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Simulating Mergers in a Vertical Supply Chain with Bargaining*

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Abstract

We model a two-level supply chain where Nash bargaining occurs upstream, while firms compete in a differentiated products logit setting downstream. The parameters of this model can be calibrated with a discrete set of data on prices, margins, and market shares. Using a series of numerical experiments, we illustrate how the model can simulate the outcome of both horizontal and vertical mergers. In addition, we extend the framework to allow for downstream competition via a second score auction.

Keywords: bargaining models; merger simulation; vertical markets

JEL classification: L13; L40; L41

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1 Introduction

The use of merger simulations has become increasingly common in antitrust analysis. Such simulations have featured prominently in a diverse set of cases, covering industries ranging from tax software (*US v. H&R Block, Inc., et al.*), to foodservice distribution (*FTC and Plaintiff States v. Sysco Corporation, et al.*), and health insurance (*US and Plaintiff States v. Anthem, Inc., et al.*).[1] This trend echoes the emphasis on structural models in the empirical industrial organization literature, where counterfactual simulations are the norm.

However, despite the widespread use of merger simulations, the vast majority of these models focus on only one market at a time, ignoring interactions with potentially related upstream input or downstream output markets. This can be a serious shortcoming, as most products in today's economy are produced via long and sometimes complicated supply chains. The literature on bargaining models in vertical contexts suggests that changes in market power at one level of such a chain can affect outcomes both upstream and downstream.[2] In order to quantify these effects, a more complete model of production interactions is required.

In this paper, we build a framework that addresses these complexities while remaining simple enough to calibrate with a limited set of data. Specifically, we model downstream competition using the familiar differentiated products logit framework, as in Werden and Froeb (1994), and embed it in an upstream Nash bargaining model. The structure is similar to that in Draganska, Klapper, and Villas-Boas (2010), who model competition between multiple retailers selling in a downstream random coefficients logit demand system, while also bargaining with a series of upstream wholesalers. Our main departure from that model is the removal of the random coefficients, which allows our parameters to be identified with limited data. In addition, we extend our model to another downstream specification, the second score auction framework of Miller (2014). Whereas the Bertrand logit model is often used to study retail markets in which customers are price takers, the auction model is better suited for business-to-business transactions, where the buyer collects quotes from specialized suppliers.[3]

[1]See the *H&R Block* opinion at pages 38-39, the *Sysco* opinion at pages 89-92, and the *Anthem* district-level opinion at pages 70-71 and 139-140.

[2]Ho and Lee (2017), for example, discuss how a change in competition between health insurers can affect the fees they negotiate with hospitals upstream.

[3]Such purchasing behaviors are sometimes referred to as "request for proposal" (RFP) sales. Miller (2014) argues that a second score auction model is appropriate for many business-to-business markets. Miller applies this model to the merger between Bazaarvoice and PowerReviews, two companies that provided ratings and reviews software for e-commerce websites.

Using this model, we show how it can predict the effects of mergers. The parameters of the model can be calibrated using the types of data typically seen in merger investigations, such as market shares, prices, and margins. Once it has been calibrated, the model can simulate the effects of horizontal mergers both upstream and downstream, plus vertical mergers between upstream and downstream firms. In a series of numerical experiments, we show how the model behaves in a wide variety of competitive environments.

Our work is related to a large literature on merger simulation, including the aforementioned Werden and Froeb (1994) and Miller (2014) papers. For a review of this topic, see Whinston (2007) and Werden and Froeb (2008). We combine the simulation methods used in these papers with the literature on bargaining in order to build a model of a vertical supply chain. Bargaining models have already proven useful in analyzing a number of vertical situations, including retailer-wholesaler relationships (Draganska, Klapper, and Villas-Boas (2010)), hospital-insurer contracting (Ho and Lee (2017)), and video content owner-distributor negotiations (Crawford and Yurukoglu (2012) and Crawford, Lee, Whinston, and Yurukoglu (2017)).[4] Much of this literature draws upon the bargaining setup pioneered by Horn and Wolinsky (1988), who construct an equilibrium in a setting with multiple, simultaneous bilateral negotiations between firms. We follow these papers in using the same equilibrium concept. Although this equilibrium restricts the manner in which various negotiations interact with each other, it has the benefit of greatly simplifying the specification of the model. That simplicity is important when it comes to calibrating and simulating the model in real time.

Given that empirical work on bargaining is an active and developing area of research, our paper contributes to a greater understanding of how these models behave in a wide range of scenarios. Using a framework that is similar to the setup seen in much of the recent empirical industrial organization literature, we show the types of results that are possible as the number of firms and their relative bargaining power varies. We are one of the first papers to do such an analysis in a systematic manner.

We find that our model is flexible enough to incorporate many of the effects frequently emphasized in the literature on mergers. We see that both upstream and downstream horizontal mergers can produce harm from reduced competition between substitutes. In the case of a downstream merger, this effect is balanced against the potential for a shift in bargaining

[4]Similar models also appear in Grennan (2013) and Gowrisankaran, Nevo, and Town (2015). However, those papers lack a strong vertical component, as the downstream model is primarily a function of patient or doctor choices, rather than the actions of a price-setting firm.

leverage toward retailers, which can decrease input costs. With vertical mergers, the model incorporates efficiencies due to the elimination of double marginalization, along with possibly offsetting incentives to disadvantage rival firms by raising their costs. Thus, our merger simulation model has the potential to be a useful tool in a number of different contexts.

The paper proceeds as follows. In Section 2 we describe the model, focusing on the downstream Bertrand logit case so as to fix ideas. Section 3 shows how this model can be calibrated and used to simulate horizontal and vertical mergers. We extend the model in Section 4 to cover downstream auction competition. In Section 5 we provide results from a series of numerical experiments, which highlight the breadth of market scenarios the model can cover. Section 6 concludes.

2 Theoretical Framework

We begin by presenting the baseline version of our model, featuring downstream Bertrand logit competition and upstream Nash bargaining. After building intuition via this base case, we later extend the model to accommodate auctions in Section 4.

We label the downstream firms "retailers" and their upstream counterparts "wholesalers" in order to distinguish them. However, our model is not limited to retail settings. Rather, it can be adapted to a variety of vertical supply relationships where bargaining is a key feature.

2.1 Downstream Model

Let there be a set of consumers indexed by i who can choose to buy a single product sold by a single retailer. Retailers, indexed by r, source their merchandise from wholesalers indexed by w. Each wholesaler offers only one product (meaning the product and wholesaler indices are synonymous), but a retailer can purchase from multiple wholesalers.[5] The set of all retailers is denoted by $\mathbb{R} = \{1, \ldots, |\mathbb{R}|\}$, and the set of all wholesalers is denoted by $\mathbb{W} = \{1, \ldots, |\mathbb{W}|\}$. The set $\mathbb{W}$ is divided into $|\mathbb{R}|$ potentially overlapping subsets, each labeled $\mathbb{W}^r$, to indicate which wholesalers' products are carried by which retailers. In turn, the set of retailers $\mathbb{R}$ is divided into $|\mathbb{W}|$ potentially overlapping subsets, each labeled $\mathbb{R}^w$, which indicate the retailers that carry the product sold by each wholesaler.

[5] Although we restrict our attention to single-product upstream firms for expositional simplicity, the model can be extended to include multiproduct wholesalers. That case is discussed in more detail by Draganska, Klapper, and Villas-Boas (2010).

We assume that consumers choose which product to buy according to the familiar multinomial logit discrete choice model. The indirect utility function for consumer i purchasing from retailer r the product owned by wholesaler w has the form,

$$u_{irw} = \delta_{rw} - \alpha p_{rw} + \epsilon_{irw}. \tag{1}$$

The parameter α measures consumer sensitivity to the retail price, denoted by p_{rw}. The δ_{rw} is a demand shifter that captures average consumer tastes for the non-price aspects of product w when purchased at retailer r. The final term, ϵ_{irw}, is an independent and identically distributed Type I extreme value error with a scale parameter of 1. We normalize the utility of the outside good to be $u_{i00} = \epsilon_{i00}$. Integrating over the error term gives the market share among all available product-retailer combinations,

$$s_{rw} = \frac{\exp(\delta_{rw} - \alpha p_{rw})}{1 + \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} \exp(\delta_{tx} - \alpha p_{tx})}, \tag{2}$$

for product w sold by retailer r.

We assume that retailers simultaneously choose prices in Nash-Bertrand competition in order to maximize profits. The retailer's profit function takes the form

$$\pi^r = \sum_{w \in \mathbb{W}^r} [p_{rw} - p^W_{rw} - c^R_{rw}] s_{rw} M, \tag{3}$$

where p^W_{rw} is the unit fee charged by wholesaler w to retailer r, c^R_{rw} captures any additional marginal costs borne by the retailer, and M is the market size. The resulting first order condition for the price p_{rw} takes the typical form,

$$\sum_{x \in \mathbb{W}^r} [p_{rx} - p^W_{rx} - c^R_{rx}] \frac{\partial s_{rx}}{\partial p_{rw}} + s_{rw} = 0. \tag{4}$$

The series of first order conditions for each of the downstream prices together form a system of equations that relates retail margins to market shares. These equations can be solved for the equilibrium outcome.

2.2 Upstream Model

Each retailer must procure its products from wholesalers. We characterize the profits of wholesaler w as

$$\pi^w = \sum_{r \in \mathbb{R}^w} [p_{rw}^W - c_{rw}^W] s_{rw} M, \tag{5}$$

where c_{rw}^W is the marginal cost borne by the wholesaler, and p_{rw}^W is the wholesale price charged for this product to retailer r. The level of this price is determined via a bilateral negotiation between wholesaler w and retailer r.

Specifically, we assume that bargaining over the price p_{rw}^W is characterized by the following maximization problem:

$$\max_{p_{rw}^W} (\pi^r - d^r(\mathbb{W}^r \setminus \{w\}))^{\lambda_{rw}} (\pi^w - d^w(\mathbb{R}^w \setminus \{r\}))^{1-\lambda_{rw}}, \tag{6}$$

where $d^r(\mathbb{W}^r \setminus \{w\})$ is the disagreement payoff for the retailer and $d^w(\mathbb{R}^w \setminus \{r\})$ is the disagreement payoff for the wholesaler. The λ_{rw} measures the bargaining power of the retailer relative to the wholesaler. In words, the wholesale price is chosen to maximize the Nash product of two terms. The first term is the difference between the profits of the retailer when it offers wholesaler w's product versus when it does not. The second term is the difference between the profits of the wholesaler when it sells to this retailer versus when it does not. The disagreement payoffs are sometimes referred to as the retailer's and wholesaler's outside options. The first order condition of this problem (after taking the natural log of the maximand and rearranging) is

$$\begin{aligned} &\lambda_{rw}[\pi^w - d^w(\mathbb{R}^w \setminus \{r\})] \left(\frac{\partial \pi^r}{\partial p_{rw}^W} - \frac{\partial d^r(\mathbb{W}^r \setminus \{w\})}{\partial p_{rw}^W} \right) + \\ &(1-\lambda_{rw})[\pi^r - d^r(\mathbb{W}^r \setminus \{w\})] \left(\frac{\partial \pi^w}{\partial p_{rw}^W} - \frac{\partial d^w(\mathbb{R}^w \setminus \{r\})}{\partial p_{rw}^W} \right) = 0, \end{aligned} \tag{7}$$

which characterizes a system of equations that determines equilibrium wholesale prices.

The disagreement payoff for the retailer is

$$d^r(\mathbb{W}^r \setminus \{w\}) = \sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] s_{rx}(\mathbb{W}^r \setminus \{w\}) M. \tag{8}$$

The market share $s_{rx}(\mathbb{W}^r \setminus \{w\})$ is computed in the case where retailer r does not offer wholesaler w's product.[6] The disagreement payoff of the wholesaler when it does not offer its product to retailer r is

$$d^w(\mathbb{R}^w \setminus \{r\}) = \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] s_{tw}(\mathbb{W}^r \setminus \{w\}) M. \tag{9}$$

From these equations, we see that both the retailer's and the wholesaler's outside options exhibit forms of "recapture." That is, when the two firms fail to come to an agreement, the retailer can recoup some of its lost sales if customers substitute to other products instead of w, but do not change which retail outlet they visit. Meanwhile, the wholesaler can regain some of its lost sales if customers stay with the same product but switch to other retailers. Thus, customer substitution patterns dictate the strength of each outside option. In so far as one or the other firm has a better outside option, that increases its relative bargaining leverage.

The bargaining setup as detailed above involves a separate negotiation for each wholesaler-retailer pair. However, the payoffs from the outcome of one negotiation are clearly related to those from all other negotiations due to competition in the downstream market. In order to simplify the multilateral complexities this situation raises, we make two assumptions,

1. *Simultaneous negotiations*: when bargaining over a single input price, the wholesaler and retailer act as if all other input price negotiations are taking place simultaneously. Thus, all other wholesale prices are treated as fixed.

2. *Simultaneous downstream pricing*: when bargaining over a single input price, the wholesaler and retailer act as if downstream prices are being set simultaneously. Therefore, all retail prices are treated as fixed.

The benefit of both of these assumptions is that they lead to a tractable solution to the series

[6] That is, $s_{rx}(\mathbb{W}^r \setminus \{w\})$ is calculated as in expression (2), but removing the term $\exp(\delta_{rw} - \alpha p_{rw})$ from the denominator. Note that implicitly downstream prices and wholesale prices besides p^W_{rw} are treated as fixed in the disagreement payoff. We discuss this assumption in more detail later.

of first order conditions characterized by equation (7). We discuss each of these assumptions in turn.

The *simultaneous negotiations* assumption was developed by Horn and Wolinsky (1988) in order to study situations with multiple firms engaged in bilateral contracting, where the outcome of one negotiation affects the payoffs from other contracts. This results in a "contract equilibrium" as seen in Crémer and Riordan (1987). When firms in one bilateral negotiation treat all other contracts as fixed, this means that the terms of these other agreements are viewed as unchanged even if one negotiation breaks down. Therefore, this simplifies the first order condition in equation (7) by removing the partial derivatives of the outside options, since $\partial d^w / \partial p^W_{rw} = \partial d^r / \partial p^W_{rw} = 0$. This assumption is admittedly restrictive, as it implies that a firm that is party to multiple contracts treats each separately. However, such simplification is important in our setting, where we are calibrating our model with limited data. This assumption has also proven important in maintaining tractability even in environments where more data are available, as seen in Crawford and Yurukoglu (2012), Grennan (2013), and Gowrisankaran, Nevo, and Town (2015), among others.

The *simultaneous downstream pricing* assumption is common in the vertical bargaining literature, appearing in, for example, Draganska, Klapper, and Villas-Boas (2010), Ho and Lee (2017), and Crawford, Lee, Whinston, and Yurukoglu (2017). If the firms engaged in bilateral bargaining assume that downstream prices are being set at the same time as upstream prices, then these firms will view downstream prices as fixed. This means that the partial derivatives of profits are treated as if $\partial \pi^w / \partial p^W_{rw} = -\partial \pi^r / \partial p^W_{rw} = s_{rw} M$, which greatly simplifies the upstream first order conditions. Although this assumption is strong, it has some appeal in settings where upstream firms lack an obvious first-mover advantage in pricing. It has been applied in situations as varied as hospital-insurer contracting (Ho and Lee (2017)) to coffee manufacturer-grocery store negotiations (Draganska, Klapper, and Villas-Boas (2010)).[7]

An alternative assumption would be to model upstream fee negotiations as taking place before downstream prices are chosen. In such a sequential framework, wholesalers could strategically raise their prices, perhaps beyond the optimal level indicated by the simultaneous solution, in order to encourage retailers to increase their prices. In the simultaneous setup, upstream firms have no incentive to pursue such a strategy, since downstream firms

[7]Note that, although this assumption limits the way in which upstream and downstream prices interact, retail prices still affect wholesale fees in equilibrium. When bargaining upstream, firms still take into account how downstream prices will be set via the first order condition in equation (4).

are unable to adjust in response. Given that downstream firms often cannot immediately adjust their prices in many real world markets, the previous literature has argued that the downstream simultaneity assumption is appropriate. Furthermore, as discussed by Draganska, Klapper, and Villas-Boas (2010), relaxing this assumption creates a tension with the assumption that all upstream negotiations are happening simultaneously and can therefore be treated separately. Once wholesalers have the ability to affect downstream prices, this naturally allows them to affect the distribution of sales between different retailers, which may, in turn, affect the outcomes for other upstream negotiations.

Under these assumptions, the bargaining first order condition simplifies to

$$\pi^w - d^w(\mathbb{R}^w \setminus \{r\}) = \frac{1-\lambda_{rw}}{\lambda_{rw}}(\pi^r - d^r(\mathbb{W}^r \setminus \{w\})).$$

Define the following:

$$\Delta s_{tx}(\mathbb{W}^r \setminus \{w\}) = s_{tx}(\mathbb{W}^r \setminus \{w\}) - s_{tx}, \tag{10}$$

which is the difference in the share of good x sold by retailer t when good w is not offered by retailer r versus when good w is offered by retailer r. Then substituting into the first order condition gives

$$\begin{aligned} &[p^W_{rw} - c^W_{rw}]s_{rw} - \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}]\Delta s_{tw}(\mathbb{W}^r \setminus \{w\}) = \\ &\frac{1-\lambda_{rw}}{\lambda_{rw}} \left([p_{rw} - p^W_{rw} - c^R_{rw}]s_{rw} - \sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}]\Delta s_{rx}(\mathbb{W}^r \setminus \{w\}) \right). \end{aligned} \tag{11}$$

Equation (11) characterizes a system of first order conditions for upstream prices that relates wholesale and retail margins to market shares. Together with the analogous conditions for the downstream problem (appearing in equation (4)), this system can be solved for the equilibrium outcome.

3 Merger Simulation

We now demonstrate how mergers, both horizontal and vertical, can be analyzed within this framework. We start by showing how the model parameters can be identified, and then discuss how mergers affect the firms' optimization problems. In what follows, we ignore the presence of efficiencies that cause marginal costs, c^R_{rw} and c^W_{rw}, to decrease. However, incorporating such efficiencies can be done immediately by adjusting those costs inside the first order conditions we derive.

3.1 Identification

We begin by explaining how one can calibrate the parameters of the downstream model using data on margins, prices, and market shares. Assume that the researcher observes market shares $\{s_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$, retail prices $\{p_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$, and one retail margin, $m^R_{rw} = p_{rw} - p^W_{rw} - c^R_{rw}$. Then the objects to be recovered in the downstream model are the price coefficient α, the demand shifters $\{\delta_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$, the remaining margins, and their associated marginal costs.

Calibration proceeds following the methods used in a typical logit merger simulation, as seen in Werden and Froeb (1994). The market share equation (2) has the following partial derivatives:

$$\frac{\partial s_{rx}}{\partial p_{rw}} = \begin{cases} \alpha s_{rx} s_{rw} & \text{if } x \neq w \\ -\alpha s_{rw}(1 - s_{rw}) & \text{if } x = w. \end{cases} \tag{12}$$

Thus, if shares and one margin are observed, the downstream first order conditions provide a system of equations where the only unknowns are the parameter α and the other margins. Solving these equations yields the coefficient α and the remaining unobserved margins. Once margins have been computed, the underlying marginal costs (inclusive of wholesale prices) are given by $p^W_{rw} + c^R_{rw} = p_{rw} - m^R_{rw}$. Then the demand shifters can be recovered using the typical Berry (1994) relationship,

$$\ln(s_{rw}) - \ln(s_{00}) = \delta_{rw} - \alpha p_{rw}, \tag{13}$$

since retail prices are observed.

Turning to the upstream model, assume that the researcher additionally observes wholesale prices $\{p^W_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$ and margins $\{m^W_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$ for each retailer-wholesaler pair.[8] If all of the downstream parameters have been recovered, the remaining unknown objects are the bargaining parameters $\{\lambda_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$.

The form of the logit share equation implies that

$$\Delta s_{tx}(\mathbb{W}^r \setminus \{w\}) = s_{rw}\left(\frac{s_{tx}}{1-s_{rw}}\right). \tag{14}$$

For those familiar with the terminology of diversion ratios, the term in parentheses is the diversion according to share from the excluded product w sold by retailer r to product x sold by retailer t. Given this equation, expression (11) is a function of observed market shares, margins, and the unknown bargaining parameters. Solving these first order conditions allows for the recovery of the bargaining parameters.

3.2 Downstream Horizontal Mergers

Once the parameters of the model have been recovered, counterfactual merger simulations can be performed. We begin with the situation where two retailers, firms r and s, merge. Their joint profit function is

$$\pi^r + \pi^s = \left\{\sum_{w\in\mathbb{W}^r}[p_{rw} - p^W_{rw} - c^R_{rw}]s_{rw} + \sum_{v\in\mathbb{W}^s}[p_{sv} - p^W_{sv} - c^R_{sv}]s_{sv}\right\}M, \tag{15}$$

which is just the sum of their individual profits. When setting downstream prices, the merged retailers now take into account the effect they have on each other's profits, as can be seen in the first order condition given by

$$\sum_{x\in\mathbb{W}^r}[p_{rx} - p^W_{rx} - c^R_{rx}]\frac{\partial s_{rx}}{\partial p_{rw}} + s_{rw} + \sum_{v\in\mathbb{W}^s}[p_{sv} - p^W_{sv} - c^R_{sv}]\frac{\partial s_{sv}}{\partial p_{rw}} = 0, \tag{16}$$

which is computed for a product sold by firm r.[9] Compared to equation (4), the expression above has an additional term that captures the effect that raising the price of one of retailer r's products has on the profits of retailer s. As the price p_{rw} increases, sales shift to retailer s,

[8] Wholesale margins are defined as $m^W_{rw} = p^W_{rw} - c^W_{rw}$.

[9] The condition for firm s can be derived analogously.

which is reflected in the partial derivative $\partial s_{sv}/\partial p_{rw}$. These increased sales earn the margin given by $p_{sv} - p^W_{sv} - c^R_{sv}$. Greater sales recapture and higher margins increase the incentive to raise price after the merger. This effect is sometimes referred to as "upward pricing pressure" (UPP). The UPP effect is typical of most horizontal merger simulation models.

The effects on upstream prices are a little different. With the merger, the retailer disagreement payoff when firm r fails to reach an agreement with wholesaler w becomes

$$\begin{aligned} d^r(\mathbb{W}^r \setminus \{w\}) + d^s(\mathbb{W}^r \setminus \{w\}) = \Bigg\{ &\sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] s_{rx}(\mathbb{W}^r \setminus \{w\}) \\ &+ \sum_{v \in \mathbb{W}^s} [p_{sv} - p^W_{sv} - c^R_{sv}] s_{sv}(\mathbb{W}^r \setminus \{w\}) \Bigg\} M. \end{aligned} \tag{17}$$

Thus, the combined firm takes into account the profits of retailer s, as reflected in the last additional term in the expression above.[10] Substituting back into the bargaining first order condition gives

$$\begin{aligned} &[p^W_{rw} - c^W_{rw}] s_{rw} - \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] \Delta s_{tw}(\mathbb{W}^r \setminus \{w\}) = \\ &\frac{1 - \lambda^*_{rs,w}}{\lambda^*_{rs,w}} \Bigg([p_{rw} - p^W_{rw} - c^R_{rw}] s_{rw} - \sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] \Delta s_{rx}(\mathbb{W}^r \setminus \{w\}) \\ &- \sum_{v \in \mathbb{W}^s} [p_{sv} - p^W_{sv} - c^R_{sv}] \Delta s_{sv}(\mathbb{W}^r \setminus \{w\}) \Bigg), \end{aligned} \tag{18}$$

where we allow the bargaining parameter $\lambda^*_{rs,w}$ to potentially change due to the merger. We assume that $\lambda^*_{rs,w} = \max\{\lambda_{rw}, \lambda_{sw}\}$, so that the merged firm has the maximum bargaining power of its two constituent retailers.[11] The first order conditions characterized by equations (16) and (18), together with those for the non-merged firms (which still have the form seen in equations (4) and (11)), determine equilibrium prices and market shares.

[10] The payoff for a negotiation by retailer s is similar. Here we assume that when retailer r fails to reach an agreement with wholesaler w, retailer s's contract with wholesaler w remains in place. The model could easily be extended such that wholesaler w withholds its product from both of the merged retailers, which would remove good w from the set $\mathbb{W}^s$ in the disagreement payoff.

[11] Alternatively, one could also assume that the parameter was fixed at the pre-merger value λ_{rw}.

Comparing equation (18) to equation (11), we see that the main difference is the additional term reflecting the profits that the merged firm earns from retailer s. If retailer r and s sell substitutes, a situation where retailer r loses access to product w can increase sales for its partner. This in turn can increase the merged retailers' bargaining leverage, since the value of their disagreement payoff has risen, which can then lead to lower input prices.

3.3 Upstream Horizontal Mergers

Assume that two wholesalers, firms w and v, merge. Their joint profit function is given by

$$\pi^w + \pi^v = \left\{ \sum_{r \in \mathbb{R}^w} [p^W_{rw} - c^W_{rw}] s_{rw} + \sum_{s \in \mathbb{R}^v} [p^W_{sv} - c^W_{sv}] s_{sv} \right\} M. \tag{19}$$

The merged firms' disagreement payoff when wholesaler w fails to reach an agreement with retailer r becomes

$$\begin{aligned} d^w(\mathbb{R}^w \setminus \{r\}) + d^v(\mathbb{R}^w \setminus \{r\}) = \Bigg\{ & \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] s_{tw}(\mathbb{W}^r \setminus \{w\}) \\ & + \sum_{s \in \mathbb{R}^v} [p^W_{sv} - c^W_{sv}] s_{sv}(\mathbb{W}^r \setminus \{w\}) \Bigg\} M. \end{aligned} \tag{20}$$

Here we see that if wholesaler w stops offering its product to retailer r, it has the possibility of recapturing profits through sales by wholesaler v.[12]

Substituting back into the first order condition gives

$$\begin{aligned} & [p^W_{rw} - c^W_{rw}] s_{rw} - \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] \Delta s_{tw}(\mathbb{W}^r \setminus \{w\}) - \sum_{s \in \mathbb{R}^v} [p^W_{sv} - c^W_{sv}] \Delta s_{sv}(\mathbb{W}^r \setminus \{w\}) = \\ & \frac{1 - \lambda^*_{r,wv}}{\lambda^*_{r,wv}} \left([p_{rw} - p^W_{rw} - c^R_{rw}] s_{rw} - \sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] \Delta s_{rx}(\mathbb{W}^r \setminus \{w\}) \right), \end{aligned} \tag{21}$$

[12]The expression for negotiations by wholesaler v is similar. The model could be easily extended to the case where both wholesalers w and v withhold their products from retailer r.

where $\lambda^*_{r,wv} = \max\{\lambda_{rw}, \lambda_{rv}\}$. As with a downstream merger, an upstream horizontal merger increases the merged firms' bargaining leverage insofar as they are able to recapture lost sales via their merging partner. This will tend to be the case if these products are substitutes. The precise effects can be calculated by jointly solving the first order conditions in equation (21) with those for the non-merging firms and for the downstream market.

3.4 Vertical Mergers

Assume that retailer r and wholesaler w merge. Their joint profit function becomes

$$\begin{aligned}\pi^r + \pi^w = \Bigg\{ &\sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] s_{rx} + [p_{rw} - c^R_{rw} - c^W_{rw}] s_{rw} \\ &+ \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] s_{tw} \Bigg\} M. \end{aligned} \tag{22}$$

The wholesale price of good w to retailer r is only a transfer price between the merging parties, so its effective marginal cost becomes the sum of the upstream and downstream costs, $c^R_{rw} + c^W_{rw}$. In this way, the merger eliminates double marginalization between the merging partners.

When deciding what downstream price to set for product w, the merged firm now has a first order condition given by

$$\sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] \frac{\partial s_{rx}}{\partial p_{rw}} + s_{rw} + [p_{rw} - c^R_{rw} - c^W_{rw}] \frac{\partial s_{rw}}{\partial p_{rw}} + \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] \frac{\partial s_{tw}}{\partial p_{rw}} = 0. \tag{23}$$

This expression has two differences relative to the first order condition in equation (4). First, the lower marginal cost due to the elimination of double marginalization appears in the second to last term. This tends to lower the resulting retail price p_{rw}. Second, the merged firm now takes into account the effect that lowering p_{rw} can have on the wholesale profits made by selling to other retailers besides firm r. This effect appears in the last term, and tends to raise the retail price p_{rw} if other retailers offer substitutes. The net effect balances these two forces. The first order condition for products sold by the merged firm besides w can be derived analogously.

Turning to the upstream market, when the merged firm is bargaining with a retailer besides r over what wholesale price to set, it has a disagreement payoff of

$$\begin{aligned}d^r(\mathbb{W}^s \setminus \{w\}) + d^w(\mathbb{R}^w \setminus \{s\}) = \Bigg\{ &\sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] s_{rx}(\mathbb{W}^s \setminus \{w\}) \\ &+ [p_{rw} - c^R_{rw} - c^W_{rw}] s_{rw}(\mathbb{W}^s \setminus \{w\}) \\ &+ \sum_{t \in \mathbb{R}^w \setminus \{r,s\}} [p^W_{tw} - c^W_{tw}] s_{tw}(\mathbb{W}^s \setminus \{w\}) \Bigg\} M.\end{aligned} \tag{24}$$

The upstream firm's disagreement payoff now has additional terms due to its affiliated retailer. The wholesale price first order condition becomes

$$\begin{aligned}&[p^W_{sw} - c^W_{sw}] s_{sw} - \sum_{t \in \mathbb{R}^w \setminus \{r,s\}} [p^W_{tw} - c^W_{tw}] \Delta s_{tw}(\mathbb{W}^s \setminus \{w\}) \\ &- \sum_{x \in \mathbb{W}^r \setminus \{w\}} [p_{rx} - p^W_{rx} - c^R_{rx}] \Delta s_{rx}(\mathbb{W}^s \setminus \{w\}) - [p_{rw} - c^R_{rw} - c^W_{rw}] \Delta s_{rw}(\mathbb{W}^s \setminus \{w\}) = \\ &\frac{1-\lambda_{sw}}{\lambda_{sw}} \left([p_{sw} - p^W_{sw} - c^R_{sw}] s_{sw} - \sum_{v \in \mathbb{W}^r \setminus \{w\}} [p_{sv} - p^W_{sv} - c^R_{sv}] \Delta s_{sv}(\mathbb{W}^s \setminus \{w\}) \right).\end{aligned} \tag{25}$$

Compared to the pre-merger first order condition in equation (11), here when the merged firm fails to agree with retailer s, it can recapture some of these lost sales through the increased profits of retailer r. These extra profits will tend to be larger if the products sold by retailer r are closer substitutes to the product w offered by retailer s. This "raising rivals' cost" (RRC) effect will tend to increase the wholesale price firm w charges to firm s.[13]

When the merged firm is bargaining with a wholesaler besides w over what input price to

[13] In the specification presented here, we have left the bargaining parameter at its pre-merger level, λ_{sw}, since the merger is not combining two firms that bargain on the same side of the vertical supply chain. If a change in bargaining power did result from the merger, it could be captured by varying this parameter.

pay, it has a disagreement payoff of

$$
\begin{aligned}
d^r(\mathbb{W}^r \setminus \{v\}) + d^w(\mathbb{R}^v \setminus \{r\}) = \Bigg\{ & \sum_{x \in \mathbb{W}^r \setminus \{w,v\}} [p_{rx} - p^W_{rx} - c^R_{rx}] s_{rx}(\mathbb{W}^r \setminus \{v\}) \\
& + [p_{rw} - c^R_{rw} - c^W_{rw}] s_{rw}(\mathbb{W}^r \setminus \{v\}) \\
& + \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] s_{tw}(\mathbb{W}^r \setminus \{v\}) \Bigg\} M.
\end{aligned} \tag{26}
$$

Then the bargaining first order condition becomes

$$
\begin{aligned}
& [p^W_{rv} - c^W_{rv}] s_{rv} - \sum_{s \in \mathbb{R}^v \setminus \{r\}} [p^W_{sv} - c^W_{sv}] \Delta s_{sv}(\mathbb{W}^r \setminus \{v\}) = \\
& \frac{1-\lambda_{rv}}{\lambda_{rv}} \Bigg([p_{rv} - p^W_{rv} - c^R_{rv}] s_{rv} - \sum_{x \in \mathbb{W}^r \setminus \{w,v\}} [p_{rx} - p^W_{rx} - c^R_{rx}] \Delta s_{rx}(\mathbb{W}^r \setminus \{v\}) \\
& + [p_{rw} - c^R_{rw} - c^W_{rw}] \Delta s_{rw}(\mathbb{W}^r \setminus \{v\}) + \sum_{t \in \mathbb{R}^w \setminus \{r\}} [p^W_{tw} - c^W_{tw}] \Delta s_{tw}(\mathbb{W}^r \setminus \{v\}) \Bigg).
\end{aligned} \tag{27}
$$

Compared to equation (11), the above expression includes extra terms that capture the merged entity's wholesale profits. In so far as sales shift to wholesaler w's clients when retailer r loses access to firm v's product, the merged firm has a better outside option than without the merger. This increases the merged firm's bargaining leverage, and can cause the fees it pays other wholesalers to fall. Combining the first order condition in expressions (25) and (27) with those for the other firms and for the downstream market allows one to solve for the new post-merger equilibrium.

3.5 Welfare Effects

Once we have recovered the predicted post-merger prices from the merger simulation, we can then turn to quantifying the resulting effect on consumers. We define the compensating variation between pre-merger prices (denoted by the subscript "pre") and post-merger prices (denoted by the subscript "post") as follows:

$$CV = \frac{1}{\alpha} \log \left(\frac{1 + \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} \exp(\delta_{tx} - \alpha p_{tx,post})}{1 + \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} \exp(\delta_{tx} - \alpha p_{tx,pre})} \right). \tag{28}$$

This expression is the difference in the logit inclusive values before the merger versus after. The inclusive value is derived from the expected value of the logit utility function in equation (1).

4 Downstream Auctions

Now that we have discussed the methodology behind simulating mergers in an environment with upstream bargaining, we extend the framework to incorporate downstream auctions. We follow Miller (2014), in using a "second score auction" setting. Similar auction models have been used to study mergers in the past.[14] In what follows, the upstream model described in Section 2 remains the same.

4.1 Basic Framework

In the second score auction model, we assume that consumer i has an indirect utility function for product w supplied by retailer r of

$$u_{irw} = \beta_{rw} - p_{rw} + e_{irw}, \tag{29}$$

where e_{irw} is an independent and identically distributed Type I extreme value error term with a scale parameter of σ. We normalize the value of the outside good such that $u_{i00} = e_{i00}$.

Each consumer selects a single product to purchase by soliciting product-specific bids, b_{rw}, from each retailer. The buyer chooses the option with the highest utility according to equation (29), substituting bids for prices. The probability that product w from retailer r is the best bid among all product-retailer pairs is

$$s_{rw} = \frac{\exp\left(\frac{\beta_{rw}}{\sigma} - \frac{b_{rw}}{\sigma}\right)}{1 + \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} \exp\left(\frac{\beta_{tx}}{\sigma} - \frac{b_{tx}}{\sigma}\right)}.$$

[14]See the *Anthem* district-level opinion at pages 66-67 and 70-71.

If we let $\delta_{rw} = \beta_{rw}/\sigma$ and $\alpha = 1/\sigma$, this probability becomes

$$s_{rw} = \frac{\exp(\delta_{rw} - \alpha b_{rw})}{1 + \sum_{t\in\mathbb{R}} \sum_{x\in\mathbb{W}^t} \exp(\delta_{tx} - \alpha b_{tx})}, \tag{30}$$

which is analogous to the market share function in equation (2). The expected value of the maximum of all these bids is

$$\frac{1}{\alpha} \ln \left(1 + \sum_{t\in\mathbb{R}} \sum_{x\in\mathbb{W}^t} \exp(\delta_{tx} - \alpha b_{tx})\right). \tag{31}$$

The profit of a retailer r can again be written as appears in equation (3).

In a second score auction, the customer sets the final realized price so that they receive the same utility from the best bidder as would have been achieved from the second best bid. We assume that each retailer knows the value of e_{irw} for a prospective customer of any of its products. The retailer does not observe this value for products sold by other retailers. As shown in Miller (2014), the dominant strategy for any retailer in this auction is to supply only the product $w \in \mathbb{W}^r$ to consumer i that gives the maximum possible utility net of marginal cost. That is, a retailer will not outbid itself. Then price is such that

$$p_{rw} = \beta_{rw} + e_{irw} - \max_{s\in\mathbb{R}\setminus\{r\}, v\in\mathbb{W}^s} \left\{\beta_{sv} + e_{isv} - b_{sv}\right\}, \tag{32}$$

in the case where product w from retailer r wins the auction. Furthermore, the retailer will set its bid equal to its marginal cost, $b_{rw} = p^W_{rw} + c^R_{rw}$. This can be seen by examining the retailer's expected margin, assuming w is the product it offers via bid,

$$E\left[m^R_{rw}\right] = -\frac{1}{\alpha} \ln \left(1 - \sum_{x\in\mathbb{W}^r} s_{rx}\right) + b_{rw} - p^W_{rw} - c^R_{rw}.$$

Taking the derivative of this expression with respect to b_{rw}, we find that it is always positive. This pushes the retailer to a corner solution, where the firm lowers its bid as much as possible, to its marginal cost.

This auction structure gives the following expression for the conditional expected margin when product w sold by retailer r wins,

$$E\left[m^R_{rw} \middle| rw \text{ wins}\right] = -\frac{1}{\alpha \sum_{x\in\mathbb{W}^r} s_{rx}} \ln\left(1 - \sum_{x\in\mathbb{W}^r} s_{rx}\right), \tag{33}$$

where we have leveraged the expected value of the maximum from equation (31). The expression above relates margins to market shares, analogous to equation (4). Along with the upstream first order conditions discussed in Section 2, we can solve this series of equations in order to determine the equilibrium.

4.2 Calibration and Merger Simulation

The auction model does not introduce any additional parameters relative to the downstream logit model. Therefore, it can be calibrated using the same data on shares, prices, and margins: $\{s_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$, $\{p_{rw}; \forall r \in \mathbb{R}, \forall w \in \mathbb{W}\}$, and one m^R_{rw} for some retailer r and product w. Using shares and the one margin, the price coefficient α can be identified using equation (33). Once α has been recovered, the same equation can be used to calculate the remaining margins. These margins, when combined with observed retail prices, in turn identify the underlying marginal costs.[15] The demand shifter parameters can be recovered using an analogous equation as in (13), but substituting bids for prices,

$$\ln(s_{rw}) - \ln(s_{00}) = \delta_{rw} - \alpha(p^W_{rw} + c^R_{rw}), \tag{34}$$

where we have used the fact that, in equilibrium, bids are equal to the retailer's marginal costs. The upstream parameters can be recovered as described in Section 3.1.

Once the model parameters have been recovered, the effects of potential mergers can be simulated. Starting with a downstream horizontal merger, if two retailers r and s combine, as shown by Miller (2014), they will cease to bid against each other. That is, the merging companies will only offer each customer the product out of both of their portfolios that has the largest utility compared to marginal cost. Assume, without loss of generality, that this best product is sourced from wholesaler w and sold by retailer r. Then the merged firms'

[15] Note that identifying marginal costs is not necessary for some applications, as equilibrium shares and margins in the model depend on the combination of the demand shifter and marginal costs, not on marginal cost separately.

expected margin conditional on winning the auction is

$$E\left[m_{rw}^R | rw \text{ wins}\right] = -\frac{1}{\alpha \sum_{t\in\{r,s\}} \sum_{x\in\mathbb{W}^t} s_{tx}} \ln\left(1 - \sum_{t\in\{r,s\}} \sum_{x\in\mathbb{W}^t} s_{tx}\right). \tag{35}$$

The merger will tend to raise prices for those customers for whom both retailers r and s are highly valued. Combining the above expression with the analogous equations for the non-merged retailers and with the upstream first order conditions seen in Section 3.2 allows one to solve for the equilibrium. As for an upstream merger, in this case the downstream first order conditions are as in equation (33), while the upstream are as discussed in Section 3.3.

Turning to vertical mergers, if retailer r and wholesaler w merge, then, as in Section 3.4, they take into account both their upstream and downstream profits when setting prices. In terms of deciding on a downstream bid, the combined firm must balance two forces: lowering its bid increases the probability of its retailer winning, but decreases the probability of other retailers who purchase its wholesale product from winning. The expected profit if product w sold by some rival retailer wins is given by

$$\sum_{s\in\mathbb{R}\setminus\{r\}} (p_{sw}^W - c_{sw}^W) s_{sw}$$

and its derivative with respect to a bid by retailer r for any of its products is always positive. Thus, the firm will again face a corner solution. If the possible profits from selling retailer r's product are higher than those that can be earned from the wholesale market, then the merged firms will lower their bid to marginal cost. If instead the profits from the wholesale market are greater, the merged firms will raise their bid, effectively removing themselves from the retail choice set for this auction. Given these equilibrium price decisions, the upstream first order conditions in Section 3.4 can be used to derive the resulting effects on wholesale prices.

Once we have simulated the predicted post-merger prices, we can calculate compensating variation using the following expression:

$$CV = \frac{1}{\alpha} \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} s_{tx,post} \left(E \left[m^R_{tx,post} \middle| tx \text{ wins post-merger} \right] + p^W_{tx,post} + c^R_{tx} \right)$$
$$- \frac{1}{\alpha} \sum_{t \in \mathbb{R}} \sum_{x \in \mathbb{W}^t} s_{tx,pre} \left(E \left[m^R_{tx,pre} \middle| tx \text{ wins pre-merger} \right] + p^W_{tx,pre} + c^R_{tx} \right). \tag{36}$$

The above equation comes from comparing the consumer surplus that the second score auction generates pre-merger versus post-merger.

5 Numerical Simulations

Here, we describe how we use the model to simulate the consumer welfare effects of three different types of mergers: a downstream horizontal merger of two retailers, an upstream horizontal merger of two wholesalers, and a vertical merger between a wholesaler and a retailer. In the pre-merger world, each wholesaler has reached an agreement with every retailer to supply its product, and upstream and downstream prices have been set as described in Section 2. Then the simulations allow us to study how mergers shift equilibrium prices and outcomes.

The aim of these simulations is to explore how mergers impact consumer welfare starting from a variety of pre-merger market scenarios. Specifically, we ask how changing the pre-merger number of upstream firms, the pre-merger number of downstream firms, and the relative bargaining power between retailers and wholesalers affects consumer welfare under each merger type. We compare and contrast the results that obtain using both the Bertrand logit and the second score auction as the assumed downstream framework.

5.1 Data Generating Process

We begin by constructing a large number markets under two different scenarios. The first is what we call our "Firm Count" scenario, where we wish to study the impact of varying the number of pre-merger firms present in the market, both upstream and downstream. The second is our "Bargaining Power" scenario, where we trace out the effects of varying the upstream bargaining power of the retailer relative to the wholesaler.

In the Firm Count setup, we simulate markets with either 2, 3, 4, 6, or 12 wholesalers or retailers but equal bargaining power (i.e. the bargaining parameter is set to 0.5). For each combination of number of wholesalers and retailers, we draw 1,000 different sets of market primitives. This results in 150,000 merger simulations.[16] Separately, in the Bargaining Power setup, we simulate markets with 3 wholesalers and either 2, 3, 4, 6, or 12 retailers, each with a bargaining parameter ranging from 0.3 (wholesalers have the advantage) to 0.9 (retailers have the advantage). Again, for each combination of number of retailers and bargaining parameter, we draw 1,000 different sets of market primitives. This results in 210,000 merger simulations.[17] All 360,000 markets treat as primitives the number of wholesalers, the number of retailers, the bargaining parameter, and the wholesaler and retailer marginal costs. Marginal costs are set to be a fixed percentage of wholesaler and retailer margins in the pre-merger world (25% and 10%, respectively), and are assumed to remain unchanged post-merger.

We assume that consumer demand for a particular wholesaler-retailer product follows a logit (equation (1)) where product shares are randomly sampled from a Dirichlet distribution with a concentration parameter vector whose elements equal 2.5.[18] The price coefficient α is calibrated by assuming that in the pre-merger world, there is a vertically integrated outside option available to customers. This outside option has a 15% market share, earns a $25 margin per unit sold, and is produced at zero marginal cost.[19] The product-specific demand shifters δ_{rw} are calibrated relative to the outside good by first using the calibrated price coefficient α and shares to impute pre-merger product margins, and then using the previously discussed assumptions on marginal cost to calculate marginal costs and pre-merger prices. Shares, pre-merger prices (or, in the case of the second score auction model, marginal costs), and the price coefficient are then used to impute the product-specific shifters.

In order to simulate a horizontal merger (either among wholesalers or among retailers), we assign all the products produced by two randomly selected firms to a single entity post-merger. Similarly, to simulate a vertical merger, we assign all the products produced by a

[16] We get 150,000 from five categories of number of wholesalers, by five categories of number of retailers, by three merger types (downstream horizontal, upstream horizontal, and vertical), by two downstream models (Bertrand logit and second score auction), and by 1,000 parameter sets.

[17] We get 210,000 from seven possible bargaining parameters (increasing by 0.1 from 0.3 to 0.9), by five categories of number of retailers, by three merger types, by two downstream models, and by 1,000 sets of market primitives.

[18] A Dirichlet distribution parameterized in this manner generates markets with reasonably asymmetric market shares, allowing our numerical simulations to better explore the space of possible market configurations.

[19] All other goods are differenced relative to this option, which maintains the outside good normalization.

randomly selected wholesaler and a randomly selected retailer to a single entity post-merger. This random assignment allows us to explore a large variety of differently sized mergers.

Table 1 provides summary statistics across our various simulations. We see that for the Firm Count scenario pre-merger HHIs are typically between 1,722 and 3,649 with a median of 2,629 and between 3,346 and 3,645 with a median of 3,435 for the Bargaining Power scenario. These pre-merger HHIs are consistent with our Firm Count markets typically containing the equivalent of between three and six equal-sized firms, and our Bargaining Power markets typically containing the equivalent of three equal-sized firms.[20] Post-merger HHIs under the the Firm Count scenario are typically between 2,197 and 5,470 points, with most simulated mergers increasing concentration from 197 to 2,000 points. Post-merger HHIs under the Bargaining Power scenario are typically between 3,544 and 5,735, with most simulated mergers increasing concentration from 310 to 2,267 points.

Our simulated markets have fairly inelastic demands. Market elasticities typically fall between -0.49 and -0.4 with a median of -0.43 for the Firm Count scenario, and between -0.55 and -0.28 with a median of -0.38 for the Bargaining Power scenario.[21] These elasticities vary as the number of firms in the market changes. Under the Firm Count scenario, our simulations predict that a merger can have a range of likely outcomes, from benefiting the typical customer by \$0.03 to harming the typical customer by \$3.5. In the Bargaining Power scenario, our simulations predict that a merger can range between benefiting the typical customer by \$0.29 to harming the typical customer by \$4.4.

5.2 Results

We summarize our findings via a series of graphs. Figures 1, 3, and 5 display results from the Firm Count scenario, which looks at changing the number of retailers and wholesalers. Figures 2, 4, and 6 give results from the Bargaining Power scenario, which studies variation in retailer bargaining strength.

These figures are divided into five panels, one each for the number of retailers listed at the top. For the Firm Count figures, there five pairs of box and whisker plots in each panel, each pair corresponding to a different number of wholesalers. For the Bargaining Power figures,

[20]The seemingly small amount of variation in the Bargaining Power HHIs is largely an artifact of fixing the number of wholesalers at three and then pooling HHIs across both retailer and wholesaler horizontal mergers.

[21]For logit demand the market elasticity is given by $-\alpha\overline{p}(1-s_{00})$, where $\overline{p}$ is the share-weighted average of non-outside good prices.

there are seven pairs of box and whisker plots within each panel, one for each different bargaining parameter value, ranging from 0.3 to 0.9. The blue box and whisker plots (on the left in each pair) depict compensating variation assuming that retailers are competing in a Bertrand logit model, while the orange box and whisker plots (on the right in each pair) show compensating variation assuming that retailers are in a second score auction. Recall that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

Downstream Horizontal Mergers We note three key features of downstream horizontal mergers in Figure 1, which examines the effect of varying the number of firms, while holding the bargaining parameter fixed at 0.5. First, the mergers in this figure are almost always harmful to consumers. Indeed, only the second score auction appears to generate any net beneficial mergers, and then only when there are few wholesalers. Second, as can be seen by looking across all five panels within this figure, mergers when there are few pre-merger retailers yield exponentially more harm to customers than mergers with many pre-merger retailers. Third, the harm from the second score specifications is typically less than the harm from the Bertrand logit model.[22]

In contrast to Figure 1, Figure 2 holds the number of wholesalers fixed and allows the bargaining power parameter to change. Now we see more instances where downstream mergers can offset upstream bargaining power, particularly when the bargaining power of the retailer is somewhat low. This offset, however, is only net beneficial to consumers when the number of retailers is greater than two. Note also that the gap in harm between the Bertrand logit and second score specifications reduces at a logarithmic rate as retailer bargaining power improves.

Upstream Horizontal Mergers Looking at Figure 3, we make three observations about how upstream horizontal mergers are affected by the number of firms present in the market. First, as can be seen by moving to the right within each of the five panels, increasing the number of wholesalers reduces consumer harm at an exponential rate. Second, as can be seen by looking across the five panels, increasing the number of retailers can modestly increase consumer harm, particularly when there are few wholesalers. Third, merger harm when retailers compete via a second score auctions is typically less than the harm when retailers compete via a Bertrand logit setup, but the gap in harm diminishes as more wholesalers are added to the market.

[22] Technically speaking, the compensating variations from the second score auction simulations appear to first-order stochastically dominate the compensating variations from the Bertrand logit model.

Turning to Figure 4, we can see the effects of varying the bargaining parameter. First, increasing retailer bargaining power from 0.3 to 0.9 mitigates – but does not eliminate – merger harm at what appears to be an approximately linear rate. Second, fixing the number of wholesalers but increasing retailer bargaining power yields almost identical consumer harm from both the Bertrand logit and second score auction models. Third, holding bargaining power fixed, increasing the number of retailers appears to modestly increase consumer harm.

Vertical Mergers Figure 5 examines how variation in the number of firms affects the outcomes of vertical mergers. We make three observations. First, while vertical mergers can substantially harm consumers by raising rivals' cost, this figure indicates that on net the average vertical merger often yields relatively little consumer harm or even modest consumer benefit. There exists substantial variation around the mean, however. Second, increasing either the number of wholesalers or the number of retailers tends to mitigate both the harm as well as the benefit from the merger. Finally, vertical mergers tend to be more beneficial – and typically net beneficial – to customers when retailers compete via a second score auction than when they compete according to the Bertrand logit model. Indeed, holding all else equal, the figure indicates that substantial harm is most likely to occur when there are few wholesalers present in the market and retailers compete according to Bertrand.

In Figure 6, we show the effect that changing the bargaining parameter has on the results of a vertical merger. This figure reveals one final interesting feature of vertical mergers: on net, these mergers tend to be beneficial to consumers when either wholesalers or retailers have substantial bargaining power. Such mergers appear to be most likely to yield anticompetitive results when wholesalers and retailers have relatively equal bargaining power, as shown by the inverted-U shape in the graph, peaking around 0.6 or 0.7.

6 Conclusion

In this paper we have developed a merger simulation model that incorporates some of the complexities of bargaining within a vertical supply chain while still remaining simple enough to calibrate with limited data. We find that the framework is highly flexible, as it captures a number of the countervailing effects highlighted in the merger literature. Horizontal mergers can harm consumers by lessening competition between substitute products. In certain instances these harms can be offset by increased retailer bargaining leverage, which allows the merged downstream firms to secure lower input prices. Vertical mergers balance both benefits and costs, in the form of the elimination of double marginalization on the one hand, and raising rivals' cost on the other. Each of these effects can be seen in the series of numerical simulations that we have run. We also find that the form of downstream competition can matter in certain situations, with the second score auction sometimes producing less harm for consumers than the Bertrand logit model.

A fruitful area for future research would be to apply this model to actual mergers, and to compare how the results differ from merger simulations that ignore interactions across the vertical supply chain. The existing literature offers a number of retrospectives of previous mergers that could be interesting to study.

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Scenario	Summary	50%	Min	25%	75%	Max
Firm Count	# Retailers	4	2	3	6	12
(150K Markets)	# Wholesalers	4	2	3	6	12
	Bargaining Power	0.5	0.5	0.5	0.5	0.5
	Pre-Merger HHI	2,629	836	1,722	3,639	9,067
	Post-Merger HHI	3,642	852	2,197	5,470	10,000
	Delta HHI	776	-2,688	197	2,000	5,435
	Market Elasticity	-0.43	-1.2	-0.49	-0.4	-0.36
	CV ($)	0.64	-12	-0.03	3.5	18
Bargaining Power	# Retailers	4	2	3	6	12
(210K Markets)	# Wholesalers	3	3	3	3	3
	Bargaining Power	0.6	0.3	0.4	0.8	0.9
	Pre-Merger HHI	3,435	840	3,346	3,645	8,099
	Post-Merger HHI	5,018	917	3,544	5,735	10,000
	Delta HHI	1,380	-2,616	310	2,267	5,700
	Market Elasticity	-0.38	-1.7	-0.55	-0.28	-0.2
	CV ($)	1.0	-27	-0.29	4.4	18

Table 1: Simulation Summary Statistics

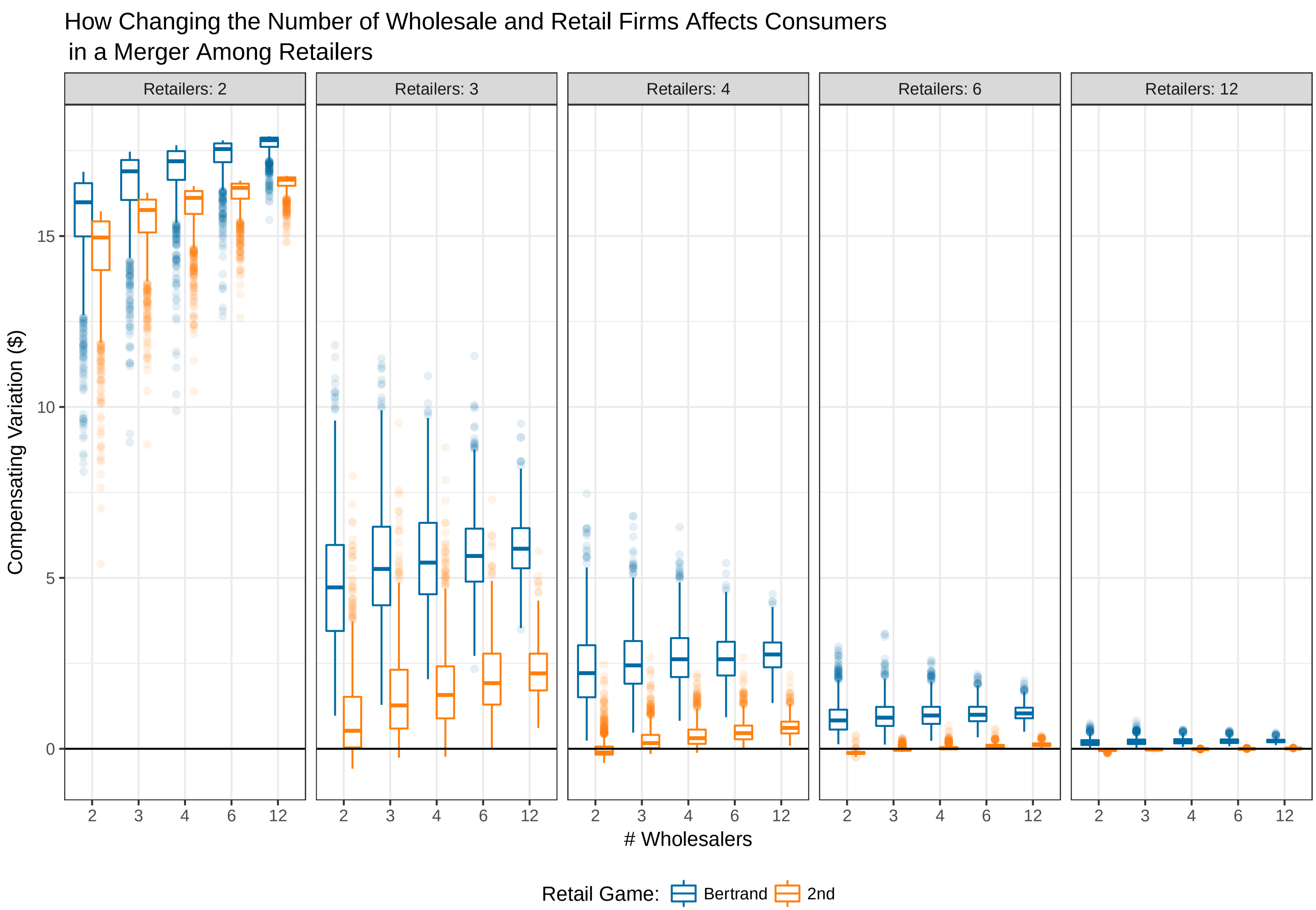

Figure 1 displays box and whisker plots summarizing the extent to which mergers among two retailers affect customers as the number of wholesalers and retailers present in a market change. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of $25, 0 marginal costs, and a 15% market share. The bargaining parameter for all firms is set to 0.5. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

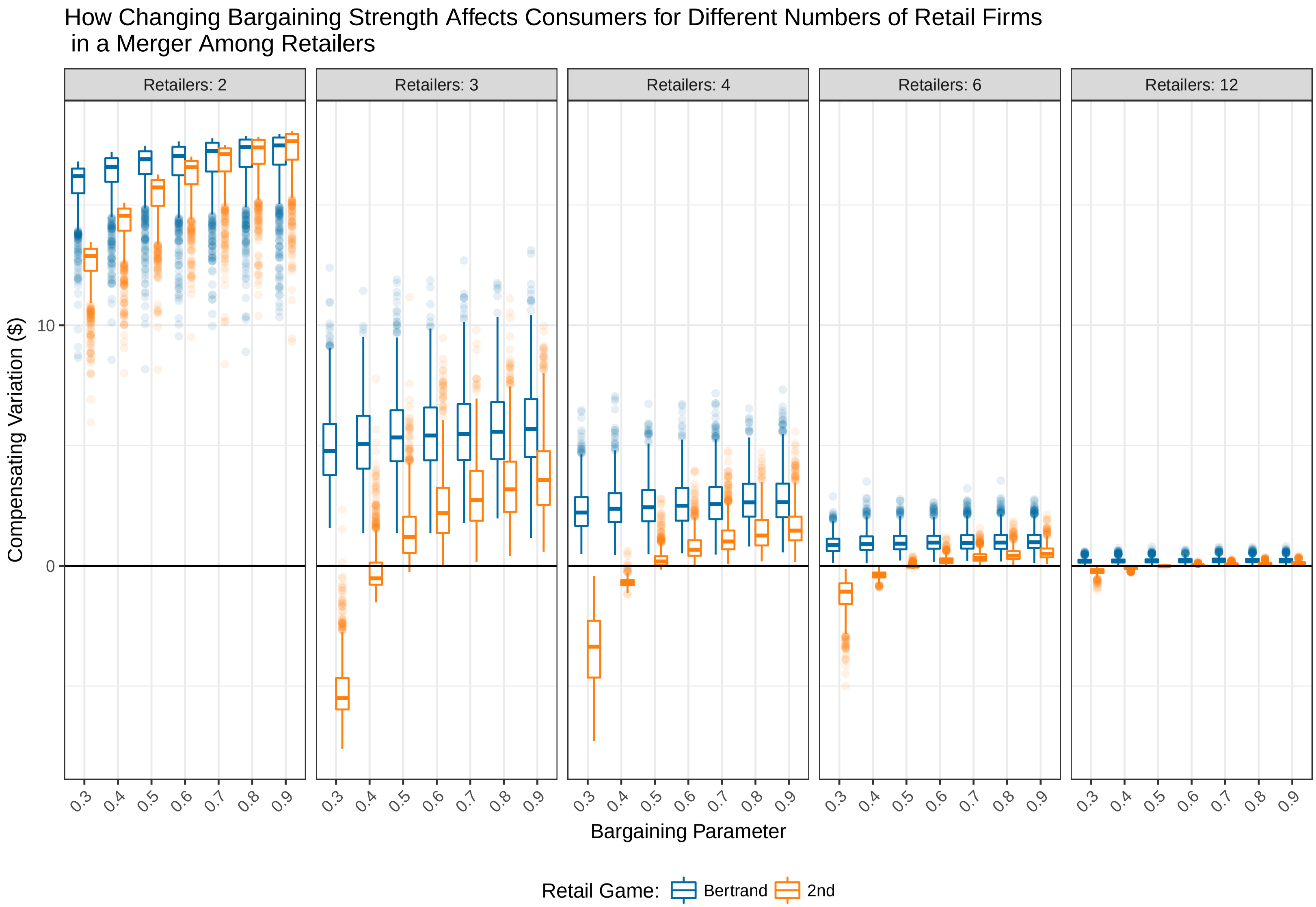

Figure 2 displays box and whisker plots summarizing the extent to which mergers among two retailers affect customers, holding fixed the number of wholesalers in the market at three, but allowing the number of retailers as well as the retailers' bargaining power to vary. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of $25, 0 marginal costs, and a 15% market share. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

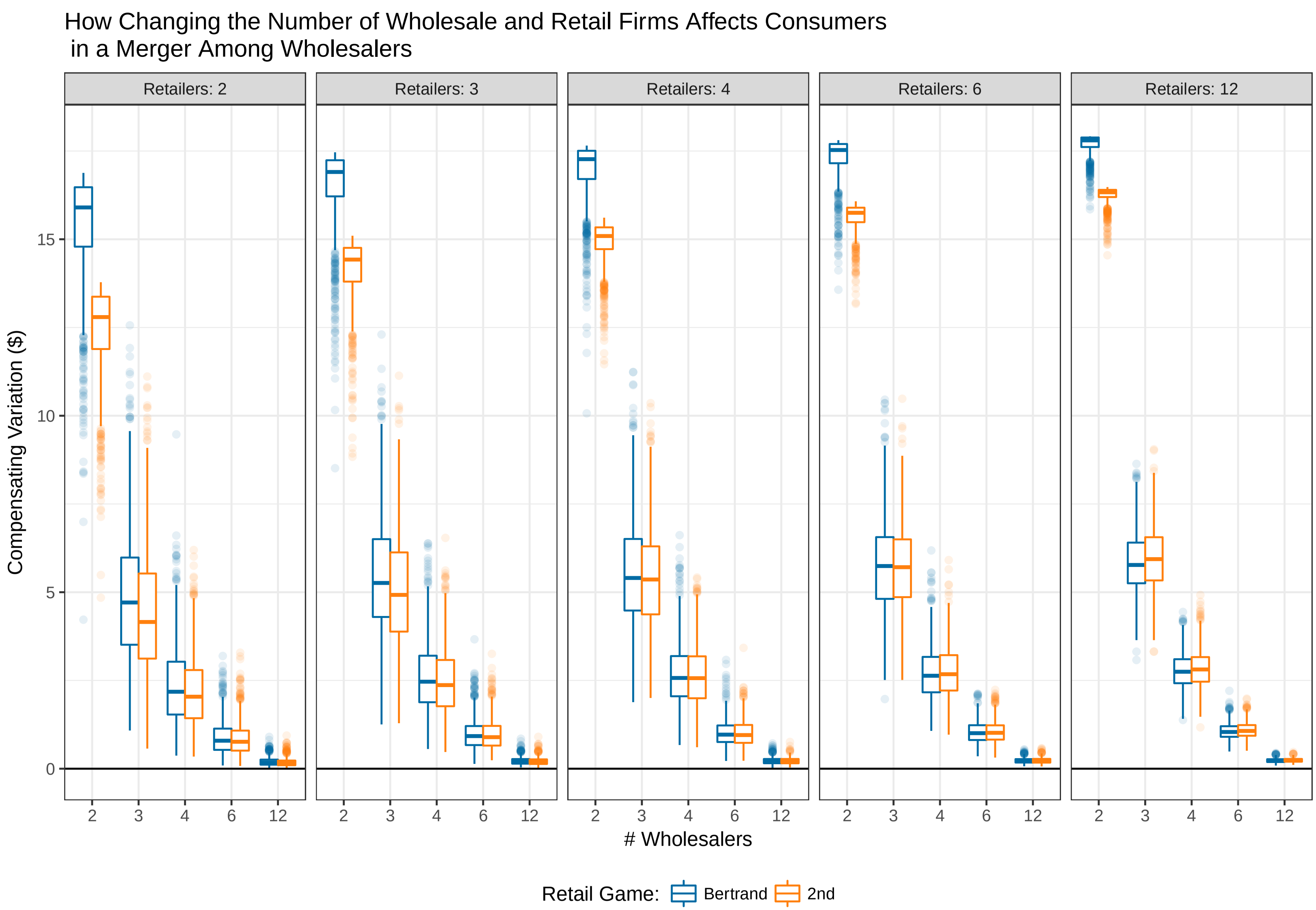

Figure 3 displays box and whisker plots summarizing the extent to which mergers among two wholesalers affect customers as the number of wholesalers and retailers present in a market change. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of \$25, 0 marginal costs, and a 15% market share. The bargaining parameter for all firms is set to 0.5. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

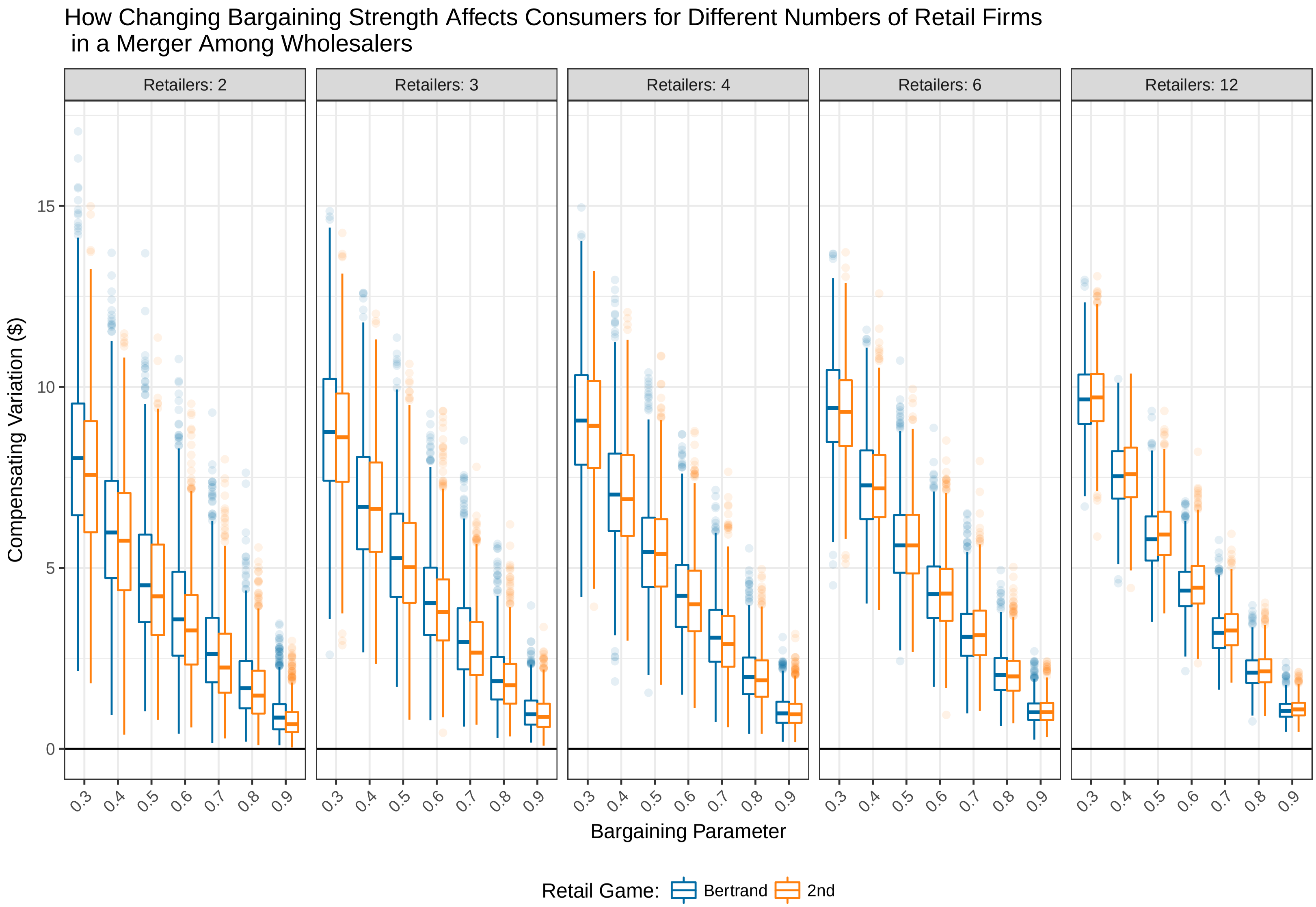

Figure 4 displays box and whisker plots summarizing the extent to which mergers among two wholesalers affect customers, holding fixed the number of wholesalers in the market at three, but allowing the number of retailers as well as the retailers' bargaining power to vary. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of $25, 0 marginal costs, and a 15% market share. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

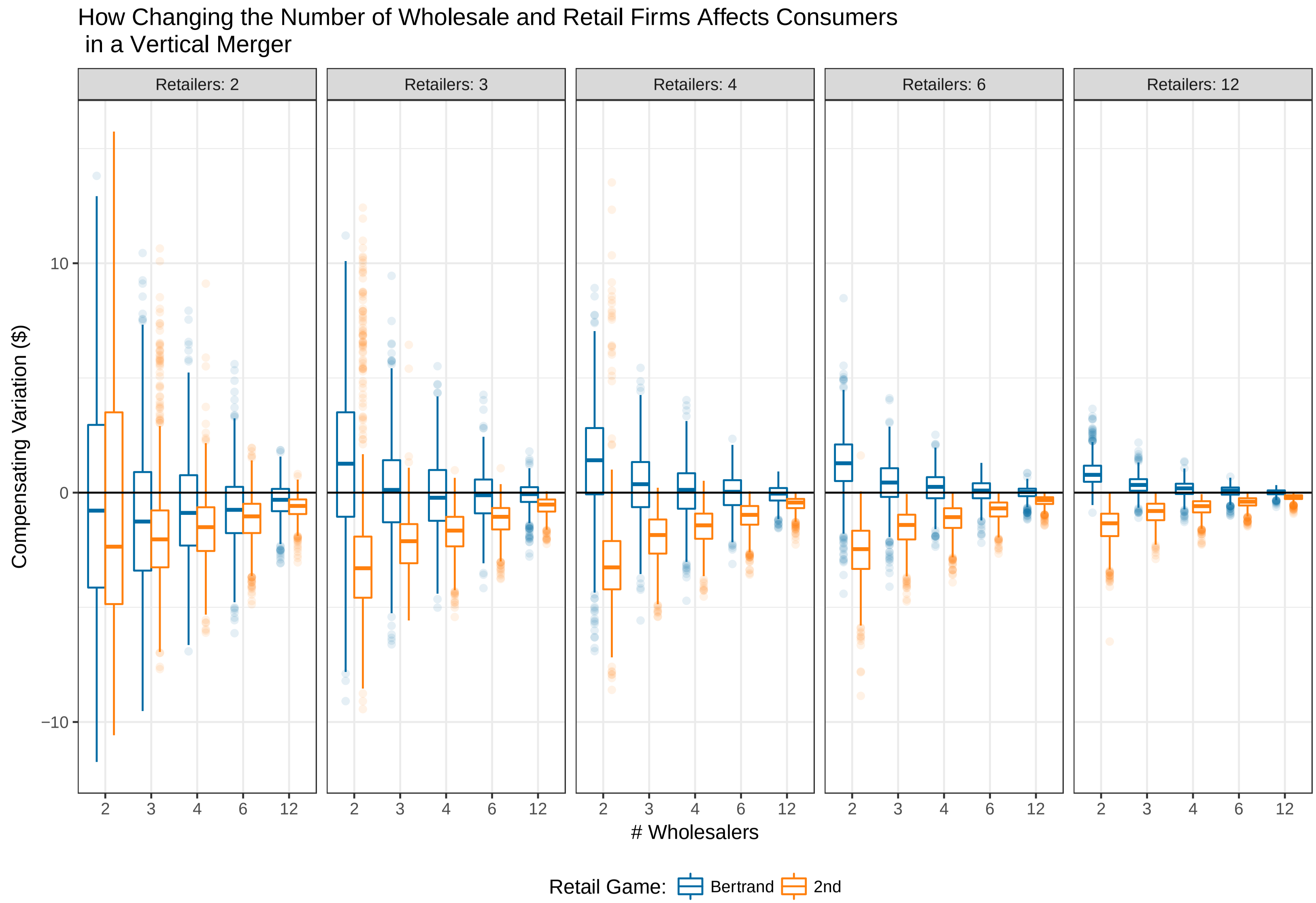

Figure 5 displays box and whisker plots summarizing the extent to which vertical mergers between a wholesaler and a retailer affect customers as the number of wholesalers and retailers present in a market change. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of $25, 0 marginal costs, and a 15% market share. The bargaining parameter for all firms is set to 0.5. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.

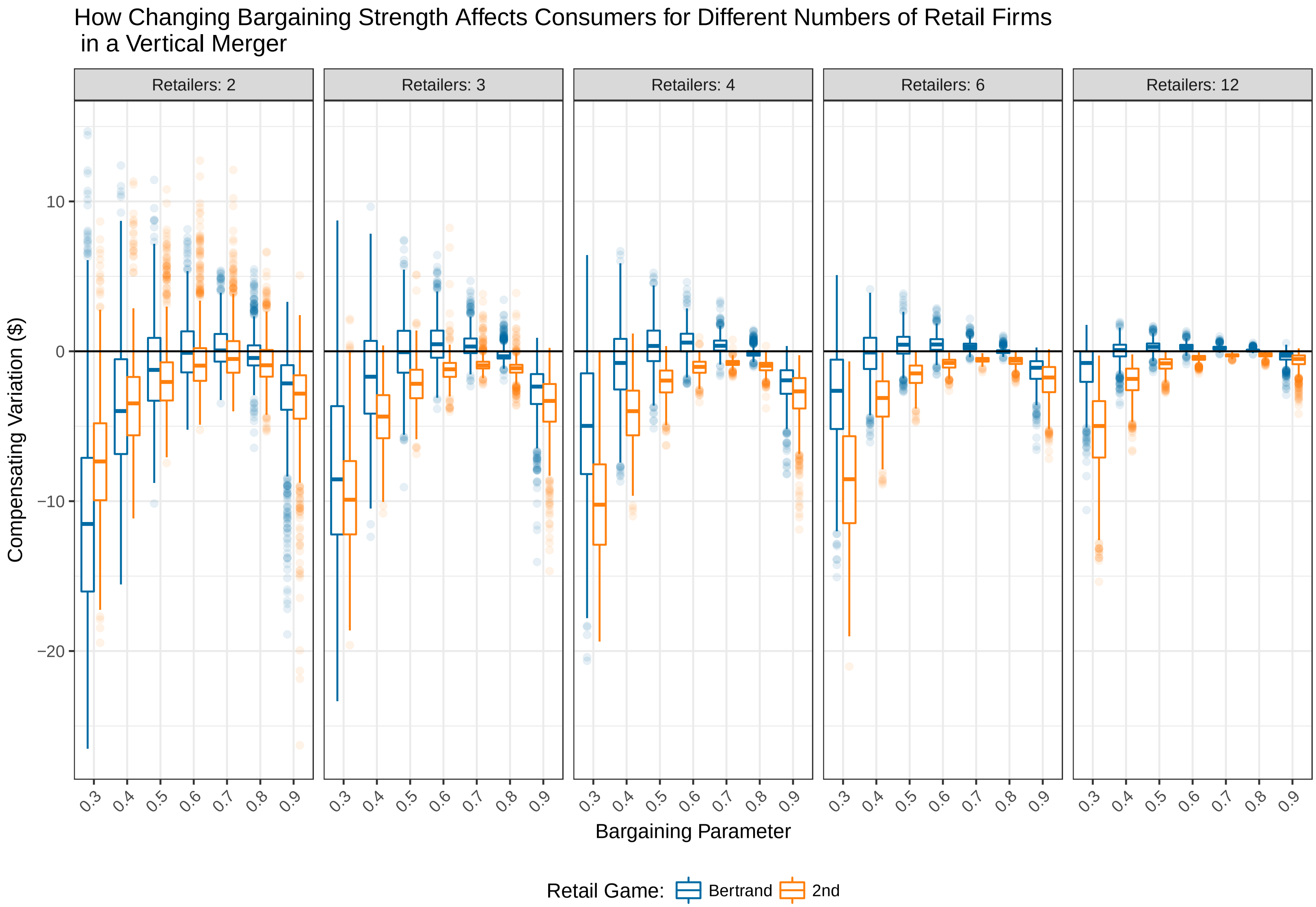

Figure 6 displays box and whisker plots summarizing the extent to which vertical mergers between a wholesaler and a retailer affect customers, holding fixed the number of wholesalers in the market at three, but allowing the number of retailers as well as the retailers' bargaining power to vary. Each blue box depicts the effects in 1,000 mergers assuming that retailers are playing a Bertrand logit pricing game, while each orange box depicts the effects in 1,000 mergers assuming that retailers are playing a second score auction game. All simulations are run assuming that the outside good has a price of $25, 0 marginal costs, and a 15% market share. Note that a negative value for compensating variation implies consumer benefit, while a positive value implies consumer harm.