1)}\right)$ (where the superscript $T-1$ reflects the number of iterations), is also larger for larger values of N. Note that in the case of monopoly ($N = 1$), the hedge rate is zero for any T as forward contracting has no strategic impact.

3.2 Hedge rates and welfare

In Proposition 1, we saw that total output is increasing in F^0, a weighted-average of forward positions. Lemma 2 showed that a firm's contracted output is increasing in its hedge rate, which itself is a function of market structure. In particular, when the market structure is more *competitive*—e.g., there are more firms or capital is distributed more symmetrically among a given number of firms—hedge rates are higher. This suggests that a forward market creates an additional channel through which market structure affects welfare.

To formalize this point, we first consider the industry-average Lerner Index, which summarizes the degree to which market output diverges from perfect competition and hence is useful as a proxy for consumer and total surplus (Shapiro (1989)). Let $s_i = q_i/Q$ denote firm i's market share and let $\epsilon = -\left(\partial Q/\partial P\right)\left(P/Q\right)$ denote the absolute price elasticity of demand.

Lemma 3 *Given a vector of hedge rates* $\mathbf{h}$*, the Lerner Index derived from firms optimizing subject to* $\mathbf{h}$ *equals*

$$LI\left(\mathbf{h}\right) \equiv \sum_i \left(\frac{P - C_i'}{P}\right) s_i = \sum_i \frac{s_i^2}{\epsilon}\left(1 - h_i\right)$$

Lemma 3 shows that each firm's price-cost margin percentage is a product of two terms, the typical Cournot term, s_i^2/ϵ, and a term reflecting the importance of forward contracting, $(1 - h_i)$. The LI can be evaluated at the SPE hedge rates, but it also holds for an arbitrary vector of hedge rates, keeping in mind that s_i and ϵ are themselves functions of the hedge rates. As hedge rates increase uniformly from zero to unity, price-cost margins and hence consumer and total surplus, span the Cournot outcome at one extreme and perfect competition at the other. Again holding T fixed, the structural conditions that give rise to larger hedge rates are the same conditions that give rise to competitive outcomes in the absence of forward contracting.[8]

3.3 Concentration and welfare

The results already established are sufficient to determine that forward markets have the greatest impact on outcomes in markets characterized by some intermediate level of competition/concentration. The $(T + 1)$-stage model is equivalent to the baseline model of Cournot

[8]When T becomes large, hedge rates approach unity and outcomes become competitive even under industry structures that look very non-competitive. Allaz and Vila (1993) showed that, in a symmetric duopoly, as $T \to \infty$, output approaches the perfectly competitive level.

competition in the monopoly case (Corollary 1), and both models converge to perfect competition as market shares approach zero (Lemma 3). Thus, if forward markets lower price and increase output (Corollary 1) then the magnitude of these effects must be maximized in markets with firms that have market shares bounded strictly by zero and unity.

We present the result using both consumer surplus (CS) and total surplus (TS) as measures of welfare. These can be expressed as functions of total output and the average price-cost margin:[9]

$$\begin{aligned} CS &= \frac{b}{2}Q^2 \\ TS &= \frac{Q}{2}\left[a - c + \sum_i s_i \left(P - C_i^{'}\right)\right] \end{aligned} \tag{5}$$

Let ρ^{CS} denote the ratio of consumer surplus in the SPE of $(T+1)$-stage model to consumer surplus in Cournot, holding constant all model parameters. Let ρ^{TS} denote the analogous ratio with respect to total surplus.

Next, we define what it means for concentration to decrease from monopoly at one extreme to the limiting case of perfect competition. Assume that there is an infinite number of potential producers, but that at any time, there are only a finite number whose capital stocks are strictly positive. We will then consider transfers of capital among a subset of potential producers, $\widetilde{\mathbf{N}}$, that reduces the absolute difference in capital between every producer in $\widetilde{\mathbf{N}}$. Suppose that a transfer changes the capital allocation from $\mathbf{k}$ to $\mathbf{k}'$ where k_i and k_i' are elements of $\mathbf{k}$ and and $\mathbf{k}'$, respectively. Following Waehrer and Perry (2003), an *equalizing transfer* is such that: (i) $|k_i - k_j| > \left|k_i' - k_j'\right|$ for every $i, j \in \widetilde{\mathbf{N}}$; (ii) $\sum_{i \in \widetilde{\mathbf{N}}} k_i = \sum_{i \in \widetilde{\mathbf{N}}} k_i'$; and (iii) $k_l = k_l'$ for all $l \notin \widetilde{\mathbf{N}}$. With respect to mergers, the pre-merger allocation of capital can be recovered from the post-merger allocation via an equalizing transfer.

We model perfect competition as the limiting case, of all allocations in which all firms

[9] Derivations are in the Appendix.

with positive capital stocks are symmetric, as the number of such firms goes to infinity. A *symmetric equalizing transfer* is an equalizing transfer moving from $\mathbf{k}$ to $\mathbf{k}'$ in which: i) $k'_i = k'_j$ for every $i, j \in \widetilde{N}$; and ii) $k'_i < k_i$ for every $i, j \in \widetilde{N}$. The industry approaches perfect competition from any arbitrary initial allocation of capital through a sequence of symmetric equalizing transfers.

Proposition 3 *If* $\mathbf{k}$ *is the monopoly allocation, then any equalizing transfer from* $\mathbf{k}$ *increases* ρ^{CS} *and* ρ^{TS}. *For any allocation* $\mathbf{k}$ *other than the monopoly allocation, there exists an allocation* $\widehat{\mathbf{k}}$, *such that any symmetric equalizing transfer to* $\widehat{\mathbf{k}}$ *causes* ρ^{CS} *and* ρ^{TS} *to decline.*

Proposition 3 is one of our core results. The idea is that when markets are sufficiently concentrated, a small decrease in concentration increases welfare more in the presence of a forward market. A large enough decrease in concentration from an intermediate allocation can increase welfare relatively more in the absence of a forward market. In the remainder of this section, we use numerical techniques to illustrate how forward markets have the greatest impact on welfare with intermediate levels of competition/concentration.

We first compare the welfare statistics obtained with $T = 1$ rounds of forward contracting to those obtained in Cournot equilibrium ($T = 0$). To do so, we create data on 9,500 "industries," evenly split between $N = 1, 2, \ldots, 20$. For each industry, we calibrate the structural parameters of the model (a, b, c, $\mathbf{k}$) such that Cournot equilibrium exactly matches randomly-allocated market shares, an average margin, and normalizations on price and total output.[10] We then obtain the welfare statistics that arise in Cournot equilibrium and with a single round of forward contracting.

Figure 1 summarizes the results. In each panel, the vertical axis provides the ratio of surplus with forward contracting to surplus with Cournot. The horizontal axes shows the Herfindahl-Hirschman index ("HHI"). The HHI is the sum of squared market shares, which

[10]We normalize $P = Q = 100$ and use an average margin of 0.40.

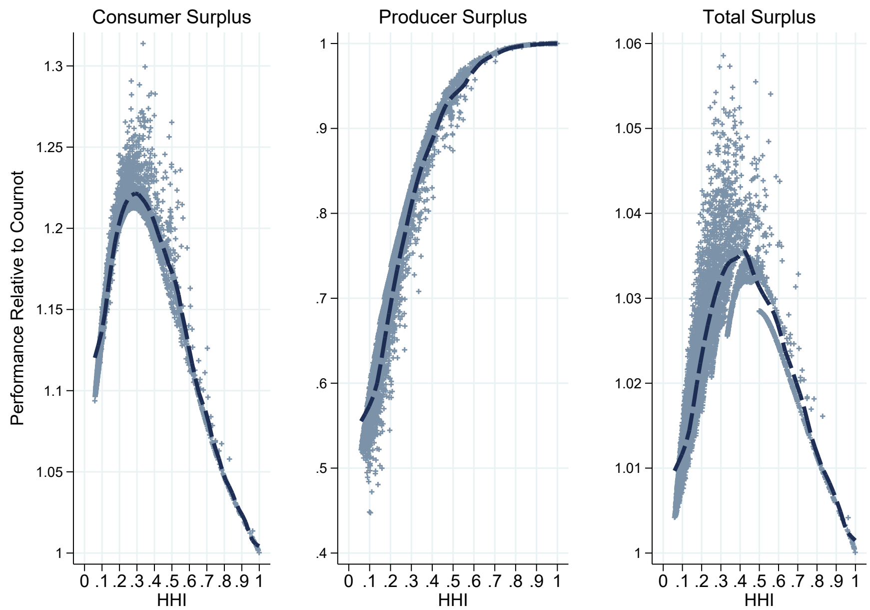

Figure 1: Welfare Statistics with Heterogeneous Capital Stocks

attains a maximum of unity in the monopoly extreme and asymptotically approaches zero as the market approaches perfect competition. The HHI is an appealing statistic due to its well-known theoretical connection to welfare in the baseline Cournot model; it also features prominently in the Merger Guidelines of the U.S. Department of Justice and Federal Trade Commission.[11] In the graphs, each dot represents a single industry, and the lines provide nonparametric fits of the data.

As shown, consumer surplus and total surplus are greater with forward contracting than with Cournot (because all dots exceed unity). Further, consistent with Corollary 1, the impact of a forward market is greatest at intermediate levels of competition.[12] The gain in consumer surplus is maximized at an HHI around 0.30, which corresponds roughly to a

[11]Notice that when all $h_i = 0$, $LI = HHI/\epsilon$.

[12]As there is not a one-to-one correspondence between HHI and consumer or total surplus, we view these results as illustrative. The advantage to using HHI to measure concentration is that it offers a complete ordering of any two capital allocations and hence allows us to plot the results. In the following section, we analyze a more theoretically-robust measure of concentration that does not offer a complete concentration-ordering of allocations.

symmetric three firm oligopoly. The gain in total surplus is maximized at an HHI around 0.40, between the symmetric triopoly and duopoly levels. The figure also shows that forward markets diminish producer surplus, particularly in non-concentrated markets.

It is also possible to compare the welfare statistics that arise with forward contracts to those obtained with perfect competition. This is especially tractable in the special case of symmetric firms and constant marginal costs ($e = 0$). The expressions in (5) can be presented as functions of the common hedge rate:

$$\begin{aligned} CS(h^{(T)}) &= \frac{(a-c)^2}{2}\left(\frac{N}{N+1-h^{(T)}}\right)^2 \\ TS(h^{(T)}) &= \frac{(a-c)^2}{2}\left(\frac{N}{N+1-h^{(T)}} - \frac{1}{2}\left(\frac{N}{N+1-h^{(T)}}\right)^2\right) \end{aligned}$$

The analogous expressions with perfect competition are $CS(1) = TS(1) = \frac{1}{2}(a-c)^2$. Thus, the levels of consumer surplus and of total surplus with forward contracts, *relative to perfect competition*, are free of the demand and cost parameters and depend only on the number of firms and the hedge rate. This holds for any given hedge rate, including the SPE rates $h^{(T)}$.

Figure 2 plots the ratios $CS(h^{(T)})/CS(1)$ and $W(h^{(T)})/W(1)$ for $T = 0, \ldots, 3$. Again, $T = 0$ corresponds to Cournot competition and $h^{(0)} = 0$. The horizontal axis in each panel is the number of firms ($N = 1, ..., 10$) which, under symmetry, is a sufficient statistic for concentration. As shown, consumer surplus and total surplus increase with N under Cournot equilibrium; in the limit as $N \to \infty$ these welfare statistics approach the perfectly competitive level. Incorporating each round of contracting adds curvature to the relationship between surplus and the number of firms, such that surplus approaches the perfectly competitive level faster as N grows large. The "gap" between surplus with Cournot and surplus with forward contracts is largest for intermediate N, again consistent with Proposition 3. Lastly, the figure is highly suggestive that forward markets amplify the impacts of market structure changes (e.g., mergers) on welfare in concentrated markets, but diminish impacts

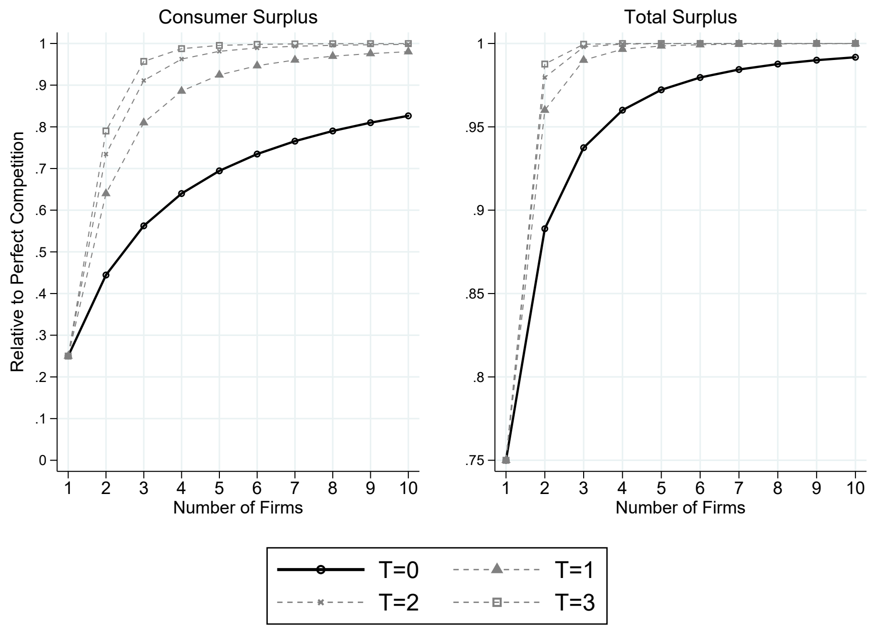

Figure 2: Welfare Statistics with Constant Marginal Costs

otherwise. We provide a more sophisticated analytical treatment of capital transfers in the next section.

4 Mergers

In this section, we analyze the welfare impacts of consolidation, which we treat as the transfer of capital stock from small to large firms. Mergers are inherently consolidating regardless of whether the larger or smaller firm is the acquirer because the merged firm's capital stock will be larger than either of the merging firms'. Our interest extends beyond mergers to partial acquisitions as many real-world applications involve the sale of individual plants. Even when evaluating full mergers, antitrust authorities must often consider whether and to what extent a partial divestiture might offset the anticompetitive harm.

4.1 Effects on consumer surplus

We begin by analyzing the effect of consolidation on consumer surplus. To the extent that antitrust agencies review mergers under a consumer surplus standard, our results should be directly applicable to antitrust policy. Our results derive from an analytic "first-order" approach which we supplement in places with simulations. The analytic approach examines effects of small consolidating transfers, restricting attention to pairwise transfers of capital from any firm 2, say, to any firm 1 whose capital stock is larger.[13] Keeping with the naming convention used in the literature, we refer to firms 1 and 2 as the "inside" firms and all other firms as the "outside" firms. Holding fixed the total capital stock controlled by the inside firms, a consolidation of capital among firms 1 and 2 is a transfer of some amount, dk, such after the transfer, firm 1 has capital stock $k_1 + dk$ while firm 2 has capital stock $k_2 - dk$, leaving the total unchanged. Our analytical approach illuminates the mechanisms underlying our results while avoiding the integer problem inherent in the analysis of full mergers.

Extrapolating to larger transfers such as full mergers involves integrating over these first-order effects. When first-order effects are insufficient to evaluate larger transfers or otherwise are aided by additional illustration, we provide simulations of full mergers. We restrict attention to a single round of forward contracting ($T = 1$) to simplify the mathematics, and remove the corresponding superscripts as appropriate. Because consumer surplus is increasing in total output (from (5)), any transfer of capital that reduces the equilibrium output reduces consumer surplus. Formally, the change in consumer surplus due to a consolidating transfer of capital is,

$$dCS = b \cdot dQ = \frac{a-c}{(1+M)^2} \sum_i d\mu_i.$$

We can deconstruct the output effect into two components, a *structural effect* (SE),

[13] Jaffe and Wyle (2013) and Farrell and Shapiro (1990) employ this approach, though they do not analyze how the merger changes firms' conjectural variations as we do.

which measures the change in output holding each firm's hedge rate fixed, and a *hedging effect* (HE), which measures the incremental change in output due to changes in how the new structure changes firms' conjectural variations. Keeping in mind that $\mu_i = \frac{\beta_i}{1+\beta_i R_i}$ (Lemma 1), we have that,

$$d\mu_i = \begin{cases} \left(\frac{\mu_i}{\beta_i}\right)^2 d\beta_i - \mu_i^2 \cdot dR_i & \text{if } i = 1,2 \\ -\mu_i^2 \cdot dR_i & \text{if } i \neq 1,2 \end{cases}$$

Collecting the $d\beta_i$ terms and the dR_i terms, respectively, the change in consumer surplus is, $dCS = SE + HE$, where,

$$\begin{aligned} SE &\equiv \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 d\beta_1 + \left(\frac{\mu_2}{\beta_2}\right)^2 d\beta_2\right] < 0 \\ HE &\equiv -\frac{a-c}{(1+M)^2}\sum_i \mu_i^2 \cdot dR_i < 0 \end{aligned}$$

This deconstruction allows us to state the following proposition.

Proposition 4 *All consolidating transfers reduce consumer surplus in the presence of a forward market. The loss of consumer surplus due to a consolidating transfer is mitigated if each firm's hedge rate remains fixed at its pre-transfer value.*

That consolidation leads to lower output should not be surprising as the result holds within the baseline model of Cournot competition. What it interesting is that the reduction is output is magnified when firms adjust their hedge rates in response to consolidation as they do in the SPE of the two-stage game. This follows from the fact that $SE, HE < 0$. The strategic effect is negative for the standard reasons: The capital transfer leads the inside firms to reduce output, while outside firms react by expanding their output. The total expansion across all outside firms only partially offsets the output reduction by the inside firms, leading to a net decrease in industry output.[14] The hedging effect is negative

[14]See Farrell and Shapiro (1990) for this result in Cournot oligopoly.

due to how the capital transfer affects firms' conjectures about competitor responses to a change in their contracted quantity. Outside firms anticipate that the inside firms will be less responsive to their contracted quantities on the basis that the inside firms produce less overall. At the same time, the inside firms have less incentive to contract since there is less productive capacity outside their control. This reduces the amount of forward contracting in equilibrium and thereby weakens a constraint on the exercise of market power.

It is natural to ask whether the effect of consolidation is more pronounced within the two-stage game relative to the baseline Cournot game.

Proposition 5 *There exists a capital allocation* $\mathbf{k}$ *such that the reduction in consumer surplus due to consolidation is greater within the SPE of the two-stage model than in Cournot. There exists a capital allocation* $\mathbf{k}'$ *that is less concentrated than* $\mathbf{k}$ *under the transfer principle such that the reduction in consumer surplus due to consolidation is greater in Cournot than in the SPE of the two-stage model.*

Proposition 5 says that the welfare effects of consolidating transfers within the two-stage model are greater than Cournot in industries that are sufficiently concentrated and smaller than Cournot in industries that are unconcentrated. The reason why contracting doesn't always lead to a greater reduction in consumer surplus is that consumer surplus depends on the pre-transaction hedge rate. Within the two-stage model, the effect of a change in structure on each firm's output is proportional to its pre-transaction output. As a result, the structural effect is damped relative to Cournot, substantially so when the industry is fairly unconcentrated. Recall from Section 3.1 that hedge rates decline in concentration. It follows that as the capital stock becomes more concentrated, hedge rates decline and each firm's output in the two-stage game converges to its output in the Cournot game. Proposition 5 establishes what amounts to a threshold level of concentration where the hedging effect exactly offsets the greater structural effect within Cournot.

We revisit the Monte Carlo experiments to illustrate and extend the analyses beyond first-order effects to full mergers.[15] We create data on 9,000 industries evenly split between $N = 2, 3, \ldots, 20$, and calibrate the structural parameters of the model to match randomly-allocated market shares, an average margin of 0.40, and normalizations on price and total output. We then simulate economic outcomes using the obtained structural parameters under the alternative assumption of Cournot competition ($T = 0$). Finally, to simulate mergers, we combine the capital stocks of the first and second firm of each industry and recompute equilibria both with the two-stage model and with Cournot.

Figure 3 summarizes the results. The vertical axis is the loss of consumer surplus in the two-stage model divided by the loss under Cournot; this is greater than unity if forward markets amplify loss. The horizontal axis is the post-merger HHI. Each dot represents a single industry, and the line provides a nonparametric fit of the data. As shown, the relative consumer surplus loss with forward contracts increases in the post-merger HHI, consistent with Proposition 5. The threshold level above which forward contracts tend to amplify consumer surplus loss is around a post-merger HHI of 0.40, roughly between symmetric triopoly and duopoly levels.

4.2 Profitability

It is notoriously difficult to analyze the effect of mergers on firm profitability in models such as ours, even the absence of forward markets (Perry and Porter (1985); Farrell and Shapiro (1990)). Thus, we begin this section with a simple numerical analysis. Revisiting the Monte Carlo exercise described above, we plot the change in the inside firms' profits against the post-merger HHI. Figure 4 shows the results for the two-stage model (Panel A) and Cournot

[15] Because Proposition 5 is a statement about first-order effects, it is theoretically ambiguous whether it extends to large transactions including full mergers. For example, it may be the case that allocation $\mathbf{k}$ is sufficiently non-concentrated that an incremental transfer would reduce consumer surplus more under Cournot but a larger transfer would reduce consumer surplus more in the contracting model.

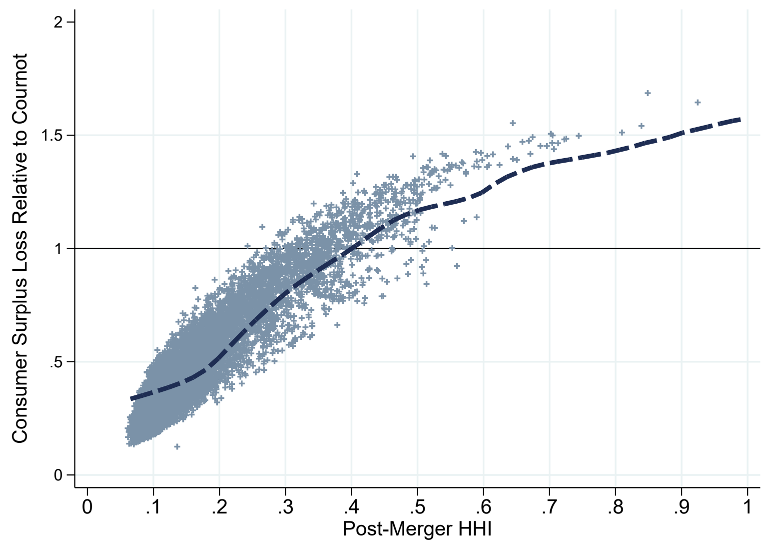

Figure 3: Relative Consumer Surplus Loss with Forward Markets

Notes: The vertical axis provides the percentage change in consumer surplus with forward markets divided by the percentage change without forward markets, given the same parameterization. Values above unity represent the effects of mergers for which forward markets amplify consumer surplus loss. The horizontal axis provides the post-merger HHI. The line provides a nonparametric fit of the data.

(Panel B). The striking result is that *all mergers within the two-stage model are profitable* whereas in Cournot, many are not. We provide the following conjecture:

Conjecture 1 *All mergers are profitable in the two-stage model.*

This may help offer a more complete response to the "merger paradox." Salant, Switzer and Reynolds (1983) examined the incentive to merge within a symmetric model of Cournot competition with constant marginal cost. They find that pairwise mergers are not profitable unless they form a monopoly. It is difficult to explain the prevalence of mergers in light of this result, hence the paradox.[16] Perry and Porter (1985) argue that the failure to explain the profitability of mergers is actually a misconception since the mergers are not well-defined

[16]Deneckere and Davidson (1985) alter the assumption that firms compete on quantity and show that mergers are always profitable when firms offer differentiated products and compete on price. The conflicting result arises because prices are strategic complements. In that case, an increase in the inside firms' prices is met by an increase in the prices of outside firms, hence mergers are profitable. But the assumption that products are differentiated may not be applicable in many settings such as the sale of commodities or wholesale electricity.

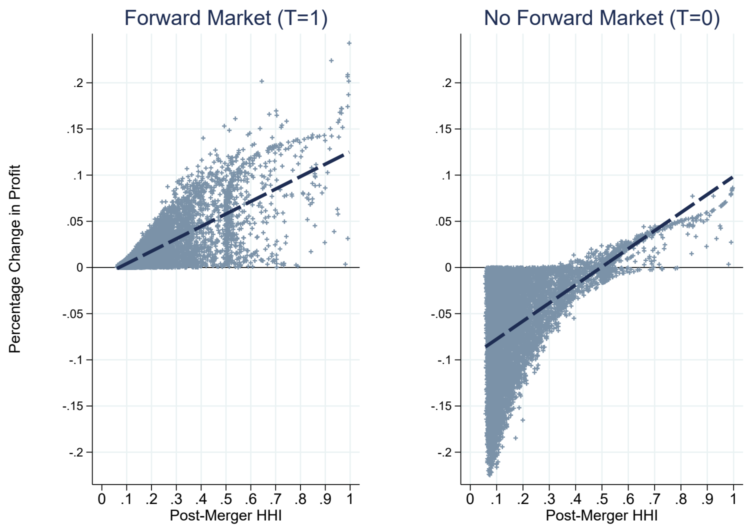

Figure 4: The Effect of Mergers on Producer Surplus

Notes: The figure provides the percentage change in producer surplus captured by the two merging firms for varying levels of post-merger HHI. Also provided are lines of best fit.

conceptually when firms can produce seemingly infinite quantities at a constant marginal cost. They propose a model of capital stocks, the same model we have adopted, and find that smaller mergers can indeed be profitable even when firms compete on quantities. Yet many mergers within their framework are unprofitable. Figure 4 suggests that supplementing Perry and Porter (1985) with a contract market is sufficient for all mergers to be profitable.

As Salant, Switzer and Reynolds (1983) demonstrate, the profitability of a merger depends on the relative strength of two forces. First, the inside firms reduce output, thereby raising the price. Outside firms respond by expanding output which counteracts somewhat the effect of the inside firms' contraction while further reducing the inside firms' share of industry output. When marginal costs are increasing as in Perry and Porter (1985), the third-party response is damped enough that highly concentrating mergers short of mergers to monopoly are profitable. As we will show, the introduction of a forward market increases the price impact of the inside firms' output reduction and further damps the third-party output expansion relative to Cournot. To see how these two forces are impacted by the presence of a forward market, we analyze the first-order effect of a small merger. Our unit of analysis is the reduction in the inside firms' output, dQ_I.[17] The expansion of outsiders' output is denoted, dQ_O.

Suppose that the acquisition of firm 2 leads to a decrease in the insiders' output of dQ_I and an increase in outsiders' output of dQ_O. The change in insiders' profits is,

$$d\pi_I = \{[P + (1 + R_I)\, Q_I P' - C_I'] + [dQ_O/dQ_I - R_I]\, Q_I P' + g/dQ_I\}dQ_I \qquad (6)$$

where: C_I' is the slope of the inside firms' marginal cost function evaluated at the pre-merger output; R_I is the inside firms' period-1 conjecture; and g (> 0) is the cost savings incurred by the inside firms upon rationalizing output across their combined capital assets. Since $dQ_I < 0$, $d\pi_I > 0$ if and only if the term in curly brackets is negative. We consider each of its components in turn.

Because insiders reduce their output in equilibrium, it must be the case that at the pre-merger equilibrium output, its marginal cost exceeds its marginal revenue. From the inside firms' period-1 first-order condition, we see that this is equivalent to $[P + (1 + R_I)\, Q_I P' - C_I'] < 0$. Since the pre-merger output puts the insiders on the downward sloping portion of π_I with respect to Q_I, a small decrease in Q_I increases profit by $-[P + (1 + R_I)\, Q_I P' - C_I']$. Since this term is decreasing in R_I, and since hedge rates decline due to consolidation (an implication of Proposition 4), it must be that this incremental profit is larger than it would be if hedge rates were kept constant or were constrained to be zero in the case of Cournot.

This incremental gain must be weighted against the effect on profit due to output expansion by outsiders, $-[dQ_O/dQ_I - R_I]\, Q_I P'$. The term inside the square brackets is the net output response from outsiders due to the merger. To derive this, consider the solution for firm j's problem in the contract market (where we have assumed $T = 1$),

[17] In the earlier sub-section on consumer surplus, we saw that a small capital transfer leads the inside firms to reduce output, so this change of variables is without loss.

$$P\left(Q\right)+q_j\left(1+R_j\right)P'\left(Q\right)=C_j'\left(q_j\right) \tag{7}$$

Differentiating both sides of (7) with respect to $Q_{-j}=\sum_{k\neq j}q_k$, we obtain,

$$\frac{dq_j}{dQ_{-j}}\equiv r_j=-\frac{\mu_j}{1+\mu_j} \tag{8}$$

This is the firm's reaction function ignoring the intertemporal effects of hedging. From $dq_j=r_jdQ_{-j}$, we have that, $dq_j\left(1+r_j\right)=r_j\left(dq_j+dQ_{-j}\right)=r_jdQ$, or equivalently,

$$dq_j=-\left(\frac{r_j}{1+r_j}\right)dQ=-\mu_jdQ \tag{9}$$

Summing (9) over all $j\in O$, we have that, $dQ_O=-M_{-I}dQ=-M_{-I}\left(dQ_O+dQ_I\right)$, or equivalently,

$$\frac{dQ_0}{dQ_I}=-\frac{M_{-I}}{1+M_{-I}} \tag{10}$$

Expression (10) is the gross change in outsiders' output to a given change in insiders' output, irrespective of intertemporal effects of hedging. However, since some of this response was already internalized by the inside firms pre-merger via Stackleberg considerations, the net impact of the merger is an expansion of $-\left[dQ_O/dQ_I-R_I\right]$.

Notice that from (10), $-dQ_O/dQ_I$ is equivalent to the inside firms' hedge rate in the game with $T=2$ rounds of contracting, $h_I^{(2)}$. It follows that the net output expansion is equal to $h_I^{(2)}-h_I^{(1)}$. In contrast, it is straightforward to show that the output expansion under Cournot is $-B_I/\left(1+B_I\right)$ which is equivalent to $h_I^{(1)}$. Since hedge rates converge to unity as the number of contracting rounds increase, we have that $h_I^{(2)}-h_I^{(1)}<h_I^{(1)}$ so that it is indeed the case that the output expansion is damped by forward trading. Furthermore, as the number of contracting rounds becomes large, the output expansion vanishes entirely.

5 Collusion

We now investigate collusion in the presence of forward markets. We place the model into a standard repeated-game setting with an infinite number of trading periods indexed $t = 0, 1, 2, \ldots$. In each period, firms simultaneously sell output in a spot market and contract for output up to T periods ahead. The discount factor is δ. Following Liski and Montero (2006), which examines the case of duopoly, we impose constant marginal costs and hence symmetry in order to improve the tractability of the incentive compatibility constraints. We advance the literature primarily by considering an arbitrary number of firms, N.

We focus on a particular set of strategies under which firms collectively produce the monopoly output, $Q^m = (a-c)/2$, in each period. Let $f_i^{t,t+\tau}$ denote the quantity contracted by firm i during period t for delivery $\tau = 1, 2, \ldots, T$ periods later. Along the collusive path, firms trade in the forward market according to $f_i^{t,t+1} = xQ^m/N$ and $f_i^{t,t+\tau} = 0$ for all $\tau > 1$ and trade in the spot market according to $q_i^s = (1-x)Q^m/N$. We consider $x \in [-1, 1]$ so that firms can be long ($x < 0$) or short ($x > 0$) in the spot market. The choice of x feeds into the incentive compatibility constraints that we develop below. If any firm deviates from this collusive path, then competition in all subsequent periods reverts to the strategies defined by Proposition 2, albeit adjusted in some periods to account for the impact of the deviation on future spot markets.

Because some fraction, x, of sales are already committed in any given period, the present value of collusion takes the form

$$V^c(\delta, x) = (1-x)\pi^m + \frac{\delta}{1-\delta}\pi^m \tag{11}$$

where $\pi^m = (a-c)^2/4N$ is the per-firm monopoly profit. The value of deviation is more complicated. Suppose the deviation occurs in period t. In the period-t spot market, firm i (the deviating firm) expands production relative to the collusive level. It also signs forward contracts which allow it to obtain a Stackleberg leadership position in spot markets in periods $t+\tau$, for $\tau \in \{1, 2, \ldots, T\}$. In the first of these periods, $\tau = 1$, competitors' contracts are fixed according to the collusive strategy. Competitors choose their output for the period $t+1$ spot market that best respond to firm i's deviation and to forward positions taken under the collusive strategy. For spot markets $\tau \in \{2, \ldots, T\}$ periods ahead, competitors choose forward quantities in periods $t+1$ through $t+\tau-1$ as well as spot quanitities in period $t+\tau$ which best respond to firm i's deviation. In light of Corollary 1, the punishment is more severe in spot markets further ahead. For spot markets $\tau > T$ periods ahead, firm i obtains no Stackleberg leadership position and all firms play according to the equilibrium of Proposition 2. Let $\pi^{d,\tau}$ denote the deviating firm's profit τ periods post-deviation. We have that,

$$\pi^{d,\tau} = \begin{cases} \pi^m \frac{(N+1-x)^2}{4N} & \text{if } \tau = 0 \\ \pi^m \left(1 - \frac{N-1}{2N}x\right)^2 & \text{if } \tau = 1 \\ \pi^m \frac{N}{1+(N-1)\mu^\tau} & \text{if } \tau \in [2, T] \\ \pi^m \frac{4N\mu^T}{(1+M^T)^2} & \text{if } \tau > T \end{cases} \tag{12}$$

where μ^τ and M^T are as defined in Proposition 2.

We now provide the main theoretical result of the section:

Proposition 6 *The aforementioned collusive strategies constitute a SPE if* $\delta \geq \underline{\delta}(x)$*, where for* $x \in [-1, 1]$*,* $\underline{\delta}(x)$ *solves,*

$$\frac{V^c(\delta, x)}{\pi^m} = \frac{\pi^{d,0}}{\pi^m} + \delta\left(\frac{\pi^{d,1}}{\pi^m}\right) + \sum_{\tau=2}^{T} \delta^\tau \left(\frac{\pi^{d,\tau}}{\pi^m}\right) + \frac{\delta^{T+1}}{1-\delta}\left(\frac{\pi^{d,T+1}}{\pi^m}\right) \tag{13}$$

The demand and cost parameters, a and c, cancel in equation (13) and thus do not affect the critical discount rate.

Of particular interest is how the critical discount rate changes with N. To make

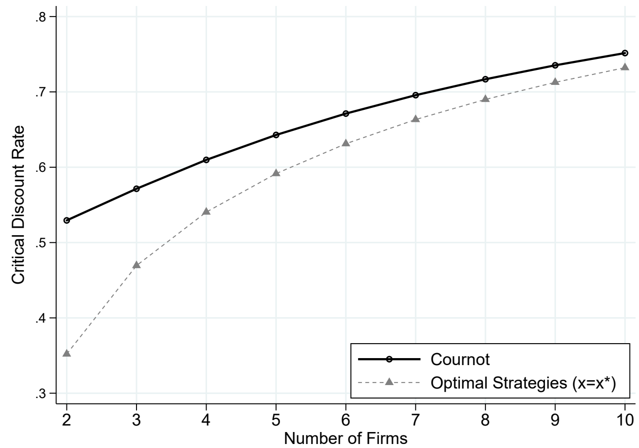

Figure 5: Effect of Forward Markets on Critical Discount Rates

progress, we use numerical techniques to calculate the "optimal" collusive strategy, $x^*(N,T)$, that minimizes the critical discount rate as a function of N and T, and obtain the corresponding critical discount rates. Figure 5 plots results for the case of $T = 1$. As one might expect, the critical discount rate $\underline{\delta}(x^*(N,1))$ increases with N, such that collusion becomes more difficult to sustain. For comparison, we also plot the critical discount rate under Cournot, which also increases with N. It is apparent that (i) forward markets decreases the critical discount rate relative to Cournot; and (ii) this effect is more pronounced for small N. This suggests that it is more likely that, in the presence of a forward market, firms will switch from competition to collusion in response to an increase in concentration. These results are robust to $T > 1$ in all of the numerical specifications we have explored: a large T discourages deviation by making punishment harsher, but encourages deviation by providing a longer period of Stackleberg leadership. The net effect appears to be small.

The relationship shown in Figure 5 derives from the "hedging effect" identified in Section 4, whereby consolidation leads firms to reduce forward sales under the strategies described by Proposition 2, thereby providing an additional boost to profits. Under Cournot, the critical discount rate decreases in concentration because greater concentration causes the incremental gain from deviation relative to cooperation to decline at a greater rate than the

Table 1: Optimal Strategies and Critical Discount Rates

N	2	3	4	5	8	10	15
$x^*(N,1)$	-0.25	-0.07	0.11	0.25	0.51	0.64	0.75
h^*	0.50	0.67	0.75	0.80	0.88	0.90	0.93
$\underline{\delta}(x^*)$	0.35	0.47	0.54	0.59	0.69	0.73	0.80
$\underline{\delta}(h^*)$	0.42	0.50	0.56	0.60	0.69	0.73	0.80

incremental gain of deviation relative to the punishment. Under contracting, the incremental gain from deviation relative to punishment declines at an even lower rate due to the hedging effect, leading to an even larger decline in the critical discount rate under contracting.

That in the presence of a forward market, the critical discount rate is increasing faster in N, is robust to the forward position dictated by the collusive strategy. Suppose that rather than x^*, firms sold a fraction $h^* = (N-1)/N$ of the collusive output in the forward market, where h^* is the hedge rate in the stationary equilibrium (see equation 3). Table 1 shows that this has little impact on the critical discount rate. It is evident that $\underline{\delta}(h^*)$ lies between $\underline{\delta}(x^*)$ and the critical discount rate under Cournot, so that our conclusion is unchanged.

6 Conclusions

We have analyzed consolidation in the presence of a forward market. Our results show that the welfare effects of consolidation are sensitive to the presence of a forward market in important ways. While our model presupposes the existence of a forward market, it is not hard to conceive of forward sales emerging organically. Whenever quantity is the strategic variable and whenever the terms of sale can be revealed to a firm's competitors, a firm will have the strategic incentive to make sales in advance of production. To the extent that such transactions do occur, the applicability of our results may well extend beyond the commodities with established futures markets.

While our results should be relevant for policy makers in the merger review process, we believe an appropriate level of caution should be exercised. The model of capital stocks which we have employed throughout is limiting as it does not reflect firms' actual marginal cost functions. In practice, consolidation may change the shape of firms' marginal cost functions in ways that exacerbate or mitigate harm from mergers.

We have also assumed the strategic variable to be quantity. In wholesale electricity markets, spot prices are determined based on price-quantity schedules submitted by firms. In the supply-function equilibrium model of Klemperer and Meyer (1989), supply functions can be strategic substitutes or complements. Mahenc and Salanie (2004) study strategic complements in the context of differentiated Bertrand spot market competition and find that forward contracting increases spot market prices. We are aware of no studies that analyze the effect of mergers within this context. If consolidation lessens the incentive to contract in advance, then harm from consolidation is mitigated relative to our results.

Finally, we have assumed that all agents have perfect foresight so that the only motive for firms to sell in the contract market is to influence spot market competition. As we do not believe this to be the case in practice, our assumption of perfect foresight was made for the sake of tractability. Allaz (1992) and Hughes and Kao (1997) show that when foresight is imperfect and firms are risk averse, equilibrium hedge rates are higher than in the perfect-foresight case. How hedge rates change in response to a merger in this setting has not been explored to our knowledge. However, it is conceivable that our basic findings would still obtain. Consolidation, by increasing market power, increases the value to the merged firm of withholding output. To the extent that forward contracting even for the sake of hedging risk comes at the expense of exercising market power, mergers may well limit the incentive for firms to forward contract. We leave this issue and the other issues posed in this section to future research.

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Appendices

A Proofs

A.1 Proof of Proposition 1

Fixing the price at a candidate equilibrium value, P, and using the definition of β_i given in the text, we can express equation (1) as,

$$\begin{aligned} q_i &= \left(\frac{k_i}{bk_i+d}\right)(P-c) + \left(\frac{bk_i}{bk_i+d}\right)q_i^0 \\ &= \frac{\beta_i}{b}(P-c) + \beta_i q_i^0 \end{aligned}$$

Using the definitions of B and F^0 from the text, we can express total output as,

$$Q = \sum_i q_i = \frac{B}{b}(P-c) + F^0$$

Substituting the identity $Q = (a-P)/b$ into the left-hand side of the above expression yields

$$\frac{a-P}{b} = \frac{B}{b}(P-c) + F^0$$

It is straightforward to solve the above for the equilibrium value of P, which we then plug into the above expressions for q_i and Q to obtain their equilibrium values.

A.2 Proof of Lemma 1

Consider $t=1$. From the expression of q_i in Proposition 1, we have that,

$$\frac{\partial q_i}{\partial f_i^1} = \frac{\beta_i(1+B_{-i})}{1+B}. \tag{A.1}$$

From the same expression of q_i, we also have that,

$$\frac{\partial q_j}{\partial f_i^1} = -\frac{\beta_i\beta_j}{1+B}.$$

so that

$$\sum_{j\neq i} \frac{\partial q_j}{\partial f_i^1} = -\frac{\beta_i B_{-i}}{1+B}. \tag{A.2}$$

Using (A.1) and (A.2), we have that,

$$R_i^1 \equiv \sum_{j \neq i} \frac{\partial q_j}{\partial f_i^1} / \frac{\partial q_i}{\partial f_i^1} = -\frac{B_{-i}}{1 + B_{-i}}$$

Now consider any $t = \tau > 1$. Fixing price at some candidate equilibrium, P, and using the definition of μ_i^τ from the statement of the lemma, we can express equation (2) as,

$$q_i = \mu_i^\tau \left(\frac{P - c}{b} \right) + \mu_i^\tau \left(1 + R_i^\tau\right) q_i^\tau \tag{A.3}$$

Define the following terms:

$$F^\tau = \sum_i \mu_i^\tau \left(1 + R_i^\tau\right) q_i^\tau, \;\; F_{-i}^\tau = \sum_{j \neq i} \mu_j^\tau \left(1 + R_j^\tau\right) q_j^\tau$$

We can then express total output as,

$$Q = \sum_i q_i = M^\tau \left(\frac{P - c}{b} \right) + F^\tau \tag{A.4}$$

Substituting $Q = (a - P)/b$ into the above yields,

$$\frac{a - P}{b} = M^\tau \left(\frac{P - c}{b} \right) + F^\tau \tag{A.5}$$

It is straightforward to solve the above expression for the equilibrium value of P, which we then plug into (A.3) to obtain,

$$q_i \left(\mathbf{q}^\tau\right) = \left(\frac{a - c}{b} \right) \frac{\mu_i^\tau}{1 + M^\tau} + \frac{\mu_i^\tau}{1 + M^\tau} \left[\left(1 + M_{-i}^\tau\right) \left(1 + R_i^\tau\right) q_i^\tau - F_{-i}^\tau \right] \tag{A.6}$$

Differentiating $q_i \left(\mathbf{q}^\tau\right)$ with respect to the firm's own forward position yields,

$$\frac{\partial q_i \left(\mathbf{q}^\tau\right)}{\partial f_i^\tau} = \frac{\mu_i^\tau \left(1 + R_i^\tau\right)}{1 + M^\tau} \left(1 + M_{-i}^\tau\right) \tag{A.7}$$

Differentiating with respect to another firm's position yields,

$$\frac{\partial q_j \left(\mathbf{q}^\tau\right)}{\partial f_i^\tau} = \frac{\mu_i^\tau \left(1 + R_i^\tau\right)}{1 + M^\tau} \mu^\tau$$

so that,

$$\sum_{j \neq i} \frac{\partial q_j \left(\mathbf{q}^\tau\right)}{\partial f_i^\tau} = \frac{\mu_i^\tau \left(1 + R_i^\tau\right)}{1 + M^\tau} M_{-i}^\tau \tag{A.8}$$

Using (A.7) and (A.8), we have that,

$$R_i^{\tau+1} \equiv \sum_{j \neq i} \frac{\partial q_j}{\partial f_i^{\tau+1}} / \frac{\partial q_i}{\partial f_i^{\tau+1}} = -\frac{M_{-i}^{\tau}}{1 + M_{-i}^{\tau}}$$

A.3 Proof of Proposition 2

Set $\tau = T$ in equation (A.5). By construction, $F^T = 0$ since T is the first period in which forward contracts are bought or sold and F^T has been defined as to reflect sales that occurred prior to period T. Solving (A.5) for P, we have,

$$P = c + \frac{a - c}{1 + M^T} \tag{A.9}$$

Set $\tau = T$ in equation (A.4), where again, $F^T = 0$ by construction. Substituting in expression (A.9) for P in equation (A.4), we have,

$$Q = \left(\frac{a - c}{b}\right) \frac{M^T}{1 + M^T}$$

Finally, set $\tau = T$ in equation (A.6), whereby $q_i^T = F_{-i}^T = 0$. We have,

$$q_i = \left(\frac{a - c}{b}\right) \frac{\mu_i^T}{1 + M^T} \tag{A.10}$$

We now proceed to characterize the firm's forward sales. In equilibrium, it must be that case that for any period $\tau > 1$, $q_i\left(\mathbf{q}^{\tau}\right) = q_i\left(\mathbf{q}^{\tau-1}\right)$. In other words period-$\tau$ behavior cannot cause firm i to deviate from its strategy; if it did, then the strategy was not an equilibrium to begin with. Since the firm's marginal cost in equation (2) is the same regardless of τ, so too is its marginal revenue.

Equating marginal revenue between periods $T - 1$ and T, while using the fact that, $q_i^T = 0$ and $q_i^{T-1} = f_i^T$, we have,

$$\left(q_i - f_i^T\right)\left(1 + R_i^{T-1}\right) = q_i\left(1 + R_i^T\right)$$

It follows that the firm's contracted quantity in period T is,

$$f_i^T = \left(\frac{R_i^{T-1} - R_i^T}{1 + R_i^{T-1}}\right) q_i,$$

where q_i is the equilibrium value from equation (A.10). It's uncommitted output at the beginning of period $T - 1$ is,

$$q_i - f_i^T = \frac{1 + R_i^T}{1 + R_i^{T-1}} \tag{A.11}$$

Continuing in this manner, we equate marginal revenue between periods $T - 1$ and $T - 2$, so that,

$$\left(q_i - f_i^T - f_i^{T-1}\right)\left(1 + R_i^{T-1}\right) = \left(q_i - f_i^T\right)\left(1 + R_i^{T-1}\right)$$

The firm's contracted quantity in period $T-1$ is,

$$f_i^{T-1} = \frac{R_i^{T-2} - R_i^{T-1}}{1 + R_i^{T-2}}\left(q_i - f_i^T\right),$$

where $q_i - f_i^T$ is the value from equation (A.11). Continuing in this manner, the expression for the firm's forward quantities is true by induction.

A.4 Proof of Corollary 1

Let $Q^{(t)}$ denote total output in a game with t rounds of forward contracting. Further, let $M^{(t)} \equiv M^T$ when there are t rounds of forward contracting. To complete the notation, suppose that $M^{(0)} = B$. From Proposition 2, we have that $Q^{(t)} > Q^{(t-1)}$ if and only if $M^{(t)} > M^{(t-1)}$.

We can construct any M^T recursively beginning with R_i^1 as given in Lemma 1. R_i^1 feeds into μ_i^1, which feeds into M_{-i}^1, which feeds into R_i^2 and so on.

Claim 1 *Outside the monopoly case, $R_i^1 \in (-1, 0)$ and $R_i^{t+1} < R_i^t$ regardless of T.*

Proof. Outside the monopoly case, $B_{-i} > 0$ for every i. It is obvious then that $R_i^1 = -B_{-i}/(1 + B_{-i}) \in (-1, 0)$. Suppose by way of induction that $R_i^t > R_i^{t-1}$ regardless of the number of contracting rounds in the game. If $R_i^t > R_i^{t-1}$, then $\mu_i^t > \mu_i^{t-1}$, which implies $M_{-i}^t > M_{-i}^{t-1}$. This implies that,

$$R_i^{t+1} = -\frac{M_{-i}^t}{1 + M_{-i}^t} < -\frac{M_{-i}^{t-1}}{1 + M_{-i}^{t-1}} = R_i^t$$

irrespective of T. ■

$R_i^T > R_i^{T-1}$ implies that $\mu_i^T > \mu_i^{T-1}$. From this we have that, $M^T > M^{T-1}$, where it is evident that $M^t = M^{(t)}$ regardless of the number of contracting rounds in the game. Since output is higher with more round of forward contracting, it is mechanically true that price is lower.

In the monopoly case, $B_{-i} = 0$ for the only producer i with strictly positive capital stock. It follows that $R_i^1 = 0$, which implies $\mu_i^1 = \beta_i$, which implies $M^1 = B$. Continuing in this manner, it is evident that for any t, $M^t = M^{t-1} = \cdots = B$, so that total output and hence price are invariant to the number of contracting rounds.

By Proposition 2, an individual producer's output is greater with $T = 1$ round of forward contracting if and only if,

$$\frac{\mu_i^1}{1 + M^1} > \frac{\beta_i}{1 + B}$$

After manipulating terms, this is equivalent to,

$$\beta_i > \frac{1}{R_i^1}\left(\frac{1+B_{-i}}{1+M_{-i}^1} - 1\right)$$

The right-hand side of the above expression is bounded above zero in all but the monopoly case. Therefore, when there are at least three firms, the right-hand side remains bounded above zero even as $\beta_i \to 0$. It follows that for β_i sufficiently close to zero, the condition fails.

A.5 Proof of Lemma 2

It was established in the proof of Proposition 2 that a producer's marginal revenue is equal across each period. Equating its period-T marginal revenue with its period-0 marginal revenue, we have,

$$q_i\left(1+R_i^T\right) = q_i - q_i^0$$

Rearranging terms, we have that,

$$\frac{q_i^0}{q_i} = \left|R_i^T\right|$$

A.6 Proof of Lemma 3

The solution to the producer's problem in period T is characterized by a modified version of equation (2) in which $\tau = T$ and $q_i^T = 0$ for all i. Rearranging terms, we have,

$$\begin{aligned}\frac{P - C_i'}{P} &= -\frac{q_i}{P}P'(Q)\left(1+R_i^T\right) \\ &= -\frac{Q}{P}P'(Q)\,s_i\left(1-h_i\right) \\ &= \frac{s_i\left(1-h_i\right)}{\epsilon}\end{aligned}$$

where the second line uses the result of Lemma 2 that $h_i = R_i^T$ and uses the substitution, $q_i = s_iQ$. The third line uses the definition of demand elasticity, ϵ. Pre-multiplying by s_i then summing over all i obtains the result.

A.7 Derivation of consumer and total surplus

Consumer surplus is social surplus net of expenditures, so that,

$$CS = \int_0^Q (a - bx - P)\,dx = (a-P)\,Q - \frac{b}{2}Q^2 = \frac{b}{2}Q^2.$$

Total surplus is social surplus net of costs, so that,

$$TS = \int_0^Q (a - bx)\, dx - \sum_i C_i = aQ - \frac{b}{2}Q^2 - \sum_i C_i \tag{A.12}$$

By construction, $C_i = cq_i + q_i^2/2k_i$, which implies that marginal cost is of the form, $C_i' = c + q_i/k_i$. It follows that,

$$\sum_i C_i = \sum_i q_i \left[c + \frac{1}{2}\left(C_i' - c\right)\right] = \frac{1}{2}\left[(c + P)\, Q - \sum_i q_i \left(P - C_i'\right)\right]$$

Substituting $q_i = Qs_i$ and $P = c + bQ/M$ (from Proposition 2), we have,

$$\sum_i C_i = \frac{Q}{2}\left[2c + \frac{b}{M}Q - \sum_i s_i \left(P - C_i'\right)\right] \tag{A.13}$$

Combining (A.12) and (A.13), we have,

$$TS = \frac{Q}{2}\left[2\,(a - c) - \frac{b\,(1 + M)}{M}Q + \sum_i s_i \left(P - C_i'\right)\right] \tag{A.14}$$

Finally, from Proposition 2, $b(1 + M)Q/M = a - c$. Substituting this into (A.14) yields the desired expression.

A.8 Proof of Proposition 3

The proof relies on the following being true: 1) $\rho^{CS} = \rho^{TS} = 1$ in the monopoly case; 2) $\rho^{CS} > 1$ and $\rho^{TS} > 1$ in all allocations of capital other than monopoly; and 3) $\rho^{CS} \to 1$ and $\rho^{TS} \to 1$ in the limit of an infinite sequence of symmetric equalizing transfers. The first of these has already been shown in Corollary 1. We turn now to the second result.

Because output is higher in the presence of a forward market outside the monopoly case (Corollary 1), so too is consumer surplus (i.e., $\rho^{CS} > 1$). The same is not necessarily true with respect to total surplus when firms are asymmetric. There are two cases to consider. If all firms expand output in the presence of a forward market, then since price exceeds marginal cost, total surplus is necessarily higher. Alternatively, if some firms reduce output, Corollary 1 showed that only small firms do so. It follows that since total output is higher in the presence of a forward market, the reduction in output by smaller firms is more than offset by the output expansion of large firms. Since larger firms are more efficient, total surplus is higher.

The third result derives from (2). Rearranging terms in (2), we have that in period T, each firm chooses q_i to solve, $(P - C_i')/P = s_i(1 - h_i)/\epsilon$. Applying an infinite sequence of symmetric equalizing transfers, $s_i \to 0$ so that $P = c$ regardless of h_i.

A.9 Proof of Proposition 4

The proof proceeds in two parts, first showing that the structural effect is negative, then showing that the hedging effect is negative. That both effects are negative is sufficient to show that the transfer reduces consumer surplus. That the hedging effect alone is negative is sufficient to say that the reduction in consumer surplus is mitigated absent the hedging effect.

Lemma 4 $SE \equiv \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 d\beta_1 + \left(\frac{\mu_2}{\beta_2}\right)^2 d\beta_2\right] \leq 0$

Proof. Using,

$$d\beta_1 = be\left(\frac{\beta_1}{bk_1}\right)^2 dk \tag{A.15}$$

and,

$$d\beta_2 = -be\left(\frac{\beta_2}{bk_2}\right)^2 dk = -\left(\frac{\beta_2}{bk_2}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2} d\beta_1 \tag{A.16}$$

SE can be expressed as,

$$SE = \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 - \left(\frac{\mu_2}{\beta_2}\right)^2\left(\frac{\beta_2}{bk_2}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2}\right] d\beta_1$$

Since $d\beta_1 > 0$, it is sufficient to show that the square-bracketed term is nonpositive. This reduces to,

$$\left(\frac{\mu_1}{bk_1}\right)^2 - \left(\frac{\mu_2}{bk_2}\right)^2 \leq 0$$

Using difference-of-squares (i.e. $x^2 - y^2 = (x+y)(x-y)$), it is sufficient that

$$\frac{\mu_1}{bk_1} - \frac{\mu_2}{bk_2} \leq 0,$$

or equivalently,

$$\mu_1 k_2 - \mu_2 k_1 \leq 0$$

By construction, $k_1 \geq k_2$. We can define $\delta \geq 0$ such that $k_2 \equiv k_1 - \delta$. The above inequality simplifies to,

$$(\mu_1 - \mu_2) k_1 - \mu_1 \delta \leq 0 \tag{A.17}$$

Using the identity,

$$\mu_i = \frac{\beta_i}{1 + \beta_i R_i} = \frac{\beta_i (1 + B - \beta_i)}{(1+B)(1-\beta_i) + \beta_i^2} \tag{A.18}$$

we have that,

$$
\begin{aligned}
(\mu_1 - \mu_2) k_1 &= \frac{(1+B)(1+B-\beta_1-\beta_2)(\beta_1-\beta_2) k_1}{[(1+B)(1-\beta_1)+\beta_1^2][(1+B)(1-\beta_2)+\beta_2^2]} \\
&= \frac{(1+B)(1+B-\beta_1-\beta_2)\beta_1(1-\beta_2)\delta}{[(1+B)(1-\beta_1)+\beta_1^2][(1+B)(1-\beta_2)+\beta_2^2]}
\end{aligned}
$$

If $\delta = 0$, then condition (A.17) holds trivially. If $\delta > 0$, condition (A.17) reduces to, $1 + B > \beta_1$, which is true by construction. ■

Lemma 5 $HE \equiv -\frac{a-c}{(1+M)^2} \sum_i \mu_i^2 dR_i \leq 0$

Proof. We have that,

$$
dR_i = -\frac{dB_{-i}}{(1+B_{-i})^2},
$$

where,

$$
dB_{-i} = \begin{cases} d\beta_2 & \text{if } i = 1 \\ d\beta_1 & \text{if } i = 2 \\ d\beta_1 + d\beta_2 & \text{if } i > 2 \end{cases}
$$

It follows that,

$$
\begin{aligned}
HE \left(\frac{a-c}{(1+M)^2} \right)^{-1} = [d\beta_1 + d\beta_2] \sum_{j \neq 1,2} \left(\frac{\mu_j}{1+B_{-j}} \right)^2 \\
+ \left[\left(\frac{\mu_1}{1+B_{-1}} \right)^2 d\beta_2 + \left(\frac{\mu_2}{1+B_{-2}} \right)^2 d\beta_1 \right]
\end{aligned}
$$

It suffices to show that each of the square-bracketed terms are negative. From equations (A.15) and (A.16) we have that,

$$
\begin{aligned}
d\beta_1 + d\beta_2 &= \left[1 - \left(\frac{\beta_2}{bk_2} \right)^2 \left(\frac{\beta_1}{bk_1} \right)^{-2} \right] d\beta_1 \\
&= \left[\left(\frac{\beta_1}{bk_1} \right)^2 - \left(\frac{\beta_2}{bk_2} \right)^2 \right] \left(\frac{\beta_1}{bk_1} \right)^{-2} d\beta_1 \\
&= \left[\frac{\beta_1}{bk_1} - \frac{\beta_2}{bk_2} \right] \left(\frac{\beta_1}{bk_1} + \frac{\beta_2}{bk_2} \right) \left(\frac{\beta_1}{bk_1} \right)^{-2} d\beta_1 \\
&= -\left(\frac{\beta_2 \delta}{bk_1 k_2} \right) \left(\frac{\beta_1}{bk_1} + \frac{\beta_2}{bk_2} \right) \left(\frac{\beta_1}{bk_1} \right)^{-2} d\beta_1 \\
&\leq 0
\end{aligned}
$$

This term is proportional to the reduction in consumer surplus in the baseline Cournot model. In the current setting, because the transfer will lead to a reduction in output among the inside firms, they will be less responsive to forward sales of outside firms.

Finally, we have that,

$$\begin{aligned}
&\left(\frac{\mu_1}{1+B_{-1}}\right)^2 d\beta_2 + \left(\frac{\mu_2}{1+B_{-2}}\right)^2 d\beta_1 \\
&\qquad = \left[\left(\frac{\mu_2}{1+B_{-2}}\right)^2 \left(\frac{\beta_2}{bk_2}\right)^{-2} - \left(\frac{\mu_1}{1+B_{-1}}\right)^2 \left(\frac{\beta_1}{bk_1}\right)^{-2}\right] \left(\frac{\beta_2}{bk_2}\right)^2 d\beta_1
\end{aligned}$$

Due to difference-of-squares, it is sufficient that,

$$\left(\frac{\mu_2}{1+B_{-2}}\right) \left(\frac{\beta_2}{bk_2}\right)^{-1} - \left(\frac{\mu_1}{1+B_{-1}}\right) \left(\frac{\beta_1}{bk_1}\right)^{-1} \leq 0$$

Using equation (A.18), this is equivalent to,

$$\left[(1+B)(1-\beta_1) + \beta_1^2\right] k_2 + \left[(1+B)(1-\beta_2) + \beta_2^2\right] k_1 \leq 0$$

Using the identity, $k_2 = k_1 - \delta$, this reduces to,

$$-(1+B-\beta_1-\beta_2)\,\beta_1(1-\beta_2)\,\delta - \left[(1+B)(1-\beta_1) + \beta_1^2\right]\delta \leq 0,$$

which is true by construction. ■

From Lemmas 4-5, we have that $dCS = SE + HE < 0$, which establishes the first argument of the proposition. The second argument is that $HE < 0$ which is shown by Lemma 5.

A.10 Proof of Proposition 5

In Cournot, the change in consumer surplus due to a consolidating transfer is,

$$dCS^0 = \frac{a-c}{(1+B)^2}(d\beta_1 + d\beta_2)$$

Lemma 6 $dCS^0 \leq SE^{CS} \leq 0$. *The first inequality is strict in all but the monopoly case. The second inequality is strict as long as* $k_1 > k_2$.

Proof. It is sufficient to show that in all but the monopoly case, $\left(\frac{\mu_1}{\beta_1}\right)^2 > \left(\frac{\mu_2}{\beta_2}\right)^2$. Using equation (A.18), this expression reduces to

$$(1+B)(B-\beta_1-\beta_2) + \beta_1\beta_2 > 0 \tag{A.19}$$

which is true by construction. In the monopoly case, $\mu_i = \beta_i$ for all i, so that $dCS^0 = SE$. ■

From Lemma 6, the loss of consumer surplus is larger under Cournot if the hedging effect is sufficiently small. Consider that in a highly unconcentrated industry, $h_i \simeq 1$ for every i. It follows that in such an industry, a small change in industry structure will have a negligible impact on each firm's hedge rate so that $HE \simeq 0$. It follows that there exists a highly unconcentrated industry structure such that $dCS^0 < SE + HE$.

We now show that the inequality is flipped in highly concentrated industries. Consider the limiting structure as all capital is consolidated in firm 1. Let κ denote the fraction of industry capital held by firm 1.

Lemma 7 $\lim_{\kappa \to 1} SE = \lim_{\kappa \to 1} \Delta CS^0 < 0$.

Proof. In the limit as $\kappa \to 1$, $\beta_j \to 0$ for all $j \neq 1$ so that $B \to \beta_1$. From (A.18), we have that,

$$\lim_{\kappa \to 1} \left(\frac{\mu_1}{\beta_1} \right) = \frac{1}{(1+B)(1-B) + B^2} = 1$$

and

$$\lim_{\kappa \to 1} \left(\frac{\mu_2}{\beta_2} \right) = \frac{1+B}{1+B} = 1.$$

It follows that,

$$\begin{aligned} \lim_{\kappa \to 1} SE = \lim_{\kappa \to 1} dCS^0 &= \lim_{\kappa \to 1} (d\beta_1 + d\beta_2) \\ &= \left(\frac{1}{(bk_1 + e)^2} - \frac{1}{e^2} \right) be \cdot dk < 0 \end{aligned}$$

■

Meanwhile, since $\lim_{\kappa \to 1} HE = -\frac{bB}{e} \cdot dk < 0$, it follows that there exist highly concentrated indsutry structures such that $dCS^0 > SE + HE$.

A.11 Proof of Proposition 6

Let $V^d(\delta, x)$ denote the present value of the most profitable deviation. It follows that the collusive strategy constitutes a SPE if $V^c(\delta, x) \geq V^d(\delta, x)$. In what follows, we derive the profit terms in expression (12).

$\tau = \mathbf{0}$: Prior to the opening of the spot market in period t (the period in which deviation takes place), each firm has a forward position of xQ^m/N from contracts signed in period $t-1$ under the collusive strategy. Firm i's spot-market deviation solves,

$$\pi^{d,0} = max \left(a - xQ^m - \frac{N-1}{N}(1-x)Q^m - q - c \right) q$$

Because the monopoly output is $Q^m = (a-c)/2$, the deviation output is,

$$q^{d,0} = \frac{(a-c)^2}{4N}(N+1-x)$$

It follows that,

$$\begin{aligned}\pi^{d,0} &= \frac{(a-c)^2}{4N}\frac{(N+1-x)^2}{4N} \\ &= \pi^m \frac{(N+1-x)^2}{4N}\end{aligned} \tag{A.20}$$

which denotes the profit from production in period t.

$\tau = \mathbf{1}$: Under the collusive strategy, the quantity traded by all firms $j \neq i$ in period t for production to be delivered in period $t+1$ is $f^{t,t+1} = xQ^m/N$. In determining the optimal deviation in the market for one-period forward quantity, firm i takes into account that rival firms will detect deviations in period t and will begin the punishment phase in the period $t+1$ spot market. Using Proposition 1, firm i's output and the spot-market price in period $t+1$ given that firm i deviates to forward quantity $f^{d,1}$ are,

$$\begin{aligned}q^{d,1} &= \frac{a-c+Nf^{d,1}-(N-1)\,f^{t,t+1}}{1+N} \\ P^{d,1} &= c+\frac{a-c+Nf^{d,1}-(N-1)\,f^{t,t+1}}{1+N}\end{aligned}$$

In period t, firm i chooses $f^{d,1}$ to solve,

$$\pi^{d,1} = max\left(P^{d,1}-c\right)q^{d,1}$$

The optimal deviation satisfies,

$$f^{d,1} = \frac{(a-c)\,(N-1)\,[2N-(N-1)\,x]}{4N^2}$$

It follows that,

$$\begin{aligned}\pi^{d,1} &= \frac{(a-c)^2}{4N}\left(1-\frac{(N-1)\,x}{2N}\right)^2 \\ &= \pi^m\left(1-\frac{(N-1)\,x}{2N}\right)^2\end{aligned} \tag{A.21}$$

$\tau \in \{\mathbf{2},\ldots,\mathbf{T}\}$: Suppose firm i's deviation involves trading $f^{d,\tau}$ in period t for production to be delivered τ periods ahead. After the deviation is detected, there are $\tau-1$ forward openings and one spot opening in which firm i can be punished. We need then to solve for a Stackleberg equilibrium in which firm i chooses $f^{d,\tau}$ followed by τ trading rounds in which all players play stationary SPE strategies. To do this, we follow the proof of Proposition 2 while requiring $F^{t'} = 0$ for all $t' > \tau$ and $F^{\tau} = \mu^{\tau}\left(1+R^{\tau}\right)f^{d,\tau}$. We have that firm i's output and the spot price in period $t+\tau$ are,

$$\begin{aligned} q^{d,\tau} &= \frac{\mu^{\tau}\left(a-c\right)+\left[1+\left(N-1\right)\mu^{\tau}\right]F^{\tau}}{1+M^{\tau}} \\ P^{d,1} &= c+\frac{a-c-F^{\tau}}{1+M^{\tau}} \end{aligned}$$

The optimal choice of $f^{d,\tau}$ satisfies,

$$\pi^{d,\tau}=max\left(P^{d,\tau}-c\right)q^{d,\tau}$$

The solution requires that,

$$F^{\tau}=\left(a-c\right)\left(\frac{1+\left(N-2\right)\mu^{\tau}}{2\left[1+\left(N-1\right)\mu^{\tau}\right]}\right)$$

It follows that,

$$\begin{aligned} \pi^{d,\tau} &= \frac{\left(a-c\right)^{2}}{4N}\frac{N}{1+\left(N-1\right)\mu^{\tau}} \\ &= \pi^{m}\frac{N}{1+\left(N-1\right)\mu^{\tau}} \end{aligned} \tag{A.22}$$

$\tau > \mathbf{T}$: For spot markets more than T periods ahead, firm i gains no Stackleberg advantage, so price and quantity are derived from the symmetric stationary SPE derived in Proposition 2. It follows that,

$$\begin{aligned} \pi^{d,T+1} &= \frac{\left(a-c\right)^{2}}{4N}\frac{4N\mu^{T}}{\left(1+M^{T}\right)^{2}} \\ &= \pi^{m}\frac{4N\mu^{T}}{\left(1+M^{T}\right)^{2}} \end{aligned} \tag{A.23}$$

Using (A.20) - (A.23), we have that,

$$V^{d}\left(\delta,x\right)=\pi^{d,0}+\delta\pi^{d,1}+\sum_{\tau=2}^{T}\delta^{\tau}\pi^{d,\tau}+\frac{\delta^{T+1}}{1-\delta}\pi^{d,T+1}$$

The result is immediate from the definition of $\underline{\delta}(x)$.