Analyzing Vertical Mergers with Auctions Upstream

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Abstract

We develop a model of vertical mergers with open auctions upstream. This setting may be appropriate for industries where inputs are procured via auction-like "requests for proposal." For example, Drennan et al (2020) reports that a model of this type was used during the CVS-Aetna merger investigation. Our approach contrasts with a growing body of work on vertical mergers where input prices are determined through Nash bargaining. We discuss how the vertical merger effects of raising rivals' costs and eliminating double markup might be quantified in our particular model.

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1 Introduction

Recent years have seen a growing need for vertical merger analyses. With the DOJ challenging AT&T's merger with Time Warner (2017) and the FTC challenging Illumina's merger with GRAIL (2021),[1] it appears the U.S. antitrust agencies have collectively sought to block more vertical mergers in the past four years than they had in the prior four decades.[2] Vigorous enforcement of vertical mergers seems likely to continue based on the recent "Executive Order on Promoting Competition in the American Economy," which encourages the DOJ and FTC to reconsider how they assess vertical as well as horizontal mergers.[3] The literature, however, offers few models that the agencies can rely on to quantify the competitive effects of vertical mergers. This research seeks to bridge that gap.

We analyze input foreclosure, specifically "raising rivals' costs" (RRC),[4] where linear input prices are determined via procurement auction. Procurement often involves solicitation of "requests for proposal" (RFPs), a process with auction-like properties that can have multiple rounds. For example, health insurance companies procure pharmacy benefits management (PBM) services through auction-like RFP processes. The resulting PBM service prices become part of insurers' marginal cost when competing for insurance (or administrative) business downstream.

As discussed in Drennan et al (2020), the 2017 merger agreement between CVS Health Corporation and Aetna Inc. had a vertical aspect in addition to a horizontal one. At the time of the merger, CVS (but not insurer Aetna) owned a major PBM. This raised the possibility that, post-merger, CVS might significantly alter its bidding behavior in

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[1] See the DOJ's complaint in U.S. v. AT&T (https://www.justice.gov/atr/case-document/file/1012916/download) and FTC's administrative complaint In the Matter of Illumina, Inc. and GRAIL, Inc. (https://www.ftc.gov/system/files/documents/cases/redacted_administrative_part_3_complaint_redacted.pdf) for the vertical concerns at issue in these merger challenges.

[2] In 1979, the FTC unsuccessfully challenged Freuhauf's acquisition of Kelsey-Hayes Company. By many accounts, this was the last vertical merger challenge prior to 2017 that was not resolved via settlement. See for example, Salop (2018) and Yde (2007).

[3] See White House, "Executive Order on Promoting Competition in the American Economy," July 9, 2021 (https://www.whitehouse.gov/briefing-room/presidential-actions/2021/07/09/executive-order-on-promoting-competition-in-the-american-economy/)

[4] The concept of "raising rivals' costs" was first introduced by Salop and Scheffman (1983).

auctions to supply PBM services to Aetna's insurance rivals to raise their PBM costs and thereby expand Aetna's book of business downstream.[5]

We describe a vertical merger simulation model of the type reportedly used by DOJ to analyze these RRC concerns). Such an analysis was reported to have contributed to DOJ's conclusion that the CVS-Aetna merger was unlikely to cause vertical harm.[6]

In our second-price open auction setting, the input supplier that submits the lowest bid wins an auction, but receives the next-higher bid as compensation in that auction. Unintegrated suppliers bid their realized costs, as doing so is a weakly dominant strategy. Integrated suppliers, on the other hand, tend to bid above realized cost in auctions to supply downstream rivals. The reason is that in case the integrated supplier's bid is the *second* lowest, the elevated bid raises the downstream rival's input price. This occurs when the integrated supplier *loses* the procurement auction but comes close enough to determine the input price.

Although bidding above cost depresses the integrated firm's expected upstream profit, it tends to increase its expected downstream profit by diverting some unit sales from the downstream rival to its own downstream division. The vertically integrated firm bids above realized cost at the point where the countervailing effects on upstream and downstream expected profits just cancel at the margin.

A vertically integrated firm in our particular open auction model also tends to gain from the elimination of double markup (EDM). The downstream division of the integrated firm runs a procurement auction just like every other participant in the downstream market. The upstream division bids its realized cost in this auction. In case the upstream division's bid is the lowest, the downstream division sources input

[5] Drennan et al (2020) also discuss another vertical aspect of CVS-Aetna that DOJ investigated: CVS's ownership of a major retail pharmacy chain.

[6] "Applying a Podwol-Raskovich type of model to the relevant markets identified various factors that contributed to our conclusion that the merger was unlikely to lead to vertical harm." (Drennan et al, 2020, Section 3.2.1). In "United States v. CVS and Aetna Questions and Answers for the General Public" (https://www.justice.gov/opa/press-release/file/1099806/download), the DOJ explained that CVS was unlikely to profitably raise the PBM costs of Aetna's insurance rivals because any resulting gain to the merged firm in its health insurance business would not offset the loss of profit in the merged firm's PBM business. Horizontal concerns with the CVS-Aetna merger were resolved by consent decree requiring divestiture.

internally at the upstream division's marginal cost, rather than the next-higher bid. This is the EDM effect of vertical merger in our setting.

The RRC and EDM effects of vertical merger tend to have countervailing effects on downstream prices. To the extent that input cost changes are passed through to output prices, RRC tends to raise rivals' (and own) downstream prices, whereas EDM tends to lower own (and rivals') downstream prices. A goal of our vertical merger simulation model is to assess the net impact of these countervailing effects.[7]

Although our auction model of procurement upstream could be paired with a variety of competition models downstream, for concreteness we treat the case of downstream competitive interactions being Nash-Bertrand price setting with discrete-choice logit demand. We derive the relationship between the input price that downstream producer j realizes through its procurement auction and the downstream profits of integrated firm i.

There is an existing literature analyzing RRC where linear input prices are determined by Nash bargaining upstream. In this setting, a vertical merger tends to improve the merged firm's bargaining position in negotiations over input prices with a downstream rival, thereby raising the input price in the post-merger bargaining outcome. Recent analyses include Crawford et al (2018), Rogerson (2014) and Sheu and Taragin (2021). These studies are well suited to industries such as video programming and distribution, where distributors (the downstream firms) routinely contract with multiple programmers (the input suppliers) for the sake of bundling several inputs into an attractive final product. The RRC effect depends on the integrated programmer-distributor reaching a supply agreement both pre- and post-merger. This is in contrast to the current setting where downstream firms contract with a single input supplier. In our setting RRC arises when an integrated supplier increases its bid above cost and in so doing, raises the lowest losing bid.

[7] We distinguish the endogenous EDM effect from merger-specific marginal cost efficiencies exogenous to the model, such as a leftward shift in the upstream merging party's cost distribution when dealing with its downstream affiliate.

The remainder of the paper proceeds as follow. In Section 2 we describe the economic setting. We derive subgame-perfect equilibrium in Section 3. In Section 4 we sketch how the vertical merger simulation might be calibrated and run and Section 5 concludes.

2 Economic Setting[8]

Let $\mathcal{M}$ be the set of independent upstream firms and upstream divisions of forward-integrated firms, indexed $i = 1, 2, \ldots, m$, where $m = |\mathcal{M}|$. Let $\mathcal{N}$ be the set of independent downstream firms and downstream divisions of backward-integrated firms, indexed $j = 1, 2, \ldots, n$, where $n = |\mathcal{N}|$. To focus on the price effects of vertical integration, we abstract from horizontal merger effects by assuming that every integrated firm is composed of a single upstream division and a single downstream division (which bear the same label). Let $\mathcal{R}$ be the set of integrated firms, with $r = |\mathcal{R}| \leq \min\{m, n\}$. For notational convenience and without loss of generality, we take the first r ordered elements of both $\mathcal{M}$ and $\mathcal{N}$ to be the integrated firms.

The game is played in two stages. In stage one, each producer secures a per-unit price for an essential input via a procurement auction held among all input suppliers. The auctions take place simultaneously and the per-unit price in each auction is revealed only at the conclusion of stage one. In stage two, producers compete for the sale of final goods to end consumers, taking as given the producer's own per-unit input price as well as the per-unit input prices of rival producers.

In what follows, we describe the features of the upstream auction market assuming a generic downstream market. For the downstream market, we require only that within an equilibrium of the stage-two subgame, an increase in a producer's per-unit input price leads to a reduction in quantity demanded of the producer in question's final product and an increase in quantity demanded of rival producers' products, all else equal. In the next section, we model the downstream market as price competition with multinomial logit demand as this model has gained purchase in antitrust applications.

[8] Unless otherwise indicated, derivations of all equations in this section are in the Appendix.

Auctions in stage one are second-price: the input supplier with the lowest bid wins the auction, receiving the next-higher bid as payment per unit supplied.[9] This can be interpreted as applying to homogeneous inputs that differ only in their cost. But the model can be interpreted equally well as applying to vertically differentiated inputs, so long as the "quality-adjusted" costs are distributed in the way described below.[10] As the results are qualitatively the same under either interpretation, we keep to the simpler notation of second-price auctions involving homogeneous inputs.

For stage one, we adopt a modified version of the stochastic approach of Waehrer and Perry (2003) (WP),[11] which models a firm's cost as distributed according to the power function

$$G(c|k_i) = 1 - [1 - F(c)]^{k_i}, \qquad i \in \mathcal{M}, \tag{1}$$

where the firm-specific parameter $k_i > 0$ (in our notation) can be interpreted as the number of times input supplier i takes independent draws from the common underlying cdf $F(\cdot)$.[12] The minimum of the k_i draws taken from $F(\cdot)$ in a given procurement auction j is then i's realized cost of supplying input to downstream producer j.

WP describe how k_i can also be interpreted as i's "capacity" to supply input. In support of this interpretation, WP note that k_i can be thought of as the number of plants

[9] We assume procurers do not run "optimal" auctions for the essential input, where reserve prices are set such that with positive probability none of the essential input is procured. We appeal to the idea that, in a dynamic setting (outside our model), non-participation in the downstream market for even one period would entail an unacceptably high reputational loss for the producer as a credible market participant. In principle, a reserve price targeted to an unaffiliated integrated supplier could improve the procurer's expected auction outcome without risking non-supply by every supplier, but we do not incorporate this complication into the model. Our model is thus "conservative" in that it tends to over-predict harm, so that a finding of no significant anticompetitive effect is likely robust to the inclusion of targeted reserve pricing.

[10] Miller (2014) also analyzes procurement auctions in a merger context. Our analysis differs from his in two respects. First, Miller's (2014) focus is on horizontal mergers, whereas ours is on vertical mergers. Second, Miller's (2014) post-merger price effect flows from the merged firm withdrawing the less preferred of the two merging products from the auction. No options are withdrawn outright in our setting, however the merged firm tends to bid as if it is withdrawing some "effective capacity," as we discuss below.

[11] See also Froeb, Tschantz and Crooke (2001) for a broader discussion of power-related distributions in second-price auctions in horizontal merger analysis.

[12] Parameter k_i is a real number, not necessarily an integer. The number-of-plants metaphor thus is not exact.

from which i could produce input, each plant obtaining an iid cost draw from $F(\cdot)$ with respect to supplying a given output producer j.[13]

WP show that firm i's probability of winning a given procurement auction (also its expected share of wins across all auctions) is its capacity share,[14]

$$s_i^u = k_i/K, \tag{2}$$

where $K \equiv \sum_{h \in \mathcal{M}} k_h$ is upstream industry capacity (superscript u indicating upstream variables). In our vertical setting, however, equation (2) only holds in the absence of any RRC, as we explain presently.

We extend WP's capacity idea to a vertical setting, modeling RRC as a bidding rule akin to a capacity restriction chosen by integrated supplier i in the auction to supply a downstream rival j.

Our modeling approach to RRC involves two steps. First, we focus on the special case of $F(c) = c$, $c \in [0,1]$.[15] Absent evidence to the contrary, we consider uniformity in the underlying cdf $F(\cdot)$ to be a reasonable prior, keeping in mind that the supplier-specific capacity parameters k_i in the overarching cost distributions $G(\cdot)$ offer flexibility to capture observed asymmetries across input suppliers.

Second, we characterize integrated input suppliers' bid functions as

$$b(c_{ij};\, \theta_{ij}) = 1 - (1 - c_{ij})^{\theta_{ij}}, \qquad i \in \mathcal{M},\ j \in \mathcal{N}, \tag{3}$$

where c_{ij} is supplier i's cost draw in auction j and $\theta_{ij} \geq 1$ is a choice variable. Now define

$$\tilde{k}_{ij} \equiv k_i/\theta_{ij}. \tag{4}$$

In the Appendix, we show that

$$G\big(b(c;\theta_{ij})\big|k_i\big) = G\big(c\big|\tilde{k}_{ij}\big) = 1 - (1 - c)^{\tilde{k}_{ij}}. \tag{5}$$

[13] We note that this interpretation is only approximate as each supplier has limitless capacity to produce the input at its cost draw.

[14] Waehrer and Perry (2003, 291), Lemma 1(i).

[15] Support on $[0,1]$ is a convenient normalization for now. In simulation, we linearly transform c to a marginal cost that is uniformly distributed on $[\overline{c}, \underline{c}]$, for appropriately chosen endpoints to this interval.

Thus comparing the variant of equation (1), $G(c|k_i) = 1 - (1-c)^{k_i}$, to equation (5), a supplier i with capacity k_i that adheres to the bid function in equation (3) and chooses $\theta_{ij} > 1$ generates a distribution of bids in auction j identical to what i's distribution of costs would be if its capacity were $\tilde{k}_{ij} < k_i$.

We refer to $\tilde{k}_{ij}$ as the *effective capacity* that supplier i provides in auction j. If i chooses $\theta_{ij} > 1$, i bids *as if* it were unintegrated (bidding realized cost) but with smaller capacity $\tilde{k}_{ij}$. In the number-of-times-drawn interpretation, the integrated firm obtains its lowest cost realization by taking k_i draws from the underlying cost distribution (not $\tilde{k}_{ij}$ draws) but bids above the realized cost *as if* it had taken only $\tilde{k}_{ij}$ draws and bid the corresponding realized cost, which is higher in expectation.

Choosing $\theta_{ij} > 1$ induces a distribution of bids for downstream rival j which first-order stochastically dominates the distribution of bids for $\theta_{ij} = 1$, thereby raising j's expected input price relative to the $\theta_{ij} = 1$ baseline. Doing so also reduces i's expected profit in the auction. For this reason, unintegrated suppliers choose $\theta_{ij} = 1$. Integrated suppliers, on the other hand, choose $\theta_{ij} > 1$. We sometimes refer to the magnitude of θ_{ij} (relative to one) as the *stringency* of integrated supplier i's RRC in auction j, or synonymously as the stringency of i's "capacity restriction" to j. Although restricting capacity to a downstream rival j reduces the integrated firm's expected profit in auction j upstream, it increases expected profit downstream.

The bid function in equation (3) can be interpreted as integrated supplier i applying θ_{ij} in auction j *ex ante* of observing its cost realization in that auction. This would not be optimal if it were feasible for i to submit its bid *ex post* of observing its cost realization. In the spirit of Choi (2001), however, we treat integrated firms as consisting of upstream and downstream profit centers whose actions are coordinated by simple heuristic rules set by "headquarters," which is less well informed than its divisions about reigning market conditions.[16] At the time any auction is run, headquarters knows only its own and rival bidders' cost distributions and bidding strategies, whereas the upstream division, which submits the bid, also observes its own cost realization. In this setting, we posit that

[16] This assumption greatly simplifies the analysis as it yields an analytical distribution of each suppliers' bids as per expression (5).

headquarters directs the upstream subsidiary to bid according to equation (3),[17] where headquarters chooses θ_{ij} to maximize integrated profits.

Given that our model of the upstream stage is a special case of WP's single-stage model, ours inherits several of the equilibrium results in WP. In particular, an analog to equation (2) holds: the probability that supplier i wins auction j is

$$s_{ij}^{u} = \tilde{k}_{ij}/\tilde{K}_{j}, \qquad i \in \mathcal{M},\ j \in \mathcal{N}, \tag{6}$$

where

$$\tilde{K}_j \equiv \sum_{h\in\mathcal{M}} \tilde{k}_{hj} \tag{7}$$

is the total effective capacity provided in auction j. Note from equations (4), (6) and (7) that, holding fixed θ_{hj} for input suppliers $h \neq i$, an increase in θ_{ij} reduces the likelihood that i wins the auction to supply j. Conversely, holding fixed θ_{ij}, an increase in θ_{hj}, which tends to lower $\tilde{K}_j$, increases the likelihood that i wins auction j.

The outcome of auction j is an input price w_j for downstream producer j, equal to the second-lowest bid submitted in the auction. $\mathrm{E}[w_j]$, the expected per-unit input price, is given by

$$\mathrm{E}[w_j] = \frac{1}{\tilde{K}_j+1} + \sum_{i=1}^{m} \mathrm{E}[\tilde{\mu}_{ij}^{u}]\ , \qquad i \in \mathcal{M},\ j \in \mathcal{N}. \tag{8}$$

As we show in the Appendix, the first term on the right-hand side of equation (8), $1/(\tilde{K}_j + 1)$, is the expected value of the lowest bid submitted in auction j. The second set of terms is the sum of input suppliers' expected (per-unit) *net* margins. The expected net margin of supplier i in auction j, $\mathrm{E}[\tilde{\mu}_{ij}^{u}]$, is the expected difference between the second-lowest bid and i's bid, conditional on i having the lowest bid and hence winning auction

[17] We further posit that, in the fullness of time, headquarters has the ability to audit/observe the upstream division's past cost realizations to verify whether the upstream manager has followed the bidding rule, punishing the manager for deviations from the rule.

j.[18] The sum of expected net margins across all bidders in auction j is the expected value of the difference between the second-lowest and lowest bids among all bidders.[19]

The expected net margin of supplier i in auction j is given by

$$\mathrm{E}\left[\tilde{\mu}_{ij}^{u}\right] = \frac{\tilde{k}_{ij}}{(\tilde{K}_{-i,j}+1)(\tilde{K}_{j}+1)}, \quad i \in \mathcal{M},\ j \in \mathcal{N}, \tag{9}$$

where

$$\tilde{K}_{-i,j} \equiv \sum_{h\in\mathcal{M}\setminus i} \tilde{k}_{hj}, \quad i \in \mathcal{M},\ j \in \mathcal{N}, \tag{10}$$

is the sum of input suppliers' effective capacities for all $h \neq i$. Thus $\mathrm{E}\left[w_j\right]$ can be calculated from equations (8)-(10) given information on effective capacities in auction j.

We show in the Appendix that

$$\frac{\partial \mathrm{E}[w_j]}{\partial \theta_{ij}} = \frac{\tilde{k}_{ij}}{\theta_{ij}(\tilde{K}_j+1)^2} \sum_{h\in\mathcal{M}\setminus i} \frac{\tilde{k}_{hj}(\tilde{K}_j+\tilde{K}_{-h,j}+2)}{(\tilde{K}_{-h,j}+1)^2} > 0, \quad i \in \mathcal{M},\ j \in \mathcal{N}. \tag{11}$$

Restricting effective capacity thus raises a downstream rival's expected input price, and the greater the reduction in effective capacity (higher θ_{ij}), the higher the downstream rival's expected input cost, all else equal.

In evaluating the profit impact of an RRC tactic, an integrated input supplier looks to its *gross* upstream margins per-unit sale of input into product j (assumed to be in fixed proportion) which are then weighted by the sales of product j. The gross margin of supplier i in auction j is

$$\mathrm{E}\left[\mu_{ij}^{u}\right] \equiv \mathrm{E}\left[\tilde{\mu}_{ij}^{u}\right] + \xi_{ij}, \quad i \in \mathcal{M},\ j \in \mathcal{N},$$

where ξ_{ij} is the expected bid-cost spread, the expected difference between i's bid in auction j and i's realized cost in that auction, conditional on winning the auction. This

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[18] Put differently, $\mathrm{E}\left[\tilde{\mu}_{ij}^{u}\right]$ is what i's expected profit margin *would be* in auction j if i's effective capacity $\tilde{k}_{ij}$ were to equal its actual capacity k_i (i.e., as if i's bid equaled its realized cost).

[19] Equation (8) derives from WP's Lemma 1 (ii) (Waehrer and Perry (2003, at 291)), applied to our particular cost distribution, replacing actual capacities k_i with effective capacities $\tilde{k}_{ij}$ and replacing expected gross margins (discussed below) with expected net margins $\mathrm{E}\left[\tilde{\mu}_{ij}^{u}\right]$. In WP's setting, auction outcomes are allocatively efficient, because suppliers bid their costs, so the winning bidder necessarily has the lowest cost. This is no longer true in our vertical setting, where integrated firms tend to bid above cost. An integrated firm may lose an auction despite having the lowest cost. Equation (8) nonetheless holds.

expected spread is $\xi_{ij} \equiv \mathrm{E}\left[b_{ij} - c_{ij} | i \text{ wins } j\right] \times \Pr\{i \text{ wins } j\}$. We derive ξ_{ij} in the Appendix in terms of effective capacities, as

$$\xi_{ij} = \frac{k_i - \tilde{k}_{ij}}{(\theta_{ij}\tilde{K}_j + 1)(\tilde{K}_j + 1)}, \quad i \in \mathcal{M},\ j \in \mathcal{N}. \tag{12}$$

Note that $\xi_{ij} = 0$ for $\theta_{ij} = 1$ and ξ_{ij} tends to increase with θ_{ij}.[20]

Despite RRC, a vertical merger's net effect on downstream prices may be nonpositive given the merger's EDM effect, to which we now turn. If procurer j is backward integrated, it does not pay a markup over cost when purchasing from its integrated supplier, so the upstream division's gross margin from internal supply is $\mu_{ii}^{u} = 0$.[21] Moreover, if i and j merge, the merger's first-order effect[22] on j's expected input price is

$$\mathrm{EDM} = -\mathrm{E}\left[\mu_{ij}^{u}\right], \tag{13}$$

where $\mathrm{E}\left[\mu_{ij}^{u}\right]$ refers to the pre-merger value. For a previously unintegrated input supplier, equation (13) follows from the fact that the term $\mathrm{E}\left[\tilde{\mu}_{ij}^{u}\right]$ on the right-hand side of equation (8) falls to zero post-merger, and $\xi_{ij} = 0$ in this case.[23]

To sum up, restricting effective capacity depresses input supplier i's expected profit upstream from supplying a downstream rival j, but tends to raise j's expected input cost, thereby tending to divert unit sales to i's downstream division and so increasing i's downstream profit. This highlights the tradeoff faced by an integrated firm contemplating RRC: in attempting to raise a rival's cost to gain sales in the downstream

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[20] For unintegrated input suppliers, $\xi_{ij} = 0$ and so expected gross and net profits are equal. A divergence between expected gross and net profits arises from RRC by an integrated supplier. Note that ξ_{ij} is a pure transfer to (winning) supplier i that has no direct effect on j's input price, hence the absence of ξ_{ij} from the right-hand side of equation (8).

[21] In our conceptual framing, headquarters instructs the upstream division to rebate to the downstream division not only the difference between the upstream division's bid and the next-higher bid, so that $\tilde{\mu}_{jj}^{u} = 0$, but also to bid its realized cost, so that $\xi_{jj} = 0$ as well.

[22] By "first-order effect" we mean holding fixed θ_{hl}, $h \in \mathcal{M}\backslash i$, $l \in \mathcal{N}$.

[23] If i is already forward-integrated prior to its vertical merger with j, the first-order EDM effect will also include eliminating the expected pre-merger markup $\xi_{ij} > 0$. Thus, all else equal, a vertical merger by an already integrated input supplier will tend to have a larger EDM effect than one by a previously unintegrated supplier. A vertical merger by an already integrated input supplier will, however, also have a horizontal aspect to be accounted for when evaluating the merger's overall likely competitive effects.

market, it must sacrifice some profit in the upstream market. Its choice of θ_{ij} captures this tradeoff.

3 Equilibrium

Our equilibrium concept is subgame perfection. At the start of stage one, every input supplier chooses its effective capacity in every auction, characterized by an $m \times n$ matrix $\boldsymbol{\theta}$. An integrated supplier i's effective capacities across auctions are characterized by the vector $\boldsymbol{\theta}_{\boldsymbol{i}}^{*}$, which maximizes i's integrated profits given $\boldsymbol{\theta}_{\boldsymbol{h}}^{*}$, $h \in \mathcal{M} \backslash i$, taking into account the implied sacrifices to upstream profits and given the mapping of $\boldsymbol{\theta}^{*}$ onto downstream profits π_i^d.[24]

Once input suppliers have chosen their effective capacities, procurers run their auctions simultaneously. The auction outcomes form a vector of input prices $\boldsymbol{w} = (w_1, w_2, \dots, w_n)$. Procurers take $\boldsymbol{w}$ as parametric in stage-two competition when choosing their final-goods prices. This yields a particular profit realization π_i^d for every integrated firm i.

We first characterize the equilibrium in the downstream subgame given input prices $\boldsymbol{w}$. The effective capacities chosen in stage one map stochastically onto downstream profits π_i^d through outcomes $\boldsymbol{w}$. In stage one, integrated suppliers choose their effective capacities so that, at the margin, the countervailing effects of foregone upstream profit and gained downstream profit just balance.

3.1 Equilibrium in the Downstream Subgame

Producers sell differentiated products and compete over prices in Nash-Bertrand fashion. The timing is as follows: producers simultaneously choose prices; once demand is realized, producers purchase inputs at the predetermined price and produce final goods. For ease of exposition, we assume one product per producer. Consumers have

[24] Here it becomes evident that specifying the bid function as an ex-ante strategy greatly simplifies the equilibrium characterization as strategies depend only on public information. If instead, bids were to be chosen after a supplier draws its cost, strategies would depend on private information as in much of the auction literature. We would then require an equilibrium concept such as perfect-Bayesian equilibrium which incorporates beliefs about rivals' private information and a plan of action for each cost that a rival may draw.

unit demands and choose the product that maximizes utility according to the multinomial logit discrete choice model.[25] Given final goods prices $\boldsymbol{p} = (p_1, p_2, \dots, p_n)$ downstream, the probability that a consumer chooses final good j is, [26]

$$s_j^d = \frac{\exp(\delta_j - \alpha p_j)}{1 + \sum_{h \in N} \exp(\delta_h - \alpha p_h)}, \qquad j \in \mathcal{N}, \tag{14}$$

where parameter δ_j is the mean quality of good j and parameter α is consumers' common disutility of price. The mean utility of the outside good, product 0, is normalized to zero so that $\exp(\delta_0 - \alpha p_0) = 1$ in the denominator of expression (14). We write the choice probability as s_j^d in recognition that this is j's expected market share downstream.

Differentiating equation (14) with respect to a price yields

$$\frac{ds_j^d}{dp_h} = \begin{cases} -\alpha s_j^d (1 - s_j^d) & \text{for} \quad h = j; \\ \alpha s_j^d s_h^d & \text{for} \quad h \neq j. \end{cases} \tag{15}$$

Firm j's (expected) per-customer profit is then

$$\pi_j^d = \left(p_j - w_j - c_j^d\right) s_j^d, \tag{16}$$

where, as before, w_j is j's realized input cost from the outcome of the procurement stage one and c_j^d captures all of j's other marginal costs. The first-order condition for profit maximum with respect to own price is

$$\begin{aligned} 0 &= s_j^d + \left(p_j - c_j^d - w_j\right) \frac{\partial s_j^d}{\partial p_j} \\ &= s_j^d \left[1 - \left(p_j - c_j^d - w_j\right) \alpha (1 - s_j^d)\right], \end{aligned} \tag{17}$$

where the second equality makes use of expression (15). Rearranging terms within square brackets on the right-hand side of equation (17), we obtain j's per-customer variable margin as

[25] Werden and Froeb (1994) introduced the application of logit demand to (horizontal) merger policy.

[26] Indirect utility of consumer l for product j has the form, $u_{lj} = \delta_j - \alpha p_j + \epsilon_{lj}$. The rightmost term, ϵ_{lj} is an independently, identically distributed Type I extreme value taste component with scale parameter 1. Integrating over the ϵ_{lj} terms, taking as given the δ_j and p_j terms, gives rise to expression (15).

$$\mu_j^d \equiv p_j - c_j^d - w_j = \frac{1}{\alpha(1-s_j^d)}\,. \tag{18}$$

We now make use of the above expressions to characterize the impact on integrated firm i's price and gross profit from changes in its own input price via EDM and changes in rivals' prices via RRC. A change in θ_{ij} changes the distribution of w_j. Since there is no closed-form solution in final good prices (as per equations (16) and (18)), the full effect of θ_{ij} on π_i^d can only be obtained via numerical methods. For this reason, the results to follow should be thought of as "reduced form" and intended to provide intuition for the connection between upstream actions and downstream outcomes.

We get j's "reaction" to h's price by differentiating equation (18) with respect to p_h:

$$\frac{dp_j}{dp_h} = \frac{1}{\alpha\left(1-s_j^d\right)^2}\left[\frac{ds_j^d}{dp_j}\frac{dp_j}{dp_h} + \frac{ds_j^d}{dp_h}\right], \qquad j \in \mathcal{N},\ h \in \mathcal{N} \setminus j\,. \tag{19}$$

Substituting equation (15) into (19) and rearranging terms, we have the instantaneous reaction function

$$\frac{dp_j}{dp_h} = \frac{s_j^d s_h^d}{1-s_j^d}, \qquad j \in \mathcal{N},\ h \in \mathcal{N} \setminus j. \tag{20}$$

To obtain the pass-through rate of j's input price to its product price, we begin by totally differentiating equation (19) with respect to w_j:

$$\frac{dp_j}{dw_j} - 1 = \frac{1}{\alpha\left(1-s_j^d\right)^2}\left[\frac{ds_j^d}{dp_j}\frac{dp_j}{dw_j} + \sum_{h\in\mathcal{N}\setminus j}\frac{ds_j^d}{dp_h}\frac{dp_h}{dp_j}\frac{dp_j}{dw_j}\right]. \tag{21}$$

Substituting equations (15) and (19) into equation (21) and rearranging terms then yields

$$\frac{dp_j}{dw_j} = \left(1-s_j^d\right)\left[1 - \frac{\left(s_j^d\right)^2}{1-s_j^d}\sum_{h\in\mathcal{N}\setminus j}\frac{\left(s_h^d\right)^2}{1-s_h^d}\right]^{-1}. \tag{22}$$

We show in the Appendix that $dp_j/dw_j > 0$. Thus a decrease in j's input cost owing to EDM post-merger will lead j to lower its final good price. While rival producers will lower their output prices in turn as per equation (22), j's downstream market share still increases on net.

With the foregoing results in hand, we now turn to the impact of a marginal increase in a downstream rival j's input cost w_j on integrated firm i's downstream profit. Inspection of equation (18) indicates that a change in w_j affects firm i's profit through its price, p_i, and through its market share, s_i. The first effect is essentially zero due to the envelope theorem, so our focus is on the second channel. Differentiating equation (16) with respect to w_j yields

$$\frac{d\pi_i^d}{dw_j} = \left(p_i - c_i^d - w_i\right)\left[\frac{ds_i^d}{dp_i}\cdot\frac{dp_i}{dp_j} + \frac{ds_i^d}{dp_j}\cdot\frac{dp_j}{dw_j}\right]$$

$$= s_i^d s_j^d \cdot \frac{dp_j}{dw_j}$$

$$= \frac{1}{\alpha}\cdot\frac{ds_i^d}{dp_j}\cdot\frac{dp_j}{dw_j} \tag{23}$$

where the second equality is arrived at by substitution via equations (15), (18), and (20). The third equality follows by observing that $s_i^d s_j^d$ is proportional to ds_i^d/dp_j in (16).

Integrated firm i benefits from an increase in rival j's input cost as it causes product j's price to rise and some consumers to switch from product j to product i. Expression (23) shows that the marginal benefit of RRC downstream to integrated supplier i is proportional to the product of the pass-through rate of j's input cost to j's price and the rate at which consumers switch to downstream division i in response to an increase in p_j.

Inspection of equations (15) and (21) indicate that the downstream benefit of RRC to integrated input supplier i is greatest when (a) product i's market share is large, as this indicates that i is a more attractive substitute to j and (b) product j's market share is at an intermediate value. The larger is s_j (relative to the share of other rival products including the outside good), the greater is the fraction of consumers who would potentially switch from j to i for a given increase in p_j, while at the same time, the lower is the pass-through rate of an increase in w_j. The optimal value of s_j from firm i's perspective balances these competing effects.

3.2 Equilibrium in Effective Capacities Upstream

The headquarters of integrated supplier i chooses $\boldsymbol{\theta}_i^*$ to maximize expected integrated profits of

$$\mathrm{E}[\pi_i] = \sum_j \mathrm{E}\left[\mu_{ij}^u \cdot s_j^d\right] + \mathrm{E}\left[\mu_i^d \cdot s_i^d\right], \qquad i \in \mathcal{R},\ j \in \mathcal{N}, \tag{24}$$

The corresponding system of first-order conditions for integrated firms is then

$$\frac{\partial \mathrm{E}\left[\mu_{ij}^u \cdot s_j^d\right]}{\partial \theta_{ij}} + \frac{\partial \mathrm{E}\left[\mu_i^d \cdot s_i^d\right]}{\partial \theta_{ij}} = 0, \qquad i \in \mathcal{R},\ j \in \mathcal{N}, \tag{25}$$

where every equation ij holds fixed the θs of all other input suppliers and in all auctions.

The solution to the system of equations (25) is an $r \times n$ submatrix $\boldsymbol{\theta}_r^*$, whose main diagonal elements are 1 and off-diagonal elements are typically greater than 1. Upstream equilibrium is then characterized by an $m \times n$ matrix $\boldsymbol{\theta}^*$ whose elements are 1 except for the off-main-diagonal elements of the submatrix $\boldsymbol{\theta}_r^*$.

The first summand in equation (25) is the integrated firm's expected upstream profit, which consists of the gross margin per-unit sale of input into product j (assumed to be one-to-one fixed proportion) weighted by total sales of product j. Note that a more stringent capacity restriction impacts an integrated firm i's expected gross margin on the sale of the input only to the extent that it decreases i's probability of winning the auction for some producer j. This is due to the fact that the auction is second price: the price is determined by the lowest *losing* bid. Conversely, a more stringent capacity restriction impacts producer j's market share downstream, s_j^d, only to the extent that it increases the lowest losing bid in the auction for producer j. Thus, a change in θ_{ij} impacts the product of μ_{ij} and s_j^d only at the point at which an increase in $b(c_{ij};\theta_{ij})$ causes i to lose the auction to supply producer j.

Computing this derivative, we find that the first summand in equation (25) is zero evaluated at $\theta_{ij} = 1$ and strictly negative for $\theta_{ij} > 1$.[27] Thus, by an envelope theorem, at least some RRC is optimal for a vertically integrated firm, given that RRC has a first-order effect of increasing the integrated firm's expected downstream sales but a second-order effect on upstream profit evaluated at $\theta_{ij} = 1$.

The second summand in (25), $\partial \mathrm{E}\left[\mu_i^d \cdot s_i^d\right]/\partial \theta_{ij}$, requires some unpacking. Expression (16) expressed an integrated firm's downstream profit given a vector of realized input

[27] The derivation is in the Appendix.

prices, $\boldsymbol{w} = (w_1, w_2, \dots, w_n)$. At the beginning of stage one, the input prices have yet to be determined and depend stochastically on each $\boldsymbol{\theta}_{\boldsymbol{r}}^*$. To obtain an expected downstream profit $\mathrm{E}[\mu_i^d \cdot s_i^d]$, firm i integrates over all possible input price outcomes $\boldsymbol{w}$ weighted by the probability density of the outcome. The probability densities depend on $\boldsymbol{\theta}^*$, which every $i \in \mathcal{R}$ anticipates given the structure of the game. A change in θ_{ij} shifts the distribution of j's input price and thereby $\mathrm{E}[\mu_i^d \cdot s_i^d]$. We discuss the challenges to computing equilibrium in Section 4.

4 Calibration and Simulation

We treat a vertical merger as moving from a pre-merger equilibrium $\boldsymbol{\theta}^{\boldsymbol{pre}}$ with a number of integrated firms $r_0 \leq \min\{m-1, n-1\}$ to a post-merger equilibrium $\boldsymbol{\theta}^{\boldsymbol{post}}$ with $r_0 + 1$ integrated firms. Post-merger, all upstream and downstream firms re-optimize given the new integration structure.

In this section, we describe how the model may be used to predict the effects of a vertical merger using data that might reasonably be available to antitrust practitioners. The analysis involves three stages: first, using data to recover the parameters underlying the pre-merger equilibrium; next, solving for the merged entity's optimal RRC strategy; and then assessing the expected impact on downstream prices inclusive of both RRC and EDM effects. The data assumed to be available are prices and market shares for upstream and downstream market participants, their status as integrated or not, and a variable cost margin for one firm at each level (typically the merging firms). Additional details on derivations are in the Appendix.

4.1 Calibrating to the Pre-Merger Equilibrium Downstream

Expression (14) is a closed-form solution for product $j's$ market share ($j \in \mathcal{N}$). The empirical analogue of (14) is

$$\hat{s}_j^d = \frac{\exp(\gamma_j)}{1+\sum_{h \in N} \exp(\gamma_h)}, \quad h, j \in \mathcal{N}, \tag{26}$$

where $\hat{s}_j^d$ is j's observed market share and $\gamma_j \equiv \delta_j - \alpha p_j$ is its mean utility. Contained in (26) is a system of n equations with n unknowns with a well-known solution:

$$\hat{\gamma}_j = \ln \hat{s}_j^d - \ln \hat{s}_0^d. \tag{27}$$

The component parameters δ_j and α can be obtained by incorporating a supply side moment. Expression (18) provides an analytical expression for a producer's per-unit variable margin. Given data on the merging firm's per-unit gross profit downstream (call it firm 1), we can express the empirical analogue of expression (18) as

$$\hat{\mu}_1^d = \frac{1}{\hat{\alpha}(1-\hat{s}_1^d)} \tag{28}$$

where $\hat{\mu}_1^d$ is firm 1's downstream per-unit gross variable margin pre-merger, which we take to be observable. Rearranging terms in equation (28), we obtain the disutility of price parameter:

$$\hat{\alpha} = \hat{\mu}_1^d(1 - \hat{s}_1^d). \tag{29}$$

Finally, given estimates of $\{\hat{\gamma}_j\}_{j\in\mathcal{N}}$ from equation (27) along with an estimate of the average price for each producer $\{\hat{p}_j\}_{j\in\mathcal{N}}$, we obtain the mean quality parameter for each producer:

$$\hat{\delta}_j = \hat{\gamma}_j + \hat{\alpha}\hat{p}_j. \tag{30}$$

4.2 Calibrating to the Pre-Merger Equilibrium Upstream

We proceed in stages, from the simplest case to more difficult cases. In the simplest case, there are no vertically integrated firms pre-merger, every input supplier i chooses $\theta_{ij} = 1$, and thus effective capacities equal actual capacities, $\tilde{k}_{ij} = k_i$.

The no-prior integration case:

Expression (6) derives supplier i's market share as the ratio of its capacity to aggregate market capacity. With no vertically integrated firms, all buyers are treated symmetrically (i.e., $\theta_{ij} = 1$ for all j), so we drop the producer subscripts in much of what follows. Given data on upstream market shares, $\{\hat{s}_i^u\}$, we can identify the ratio of firm-to-aggregate capacity by

$$\hat{s}_i^u = k_i/K. \tag{31}$$

To separately identify the component terms, we turn to average prices for additional moments. Recall that in adopting the stochastic approach of WP in expression (1), we took $F(c)$ to be uniform on [0,1]. In fitting the model to data, we allow for greater flexibility by assuming F is uniform over some interval $[\underline{c}, \overline{c}]$, where $\underline{c}$ and $\overline{c}$ are parameters to be estimated. The ex-ante bid function from expression (3) is easily adapted to the case where F is uniform on $[\underline{c}, \overline{c}]$ so that,[28]

$$b(c_{ij}; \theta_{ij}) = \overline{c} - (\overline{c} - \underline{c}) \left(\frac{\overline{c} - c_{ij}}{\overline{c} - \underline{c}}\right)^{\theta_{ij}} \tag{32}$$

Taking $F(c) = (c - \underline{c})/(\overline{c} - \underline{c})$, we have that the expected input price for supplier i given that supplier i wins the auction is,

$$\begin{aligned} W_i &= \underline{c} + \frac{\overline{c} - \underline{c}}{K - \tilde{k}_{ij} + 1} + \frac{K - k_i}{K - k_i + 1} \frac{1}{k_i + 1} \\ &= \underline{c} + \frac{\overline{c} - \underline{c}}{K(1 - \hat{s}_i^u) + 1} + \frac{K(1 - \hat{s}_i^u)}{K(1 - \hat{s}_i^u) + 1} \frac{1}{\hat{s}_i^u K + 1}, \end{aligned} \tag{33}$$

where the second equality in (33) takes $k_i = \hat{s}_i K$ from (31). There are three unknowns in expression (33), K, $\underline{c}$, and $L = \overline{c} - \underline{c}$; W_i is monotonic in each. It follows that relying on the average price for three suppliers is sufficient to identify the upstream model pre-merger. In a market with only two suppliers (or where average prices for only two suppliers are available), a profit margin for one supplier is sufficient. The per-unit variable-cost markup for supplier i conditional on winning the auction is,

$$\begin{aligned} U_i &= \frac{(\overline{c} - \underline{c}) k_i}{[K - k_i + 1][k_i + 1]} \\ &= \frac{(\overline{c} - \underline{c}) K \hat{s}_i^u}{[K(1 - \hat{s}_i^u) + 1][K \hat{s}_i^u + 1]}. \end{aligned} \tag{34}$$

The reliance on expression (34) in calibration may impose constraints on the parameter K. The issue is that U_i is not monotonic in K when $\hat{s}_i^u$ is close to 0 or 1, whereas

[28] Expression (32) reduces to expression (3) if $\underline{c} = 0$ and $\overline{c} = 1$. In the Appendix, we show that the bid function defined by (32) is distributed $G(\cdot\,|k_i/\theta_{ij})$, appropriately defined, so that all of the analytical results from Section 2 extend to generalized uniform case. In fact, these results can be shown to extent to any F that is invertible.

we can only identify K from a monotonic region. The variable-cost markup expression is decreasing in K if

$$K > \sqrt{\frac{1}{\hat{s}_i^u(1-\hat{s}_i^u)}}. \tag{35}$$

Having identified the pre-merger model from (31) and some combination of (33) and (34), we calculate the merged firm's optimal RRC severity using a hill-climbing routine. For each θ_{ij} that integrated firm i may choose, we can simulate the n procurement auctions upstream to determine input prices and firm i's per-unit margin. The input prices are fed into equations (14) and (18) to determine downstream prices, market shares, and hence profits in the upstream and downstream markets. The simulation is repeated thousands of times to obtain expected upstream and downstream profits. We then choose an incrementally larger/smaller value of θ_{ij} and repeat the process to see if firm i's expected combined profit increases.

The following describes the basic properties of the profit function. The first summand in equation (24) is the integrated firm's expected upstream profit, which consists of the gross margin per-unit sale of input into product j (assumed to be one-to-one fixed proportion) weighted by total sales of product j. Note that a more stringent capacity restriction impacts an integrated firm i's expected gross margin on the sale of the input only to the extent that it decreases i's probability of winning the auction for some producer j. This is due to the fact that the auction is second price: the price is determined by the lowest *losing* bid. Conversely, a more stringent capacity restriction impacts producer j's market share downstream, s_j^d, only to the extent that it increases the lowest losing bid in the auction for producer j. Thus, a change in θ_{ij} impacts the product of μ_{ij}^u and s_j^d only at the point at which at increase in $b(c_{ij};\theta_{ij})$ causes i to lose the auction to supply producer j.

To make explicit the connection between the right-hand side of (14) and upstream random variables, let $s_j^d = \sigma_j(w_1, \dots, w_j, \dots, w_n)$ denote producer j's market share as a function of input prices, $\{w_l\}$. The input prices, $\{w_l\}$, are treated as random variables by integrated firm i when it is choosing its RRC severity. Letting $g(\cdot|\widetilde{K}_{-ij})$ denote the density of $b_{-i} = \min_{h\neq i}\{b_h\}$,[29] we can express the first summand in expression (25) as,

$$\frac{\partial \mathrm{E}\left[\mu_{ij}^{u} \cdot s_j^d\right]}{\partial \theta_{ij}} = -\frac{\partial b(c_{ij};\theta_{ij})}{\partial \theta_{ij}} \times$$

$$\left[\int \int_0^1 \left[b(c;\theta_{ij}) - c\right] \sigma\left(w_1, \dots, b(c;\theta_{ij}), \dots, w_m\right) g\left(b(c;\theta_{ij})|\widetilde{K}_j\right) dG(c|k_i)\, dG_{\mathbf{w}_{-j}}\right], \quad (36)$$

where w_j denotes producer j's (stochastic) input price, and

$$\frac{\partial b(c_{ij};\theta_{ij})}{\partial \theta_{ij}} = -(\overline{c} - \underline{c}) \ln\left(\frac{\overline{c} - c_{ij}}{\overline{c} - \underline{c}}\right) \cdot \left(\frac{\overline{c} - c_{ij}}{\overline{c} - \underline{c}}\right)^{\theta_{ij}} > 0 \quad (37)$$

by differentiating expression (35). Thus, a marginal increase in θ_{ij} decreases i's upstream profit, strictly so for $\theta_{ij} > 1$. We also have that successive increases in θ_{ij} must eventually lead to smaller reductions in expected upstream profit since in the limit as $\theta_{ij} \to \infty$, $b(c;\theta_{ij}) \to \overline{c}$, and hence $g\left(b(c;\theta_{ij})|\widetilde{K}_j\right) \to 0$. Thus, the first summand in (25) is well behaved.

The second summand in equation (25) is the integrated firm's expected downstream profit, which consists of the gross margin per-unit sale of firm i's final product weighted by total sales of the product. An increase in θ_{ij} changes the distribution of w_j making higher values more likely. Should a higher value of w_j be realized, firm i's downstream profit is increased via expression (23). The profit-increasing impact of increasing θ_{ij} must eventually diminish since in the limit as $\theta_{ij} \to \infty$, $b(c\colon \theta_{ij}) \to \overline{c}$, wherein firm i loses the ability to shift the distribution of w_j any further.

The prior integration case:

Now consider the case where some subset of firms, $\mathcal{R}$, are vertically integrated prior to the merger in question. There is an $r \times n$ submatrix $\boldsymbol{\theta}_r^*$, whose main diagonal elements are 1 and off-diagonal elements are typically greater than 1, which describes the pre-merger RRC severities of these integrated firms. In this case, the calibration exercise is more challenging. Because integrated firm h's bid is specific to the producer, the

[29] This derivation allows for the possibility that other firms are integrated, which will be useful later on. In the current case where no firms are integrated prior to the merger in question, $b_{-ij} = \min_{h\neq i}\{c_{hj}\}$ and its density is $g(c|K_{-i}) = dG(c|K_{-i})/dc$.

probability that it wins a given auction, $s^d_{hj} = \tilde{k}_{hj}/\widetilde{K}_j$, will vary by auction. We do not observe these win probabilities only whether or not firm h won the business. Input prices and profit margins do not help since the integration-analogue of expressions (33) and (34) replace each instance of K with $\widetilde{K}_j$, where the latter varies by auction.[30] Even if we observe all n input prices, we have $m + 2 + r(n-1) > n$ unknowns to solve for.

Incorporating supplemental data on bids placed by integrated firms, including bids in auctions they did not win, allows us to make progress. Each bid in $\widehat{\boldsymbol{b}}$, the set of observed bids, reflects the firm's choice of θ_{hj}, which is made so as to satisfy the optimality condition in expression (25). The optimal value of θ_{hj} reflects the vector of capital, $\{k_i\}_{i\in\mathcal{M}}$, the vector of product quality parameters, $\{\delta_j\}_{j\in\mathcal{N}}$, and the firm's equilibrium guess as to its rivals' RRC severities, $\{\theta_{ij}\}_{i\in\mathcal{R}\backslash h}$. The product-quality parameters can be calibrated without regard to RRC severities via equations (26)-(30). It remains to identify $\{k_i\}_{i\in\mathcal{M}}$ and $\{\theta_{ij}\}_{i\in\mathcal{R}}$, along with $\{\underline{c}, \overline{c}\}$.

We propose the following maximum likelihood routine.

(0) Begin with a guess as to each element of $\{k_i\}_{i\in\mathcal{M}}$ and $\{\underline{c}, \overline{c}\}$. A plausible starting point would be the values suggested by (31)-(33) from the no integration case.

(1) Next, solve for each integrated firm's vector of RRC severities, $\{\theta_{ij}\}_{j\in\mathcal{M}}$ under the assumption that all θ_{hj}, $h \neq i$, are equal to 1. Plug the "first round" of RRC severities for each rival $h \neq i$ into i's profits and re-solve for the optimal $\{\theta_{ij}\}_{j\in\mathcal{M}}$; continue to iterate in this way until a minimum distance threshold is met for all integrated suppliers. Let $\hat{\theta}_{ij}$ denote an element of the set of optimal RRC severities.

(2) The distributional parameters, $\{\underline{c}, \overline{c}\}$, can be identified from the integration-analogue of (32) and (33) using the $\tilde{k}_{ij}$ and $\widetilde{K}_j$ derived in Step 1.

(3) Calculate the likelihood of the observed bids, $\widehat{\boldsymbol{b}}$, given that each element, $\hat{b}_{ij}$, is drawn from $G(\cdot\,|k_i/\hat{\theta}_{ij})$.

(4) Next, return to the guess as to $\{k_i\}_{i\in\mathcal{M}}$ and update the guess. To the extent that the initial values in Step 0 understate the k_i for integrated firms, this step should adjust

[30] See expressions (A31) and (A32) in the Appendix.

k_i upward for firms for which the $\hat{\theta}_{ij}$ significantly exceed 1. Given the revised guess, repeat Steps 1-3.

(5) After sufficient repetitions of Steps 1-4, select the $\{k_i\}$ corresponding to the maximum likelihood.

The above routine identifies the pre-merger model. We then solve for the merged firm's optimal RRC severities, while also accounting for equilibrium reactions in the RRC severities of rival integrated firms, following the approach of Step (1) above.

5 Conclusion

We develop a two-stage model of vertical merger with input procurement auctions in the first (upstream) stage. Our model presents an analysis of the type that was reportedly used by DOJ as part of its assessment in the CVS-Aetna merger case of the potential for harm from raising rival insurers' costs (RRC) for pharmacy benefit management services.

We treat RRC as akin to "capacity restriction" in procurement auctions, with vertically integrated firms bidding higher than their realized costs as if their "capacities" to realize low costs are smaller than they are. In our setting, vertical merger also entails an elimination of double markup (EDM) effect. Given the interplay between RRC and EDM effects, a vertical merger's net effect on downstream prices is an empirical question. We describe how our simulation model can be implemented to answer this question, both for the case of no prior vertical integration and the more complex case of prior integration in the industry.

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Appendix

***Derivation of equations* (2) and (6)**:

WP derive equation (2) for the more general case, but it may help the reader to show the derivation for our special case of $F(c) = c$. Clearly the derivation of equation (6) is the same as for equation (2), *mutatis mutandis*.

Given a cost realization c_i, the probability that input supplier i wins any given auction (assuming no RRC for now, as in equation (2)) is

$$\Pr\{i \text{ wins} | c_i = c\} = \prod_{h \in M \setminus \{i\}} [1 - G(c_i | k_h)] = (1 - c_i)^{K - k_i}. \tag{A1}$$

Integration then yields

$$s_i^u = \int_0^1 (1 - c)^{K - k_i} dG(c | k_i), \tag{A2}$$

Now $dG(c|k_i) = k_i(1-c)^{k_i - 1}$, so (A2) becomes

$$s_i^u = k_i \int_0^1 (1 - c)^{K-1} dc. \tag{A3}$$

The antiderivative of $(1-c)^{K-1}$ is $\frac{1}{K}(1-c)^K$ and $\left[\frac{1}{K}(1-c)^K\right]_0^1 = \frac{1}{K}$, therefore $s_i^u = k_i / K$ ∎

***Derivation of equation* (5)**:

Let c_{ij} be i's realized cost in auction j. Conditional on drawing c_{ij} and bidding according to $b(c_{ij}; \theta_{ij})$, the probability that i's bid in auction j is less than some value c is

$$\Pr\{b(c_{ij}; \theta_{ij}) < c | k_i\} = \Pr\left\{1 - (1 - c_{ij})^{\theta_{ij}} < c \middle| k_i\right\}$$

$$= \Pr\left\{c_{ij} < 1 - (1 - c)^{\frac{1}{\theta_{ij}}} \middle| k_i\right\} = 1 - (1 - c)^{\frac{k_i}{\theta_{ij}}} = G\left(c \middle| \frac{k_i}{\theta_{ij}}\right) = G(c | \tilde{k}_{ij}) \;∎$$

Proof that $1/(\tilde{K}_j + 1)$ in equation (8) is the expected value of the lowest bid in auction j:

Let $b_j \equiv \min\{b_{hj}\}_{h \in M}$ denote the lowest bid (i.e., the winning bid) among all m input suppliers in auction j. To calculate $\mathrm{E}[b_j]$, we begin with its cdf,

$$\Pr\{b_j < b\} = 1 - \Pr\{b_j > b\} = 1 - \prod_{i \in M} \Pr\{b > b_j\} = 1 - \prod_{i \in M} (1 - b_j)^{\tilde{k}_{ij}}$$

$$= 1 - \left(1 - b_j\right)^{\tilde{K}_j} = G(b_j|\tilde{K}_j)\,. \tag{A4}$$

Its density is

$$dG(b_j|\tilde{K}_j) = \tilde{K}_j\left(1 - b_j\right)^{\tilde{K}_j - 1} db_j\,, \tag{A5}$$

from which it follows that

$$\mathrm{E}\left[b_j\right] = \int_0^1 b_j\, dG(b_j|\tilde{K}_j) = \int_0^1 b_j \tilde{K}_j\left(1 - b_j\right)^{\tilde{K}_j - 1} db_j\,. \tag{A6}$$

Integrating by parts then yields

$$\mathrm{E}\left[b_j\right] = \left[-b_j\left(1 - b_j\right)^{\tilde{K}_j}\right]_0^1 - \left[-\frac{1}{\tilde{K}_j+1}\left(1 - b_j\right)^{\tilde{K}_j+1}\right]_0^1 = \frac{1}{\tilde{K}_j+1}\ \blacksquare \tag{A7}$$

***Derivation of equation* (9)**:

Let $b_{-i,j} \equiv \min_{h\in M\setminus i}\{b_{hj}\}$ be the lowest bid in auction j among input suppliers $h \neq i$. Conditional on winning auction j with a bid of b_{ij}, supplier i's net margin is $b_{-i,j} - b_{ij}$. To obtain the expected net margin, we proceed in two steps. We integrate over outcomes $b_{-i,j}$, then integrate over outcomes b_{ij}, conditional on every possible realization $b_{-i,j}$.

Taking the expectation over $b_{-i,j}$ while conditioning on b_{ij}, i's ex post margin is

$$\mathrm{E}\left[b_{-i,j} - b_{ij}|b_{-i,j} > b_{ij}\right] \times \Pr\left\{b_{-i,j} > b_{ij}\right\}. \tag{A8}$$

To calculate this, we need the density of $b_{-i,j}$. We have

$$\Pr\left\{b_{-i,j} < b_{ij}\right\} = 1 - \Pi_{h\in M\setminus i} \Pr\left\{b_{-i,j} > b_{ij}\right\}$$

$$= 1 - \left(1 - b_{-i,j}\right)^{\tilde{K}_j - \tilde{k}_{ij}} = G\left(b_{ij}|\tilde{K}_j - \tilde{k}_{ij}\right). \tag{A9}$$

The density of $b_{-i,j}$ is then

$$dG\left(b_{ij}|\tilde{K}_j - \tilde{k}_{ij}\right) = \left(\tilde{K}_j - \tilde{k}_{ij}\right)\left(1 - b_{-i,j}\right)^{\tilde{K}_j - \tilde{k}_{ij} - 1} db_{-i,j}\,. \tag{A10}$$

Thus

$$\mathrm{E}\left[b_{-i,j}|b_{-i,j} > b_{ij}\right] \times \Pr\left\{b_{-i,j} > b_{ij}\right\} = \int_{b_{ij}}^1 b_{-i,j} dG\left(b_{ij}|\tilde{K}_j - \tilde{k}_{ij}\right)$$

$$= \left[b_{-i,j} G\left(b_{-i,j}|\tilde{K}_j - \tilde{k}_{ij}\right)\right]_{b_{ij}}^1 - \int_{b_{ij}}^1 G\left(b_{-i,j}|\tilde{K}_j - \tilde{k}_{ij}\right) db_{-i,j}$$

$$= b_{-i,j}\left[1-\left(1-b_{-i,j}\right)^{\widetilde{K}_j-\tilde{k}_{ij}}\right]_{b_{ij}}^{1} - \int_{b_{ij}}^{1}\left[1-\left(1-b_{-i,j}\right)^{\widetilde{K}_j-\tilde{k}_{ij}}\right]db_{-i,j}$$

$$= 1-b_{ij}\left(1-\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}}\right) - \left(1-b_{ij}\right) - \left[\frac{\left(1-b_{-i,j}\right)^{\widetilde{K}_j-\tilde{k}_{ij}+1}}{\widetilde{K}_j-\tilde{k}_{ij}+1}\right]_{b_{ij}}^{1}$$

$$= b_{ij}\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}} + \left(\frac{1}{\widetilde{K}_j-\tilde{k}_{ij}+1}\right)\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}+1}, \tag{A11}$$

where the second line in (A11) uses integration by parts. From (A11) it follows that

$$\mathrm{E}\left[\tilde{\pi}_{ij}|b_{ij}\right] = \mathrm{E}\left[b_{-i,j}|b_{-i,j} > b_{ij}\right] \times \Pr\left\{b_{-i,j} > b_{ij}\right\} - b_{ij}\left[1-G\left(b_{-i,j}|\widetilde{K}_j-\tilde{k}_{ij}\right)\right]$$

$$= \left(b_{ij}\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}} + \left(\frac{1}{\widetilde{K}_j-\tilde{k}_{ij}+1}\right)\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}+1}\right) - b_{ij}\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}}$$

$$= \left(\frac{1}{\widetilde{K}_j-\tilde{k}_{ij}+1}\right)\left(1-b_{ij}\right)^{\widetilde{K}_j-\tilde{k}_{ij}+1}. \tag{A12}$$

Finally, using equation (A12), i's unconditional expected net margin in auction j is

$$\mathrm{E}\left[\tilde{\mu}_{ij}\right] = \int_0^1 \mathrm{E}\left[\tilde{\mu}_{ij}|b_{ij}=b\right]dG\left(b|\tilde{k}_{ij}\right)$$

$$= \int_0^1\left\{\left(\frac{1}{\widetilde{K}_j-\tilde{k}_{ij}+1}\right)(1-b)^{\widetilde{K}_j-\tilde{k}_{ij}+1}\right\}\tilde{k}_{ij}(1-b)^{\tilde{k}_{ij}-1}db$$

$$= \frac{\tilde{k}_{ij}}{\widetilde{K}_j-\tilde{k}_{ij}+1}\int_0^1(1-b)^{\widetilde{K}_j}db = \frac{\tilde{k}_{ij}}{\left(\widetilde{K}_j-\tilde{k}_{ij}+1\right)\left(\widetilde{K}_j+1\right)}\left[-(1-b)^{\widetilde{K}_j+1}\right]_0^1$$

$$= \frac{\tilde{k}_{ij}}{\left(\widetilde{K}_j-\tilde{k}_{ij}+1\right)\left(\widetilde{K}_j+1\right)} = \frac{\tilde{k}_{ij}}{\left(\widetilde{K}_{-i,j}+1\right)\left(\widetilde{K}_j+1\right)} \quad \blacksquare$$

***Derivation of equation* (11)**:

Differentiating equation (8) with respect to θ_{ij} we have

$$\frac{\partial\mathrm{E}[w_j]}{\partial\theta_{ij}} = \frac{\partial}{\partial\theta_{ij}}\left(\frac{1}{\widetilde{K}_j+1}\right) + \frac{\partial\mathrm{E}[\tilde{\mu}_{ij}]}{\partial\theta_{ij}} + \sum_{h\in M\setminus i}\frac{\partial\mathrm{E}[\tilde{\mu}_{hj}]}{\partial\theta_{ij}}. \tag{A13}$$

We consider in turn each of the three derivatives on the right-hand side of (A13). First,

$$\frac{\partial}{\partial\theta_{ij}}\left(\frac{1}{\widetilde{K}_j+1}\right) = -\frac{1}{\left(\widetilde{K}_j+1\right)^2}\frac{\partial\widetilde{K}_j}{\partial\theta_{ij}} = \frac{\tilde{k}_{ij}}{\theta_{ij}\left(\widetilde{K}_j+1\right)^2} > 0\,. \tag{A14}$$

Next

$$\frac{\partial\mathrm{E}[\tilde{\mu}_{ij}]}{\partial\theta_{ij}} = \frac{-\tilde{k}_{ij}}{\theta_{ij}\left(\widetilde{K}_{-i,j}+1\right)\left(\widetilde{K}_j+1\right)} + \frac{\tilde{k}_{ij}^2}{\theta_{ij}\left(\widetilde{K}_{-i,j}+1\right)\left(\widetilde{K}_j+1\right)^2}$$

$$= \frac{-\tilde{k}_{ij}}{\theta_{ij}(\tilde{K}_{-i,j}+1)(\tilde{K}_j+1)}\left[1 - \frac{\tilde{k}_{ij}}{\tilde{K}_j+1}\right]$$

$$= \frac{-\tilde{k}_{ij}}{\theta_{ij}(\tilde{K}_j+1)^2} < 0. \tag{A15}$$

Note that the expressions in (A14) and (A15) cancel. Finally, for bidders $h \neq i$, note that

$$\frac{\partial \tilde{K}_{-h,j}}{\partial \theta_{ij}} = \frac{\partial \tilde{K}_j}{\partial \theta_{ij}} = \frac{\partial \tilde{k}_{ij}}{\partial \theta_{ij}} = -\frac{\tilde{k}_{ij}}{\theta_{ij}}.$$

Thus

$$\frac{\partial \mathrm{E}[\tilde{\mu}_{hj}]}{\partial \theta_{ij}} = \frac{\tilde{k}_{ij}\tilde{k}_{hj}}{\theta_{ij}(\tilde{K}_{-h,j}+1)(\tilde{K}_j+1)}\left(\frac{1}{\tilde{K}_{-h,j}+1} + \frac{1}{\tilde{K}_j+1}\right)$$

$$= \frac{\tilde{k}_{ij}\tilde{k}_{hj}}{\theta_{ij}(\tilde{K}_{-h,j}+1)(\tilde{K}_j+1)}\left(\frac{\tilde{K}_j+\tilde{K}_{-h,j}+2}{(\tilde{K}_{-h,j}+1)(\tilde{K}_j+1)}\right)$$

$$= \frac{\tilde{k}_{ij}\tilde{k}_{hj}(\tilde{K}_j+\tilde{K}_{-h,j}+2)}{\theta_{ij}(\tilde{K}_{-h,j}+1)^2(\tilde{K}_j+1)^2} > 0,$$

and therefore

$$\frac{\partial \mathrm{E}[w_j]}{\partial \theta_{ij}} = \frac{\tilde{k}_{ij}}{\theta_{ij}(\tilde{K}_j+1)^2} \sum_{h\in M\setminus i} \frac{\tilde{k}_{hj}(\tilde{K}_j+\tilde{K}_{-h,j}+2)}{(\tilde{K}_{-h,j}+1)^2} > 0 \blacksquare$$

***Derivation of equation* (12)**:

ξ_{ij} is the expected value of the bid-cost spread that i earns in auction j conditional on winning the auction:

$$\xi_{ij} = \mathrm{E}\left[b_{ij} - c_{ij} | i \text{ wins } j\right] \times \Pr\{i \text{ wins } j\}.$$

From equation (3), the spread for a given realization c can be written as

$$b(c;\theta_{ij}) - c = (1-c)\left(1-(1-c)^{1-\theta_{ij}}\right). \tag{A16}$$

Bidder i wins auction j when the minimum of rival bids, $b_{-i,j}$, is greater than i's bid, $b(c;\theta_{ij})$. Recall that the minimum of all m bids b_j is distributed $G(c|\tilde{K}_j)$. Likewise, the minimum of all rival bids $b_{-i,j}$ is distributed $G(c|\tilde{K}_{-i,j})$. It follows that i wins auction j with probability

$$\Pr\{b(c;\theta_{ij}) < b_{-i,j}\} = 1 - G\left(b(c;\theta_{ij})|\tilde{K}_{-i,j}\right)$$

$$= \left(1 - b\left(c; \theta_{ij}\right)\right)^{\tilde{K}_{-i,j}}$$
$$= (1-c)^{\theta_{ij}\tilde{K}_{-i,j}} \tag{A17}$$

Recall also that c_{ij} is distributed $G(c|k_i)$, so its density is

$$dG(c|k_i) = k_i(1-c)^{k_i-1}. \tag{A18}$$

Relying on (A17) and (A18), the expected spread can be written as

$$\xi_{ij} = \int_0^1 (1-c)\left(1-(1-c)^{\theta_{ij}-1}\right)\Pr\left\{b\left(c;\theta_{ij}\right) < b_{-i,j}\right\} dG(c|k_i)$$
$$= k_i \int_0^1 (1-c)^{k_i+\theta_{ij}\tilde{K}_{-i,j}}\left(1-(1-c)^{\theta_{ij}-1}\right)dc,$$
$$= k_i\left[\frac{1}{k_i+\theta_{ij}\tilde{K}_{-i,j}+1} - \frac{1}{k_i+\theta_{ij}\left(\tilde{K}_{-i,j}+1\right)}\right]$$
$$= \frac{k_i\left(\theta_{ij}-1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(k_i+\theta_{ij}\left(\tilde{K}_{-i,j}+1\right)\right)}. \tag{A19}$$

Given that $\tilde{K}_j \equiv \tilde{k}_{ij} + \tilde{K}_{-i,j}$, the term $\left(k_i + \theta_{ij}\left(\tilde{K}_{-i,j}+1\right)\right)$ in the denominator of (A19) can be written as $\theta_{ij}\left(\tilde{K}_j+1\right)$. Thus (A19) can be rewritten as

$$\xi_{ij} = \frac{k_i - \tilde{k}_{ij}}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)} \quad \blacksquare$$

***Derivation of equation* (13)**:

To begin, it is helpful to rewrite equation (12) as

$$\xi_{ij} = \left(1-\frac{1}{\theta_{ij}}\right)\frac{k_i}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)}. \tag{A20}$$

Differentiating equation (A20) with respect to θ_{ij} yields

$$\frac{\partial \xi_{ij}}{\partial \theta_{ij}} = \frac{\tilde{k}_{ij}}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)} + \left(1-\frac{1}{\theta_{ij}}\right)\left[\frac{-k_i\tilde{K}_{-i,j}}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)} + \frac{\tilde{k}_{ij}^2}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)^2}\right]$$
$$= \frac{\tilde{k}_{ij}}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)} + \frac{\theta_{ij}-1}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)}\left[\frac{-k_i\tilde{K}_{-i,j}}{k_i+\theta_{ij}\tilde{K}_{-i,j}+1} + \frac{\tilde{k}_{ij}^2}{\tilde{K}_j+1}\right]$$
$$= \frac{\tilde{k}_{ij}}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)} + \frac{\theta_{ij}-1}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)}\left[\frac{-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)+\tilde{k}_{ij}^2\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)}\right]$$
$$= \frac{\tilde{k}_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left(\tilde{K}_j+1\right)+\left(\theta_{ij}-1\right)\left[-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)+\tilde{k}_{ij}^2\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\right]}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2}. \tag{A21}$$

Now note that the derivative of net margin in equation (A15) can be rewritten as

$$\frac{\partial \mathrm{E}[\tilde{\mu}_{ij}]}{\partial \theta_{ij}} = \frac{-\tilde{k}_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2}\,. \tag{A22}$$

Summing together (A21) and (A22) then yields the derivative of gross margin:

$$\frac{\partial \mathrm{E}\left[\mu_{ij}^u\right]}{\partial \theta_{ij}} = \frac{\tilde{k}_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\left[\tilde{K}_j+1-\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\right]+\left(\theta_{ij}-1\right)\left[-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)+\tilde{k}_{ij}^2\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)\right]}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2}. \tag{A23}$$

Noting that $k_i + \theta_{ij}\tilde{K}_{-i,j} + 1 = \theta_{ij}\tilde{K}_j + 1$, equation (A23) can be rewritten as

$$\begin{aligned}
\frac{\partial \mathrm{E}\left[\mu_{ij}^u\right]}{\partial \theta_{ij}} &= \frac{-\left(\theta_{ij}-1\right)\tilde{k}_{ij}\tilde{K}_j\left(\theta_{ij}\tilde{K}_j+1\right)+\left(\theta_{ij}-1\right)\left[-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)+\tilde{k}_{ij}^2\left(\theta_{ij}\tilde{K}_j+1\right)\right]}{\theta_{ij}\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \\
&= \left(\frac{\theta_{ij}-1}{\theta_{ij}}\right)\frac{-\tilde{k}_{ij}\tilde{K}_j\left(\theta_{ij}\tilde{K}_j+1\right)-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)+\tilde{k}_{ij}^2\left(\theta_{ij}\tilde{K}_j+1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \\
&= \left(\frac{\theta_{ij}-1}{\theta_{ij}}\right)\frac{\tilde{k}_{ij}\left(\tilde{k}_{ij}-\tilde{K}_j\right)\left(\theta_{ij}\tilde{K}_j+1\right)-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \\
&= \left(\frac{\theta_{ij}-1}{\theta_{ij}}\right)\frac{-\tilde{k}_{ij}\tilde{K}_{-i,j}\left(\theta_{ij}\tilde{K}_j+1\right)-k_i\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \\
&= \left(\frac{\theta_{ij}-1}{\theta_{ij}}\right)\frac{-\tilde{k}_{ij}\tilde{K}_{-i,j}\left(\theta_{ij}\tilde{K}_j+1\right)-\theta_{ij}\tilde{k}_{ij}\tilde{K}_{-i,j}\left(\tilde{K}_j+1\right)}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \\
&= -\left(\frac{\theta_{ij}-1}{\theta_{ij}}\right)\frac{\tilde{k}_{ij}\tilde{K}_{-i,j}\left[\theta_{ij}\left(2\tilde{K}_j+1\right)+1\right]}{\left(k_i+\theta_{ij}\tilde{K}_{-i,j}+1\right)^2\left(\tilde{K}_j+1\right)^2} \quad \blacksquare
\end{aligned}$$

Proof of $\partial p_j/\partial w_j > 0$:

The proof is complete once we have shown that the expression within square brackets in equation (21) is positive, or equivalently that

$$\frac{s_j^2}{1-s_j}\sum_{h\in N\setminus j}\frac{s_h^2}{1-s_h} < 1\,, \tag{A24}$$

where superscripts d have been dropped to reduce clutter. Throughout the proof, we fix an arbitrary $s_j \in (0,1)$ and consider possible distributions of share among the remaining $n-1$ firms $h \neq i$. Consider first the case of equal shares: $s_h = \left(1-s_j\right)/(n-1),\ \forall h \neq i$. In this case, the left-hand side of inequality (A24) reduces to

$$\frac{s_j^2}{1-s_j}\sum_{h\in N\setminus j}\frac{s_h^2}{1-s_h} = \frac{s_j^2}{1-s_j}(n-1)\frac{\left(\frac{1-s_j}{n-1}\right)^2}{1-\frac{1-s_j}{n-1}} = \frac{s_j^2(1-s_j)}{n-2+s_j}. \tag{A25}$$

By inspection, the rightmost side of equation (A25) is largest when n is smallest, $n = 2$. In this case, the expression becomes $s_j(1-s_j)$, which is less than one, satisfying (A24).

Now note that any feasible distribution of shares among the $n-1$ remaining firms $h \neq i$ can be constructed from the equal-share distribution above via a sequence of share shifts from weakly smaller to weakly larger firms. Any such individual share shift between two firms increases the sum $\sum_{h\in N\setminus j} s_h^2/(1-s_h)$, given $s^2/(1-s)$ convex in s:

$$\frac{\partial}{\partial s}\left(\frac{s^2}{1-s}\right) = \frac{2s-s^2}{(1-s)^2} > 0; \quad \frac{\partial^2}{\partial s^2}\left(\frac{s^2}{1-s}\right) = \frac{2}{(1-s)^3} > 0. \tag{A26}$$

The largest that the sum $\sum_{h\in N\setminus j} s_h^2/(1-s_h)$ can get, therefore, is in case one of the firms has virtually all remaining share $1-s_j$ and all other firms $h \neq i$ have share close to zero. But in the limit this approaches the case of $n = 2$ above, so once again $s_j(1-s_j) < 1$ ■

***Proof that the bid function in equation* (32) *is distributed* $\boldsymbol{G(\cdot\,|k_i/\theta_{ij})}$:**

Given $F(c) = (c-\underline{c})/(\overline{c}-\underline{c})$, we have that,

$$G(c|k_i/\theta_{ij}) = 1-[1-F(c)]^{k_i/\theta_{ij}} = 1-\left(\frac{\overline{c}-c}{\overline{c}-\underline{c}}\right)^{k_i/\theta_{ij}}. \tag{A27}$$

The bid function, $b(c_{ij};\theta_{ij}) = \overline{c}-(\overline{c}-\underline{c})\left(\frac{\overline{c}-c_{ij}}{\overline{c}-\underline{c}}\right)^{\theta_{ij}}$ has an inverse,

$$b^{-1}(c) = \overline{c}-(\overline{c}-\underline{c})\left(\frac{\overline{c}-c}{\overline{c}-\underline{c}}\right)^{1/\theta_{ij}}. \tag{A28}$$

The bid function is distributed

$$\begin{aligned}\Pr\{b(c_{ij};\theta_{ij}) < c\} &= \Pr\{c_{ij} < b^{-1}(c)\} \\ &= G(b^{-1}(c)|k_i) = 1-\left[1-F\left(b^{-1}(c)\right)\right]^{k_i}.\end{aligned} \tag{A29}$$

From (A27) and (A29), we have that $\Pr\{b(c_{ij};\theta_{ij}) < c\} = G(c|k_i/\theta_{ij})$ if and only if

$$\left[1-F\left(b^{-1}(c)\right)\right]^{k_i} = [1-F(c)]^{k_i/\theta_{ij}},$$

which is equivalent to

$$F\left(b^{-1}(c)\right) = 1 - [1 - F(c)]^{1/\theta_{ij}}. \tag{A30}$$

From (A28), we have that the left-hand side of (A30) is

$$F\left(b^{-1}(c)\right) = \frac{\left(b^{-1}(c) - \underline{c}\right)}{\bar{c} - \underline{c}} = 1 - \left(\frac{\bar{c} - c}{\bar{c} - \underline{c}}\right)^{1/\theta_{ij}},$$

whereas the right-hand side of (A30) is

$$1 - [1 - F(c)]^{\frac{1}{\theta_{ij}}} = 1 - \left(\frac{\bar{c} - c}{\bar{c} - \underline{c}}\right)^{1/\theta_{ij}}.$$

Given that the right-hand side of (A30) equals the left-hand side for a bid function satisfying (A28), we have shown that the bid function is distributed $G\left(c|k_i/\theta_{ij}\right)$. It bears mentioning that (A30) suggests a more general result: for any F that is invertible, a bid function satisfying, $b^{-1}(c) = F^{-1}\left\{1 - [1 - F(c)]^{1/\theta_{ij}}\right\}$ is distributed $G\left(c|k_i/\theta_{ij}\right)$. ■

***Derivation of equation* (33)**:

We derive the expression W_{ij}, which is the expected input price for an integrated supplier i when supplying a particular producer j. The no-integration analogue, W_i, can be recovered through a simple change of variables. We have that,

$$W_{ij} = E\left[b_{-ij}|b_{-ij} > b_{ij}\right] = \int_{\underline{c}}^{\bar{c}} \int_{b}^{\bar{c}} y \frac{dG\left(y|\tilde{K}_{-ij}\right)}{1 - G\left(b|\tilde{K}_{-ij}\right)} dG\left(b|\tilde{k}_i\right)$$

Now using integration by parts and the substitution $1 - G\left(b|\tilde{K}_{-ij}\right) = \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}}\right)^{-\tilde{K}_{-ij}}$, we have

$$\begin{aligned} W_{ij} &= \int_{\underline{c}}^{\bar{c}} \left\{\bar{c} - bG\left(b|\tilde{K}_{-ij}\right) - \int_{b}^{\bar{c}} G\left(y|\tilde{K}_{-ij}\right)dy\right\} \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}}\right)^{-\tilde{K}_{-ij}} dG\left(b|\tilde{k}_{ij}\right) \\ &= \int_{\underline{c}}^{\bar{c}} \left\{\bar{c} - b\left[1 - \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}}\right)^{-\tilde{K}_{-ij}}\right]\right. \\ &\qquad \left. - \left[(\bar{c} - b) - \frac{\bar{c} - \underline{c}}{\tilde{K}_{-ij} + 1}\left(\frac{\bar{c} - b}{\bar{c} - \underline{c}}\right)^{\tilde{K}_{-ij} + 1}\right]\right\} \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}}\right)^{-\tilde{K}_{-ij}} dG\left(b|\tilde{k}_{ij}\right) \end{aligned}$$

$$= \int_{\underline{c}}^{\bar{c}} \left\{ b \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}} \right)^{\tilde{K}_{-ij}} + \frac{\bar{c} - \underline{c}}{\tilde{K}_{-ij} + 1} \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}} \right)^{\tilde{K}_{-ij}+1} \right\} \left(\frac{\bar{c} - b}{\bar{c} - \underline{c}} \right)^{-\tilde{K}_{-ij}} dG\left(b | \tilde{k}_{ij}\right)$$

$$= \int_{\underline{c}}^{\bar{c}} \left\{ b + \frac{\bar{c} - b}{\tilde{K}_{-ij} + 1} \right\} dG\left(b | \tilde{k}_{ij}\right)$$

$$= \int_{\underline{c}}^{\bar{c}} \left\{ \frac{\bar{c}}{\tilde{K}_{-ij} + 1} + \left[1 - \frac{1}{\tilde{K}_{-ij} + 1} \right] b \right\} dG\left(b | \tilde{k}_{ij}\right)$$

$$= \frac{\bar{c}}{\tilde{K}_{-ij} + 1} + \left[\frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij} + 1} \right] \left[\bar{c} - \int_{\underline{c}}^{\bar{c}} G\left(b | \tilde{k}_{ij}\right) db \right]$$

$$= \frac{\bar{c}}{\tilde{K}_{-ij} + 1} + \left[\frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij} + 1} \right] \left[\bar{c} - (\bar{c} - \underline{c}) + \frac{1}{\tilde{k}_{ij} + 1} \right]$$

$$= \frac{\bar{c} + \tilde{K}_{-ij}\underline{c}}{\tilde{K}_{-ij} + 1} + \frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij} + 1} \frac{1}{\tilde{k}_{ij} + 1}$$

$$= \underline{c} + \frac{\bar{c} - \underline{c}}{\tilde{K}_{-ij} + 1} + \frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij} + 1} \frac{1}{\tilde{k}_{ij} + 1}$$

$$= \underline{c} + \frac{\bar{c} - \underline{c}}{\tilde{K}_j - \tilde{k}_{ij} + 1} + \frac{\tilde{K}_j - \tilde{k}_{ij}}{\tilde{K}_j - \tilde{k}_{ij} + 1} \frac{1}{\tilde{k}_{ij} + 1}, \tag{A31}$$

where the last equality above uses $\tilde{K}_{-ij} \equiv \tilde{K}_j - \tilde{k}_{ij}$. With no prior integration, $\tilde{K}_j = K$ and $\tilde{k}_{ij} = k_i$, thus providing the first equality in equation (33). The second equality in equation (33) uses $k_i = \hat{s}_i^d$ from (31). ■

***Derivation of equation* (34)**:

We derive the expression U_{ij}, which is the expected per-unit margin for an integrated supplier i, when supplying a particular producer j. The no-integration analogue, U_i, can be recovered through a simple change of variables. We have that,

$$U_{ij} = E\left[b_{-ij} - c_{ij} | b_{-ij} > b_{ij}\right] = \int_{\underline{c}}^{\bar{c}} \int_{b(c)}^{\bar{c}} (y - c) \frac{dG\left(y | \tilde{K}_{-ij}\right)}{1 - G\left(b(c) | \tilde{K}_{-ij}\right)} dG(c | k_i),$$

where $b(c) \equiv b\left(c; \theta_{ij}\right)$ as a shorthand. Using integration by parts and the substitution, $1 - G\left(b(c) | \tilde{K}_{-ij}\right) = \left(\frac{\bar{c} - b(c)}{\bar{c} - \underline{c}}\right)^{-\tilde{K}_{-ij}}$, we have $U_{ij} =$

$$\int_{\underline{c}}^{\overline{c}} \left\{\left[\overline{c} - b(c)G\left(b(c)|\breve{K}_{-ij}\right) - \int_{b(c)}^{\overline{c}} G\left(y|\breve{K}_{-ij}\right)\right]\right.$$

$$\left. - c\left[1 - G\left(b(c)|\breve{K}_{-ij}\right)\right]\right\}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{-\breve{K}_{-ij}} dG(c|k_i)$$

$$= \int_{\underline{c}}^{\overline{c}} \left\{\left[\overline{c} - c - (b(c) - c)G\left(b(c)|\breve{K}_{-ij}\right)\right]\right.$$

$$\left. - \left[\overline{c} - b(c) - \frac{\overline{c} - \underline{c}}{\breve{K}_{-ij} + 1}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{\breve{K}_{-ij}+1}\right]\right\}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{-\breve{K}_{-ij}} dG(c|k_i)$$

$$= \int_{\underline{c}}^{\overline{c}} \left\{(b(c) - c)\left[1 - G\left(b(c)|\breve{K}_{-ij}\right)\right]\right.$$

$$\left. + \frac{\overline{c} - \underline{c}}{\breve{K}_{-ij} + 1}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{\breve{K}_{-ij}+1}\right\}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{-\breve{K}_{-ij}} dG(c|k_i)$$

$$= \int_{\underline{c}}^{\overline{c}} \left\{(b(c) - c)\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{\breve{K}_{-ij}}\right.$$

$$\left. + \frac{\overline{c} - \underline{c}}{\breve{K}_{-ij} + 1}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{\breve{K}_{-ij}+1}\right\}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)^{-\breve{K}_{-ij}} dG(c|k_i)$$

$$= \int_{\underline{c}}^{\overline{c}} \left\{(b(c) - c) + \frac{\overline{c} - \underline{c}}{\breve{K}_{-ij} + 1}\left(\frac{\overline{c} - b(c)}{\overline{c} - \underline{c}}\right)\right\} dG(c|k_i)$$

$$= \int_{\underline{c}}^{\overline{c}} \left\{\left[(\overline{c} - c) - \left(\overline{c} - \underline{c}\right)\left(\frac{\overline{c} - c}{\overline{c} - \underline{c}}\right)^{\theta_{ij}}\right] + \frac{\overline{c} - \underline{c}}{\breve{K}_{-ij} + 1}\left(\frac{\overline{c} - c}{\overline{c} - \underline{c}}\right)^{\theta_{ij}}\right\}\left(\frac{k_i}{\overline{c} - \underline{c}}\right)\left(\frac{\overline{c} - c}{\overline{c} - \underline{c}}\right)^{k_i - 1} dc,$$

where the above expression makes the substitutions

$$b(c) - c = (\overline{c} - c) - \left(\overline{c} - \underline{c}\right)\left(\frac{\overline{c} - c}{\overline{c} - \underline{c}}\right)^{\theta_{ij}},$$

$$\overline{c} - b(c) = \left(\overline{c} - \underline{c}\right)\left(\frac{\overline{c} - c}{\overline{c} - \underline{c}}\right)^{\theta_{ij}}, \text{ and}$$

$$dG(c|k_i) = \left(\frac{k_i}{\bar{c}-\underline{c}}\right)\left(\frac{\bar{c}-c}{\bar{c}-\underline{c}}\right)^{k_i-1} dc.$$

We then have

$$U_{ij} = \left(\frac{k_i}{\bar{c}-\underline{c}}\right)\int_{\underline{c}}^{\bar{c}}\left[(\bar{c}-c)\left(\frac{\bar{c}-c}{\bar{c}-\underline{c}}\right)^{k_i-1} - \left(\frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij}+1}\right)(\bar{c}-\underline{c})\left(\frac{\bar{c}-c}{\bar{c}-\underline{c}}\right)^{\theta_{ij}+k_i-1}\right]dc$$

$$= \left(\frac{k_i}{\bar{c}-\underline{c}}\right)\left\{\left[\frac{(\bar{c}-c)^2}{k_i} - \frac{\bar{c}-\underline{c}}{k_i}\int_{\underline{c}}^{\bar{c}}\left(\frac{\bar{c}-c}{\bar{c}-\underline{c}}\right)^{k_i} dc\right] - \left(\frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij}+1}\right)\left(\frac{(\bar{c}-c)^2}{\theta_{ij}+k_i}\right)\right\}$$

$$= \left(\frac{k_i}{\bar{c}-\underline{c}}\right)\left\{\left[\frac{(\bar{c}-c)^2}{k_i} - \frac{(\bar{c}-c)^2}{k_i(k_i+1)}\right] - \left(\frac{\tilde{K}_{-ij}}{\tilde{K}_{-ij}+1}\right)\left(\frac{(\bar{c}-c)^2}{\theta_{ij}+k_i}\right)\right\}$$

$$= (\bar{c}-\underline{c})k_i\left[\frac{1}{k_i+1} - \frac{\tilde{K}_{-ij}}{(\tilde{K}_{-ij}+1)(k_i+\theta_{ij})}\right]$$

$$= (\bar{c}-\underline{c})k_i\left[\frac{1}{k_i+1} - \frac{\tilde{K}_j-\tilde{k}_{ij}}{(\tilde{K}_j-\tilde{k}_{ij}+1)\theta_{ij}(\tilde{k}_{ij}+1)}\right], \qquad \text{(A32)}$$

where the last equality uses $\tilde{K}_{-ij} = \tilde{K}_j - \tilde{k}_{ij}$ and $k_i + \theta_{ij} = \theta_{ij}(\tilde{k}_{ij}+1)$. With no prior integration, $\tilde{K}_j = K$ and $\tilde{k}_{ij} = k_i$, thus providing the first equality in equation (34). The second equality in equation (34) uses $k_i = \hat{s}_i^d$ from (31). ■

***Derivation of equation* (36) *and proof of first-order impact of RRC*:**

As before, let $b_{-ij} = \min_{h\neq i}\{b_{hj}\}$. By construction, $\mu_{ij}^u = (b_{-ij} - c_{ij})\mathbb{I}\{b_{-ij} > b_{ij}\}$, where $\mathbb{I}\{b_{-ij} > b_{ij}\}$ is an indicator equal to one if $b_{-ij} > b_{ij}$ and zero otherwise. Also by construction, $\mu_{ij} \cdot s_j^d$ is nonzero only when $b_{-ij} > b_{ij}$, in which case, $w_j = b_{-ij}$. Let $G_{\mathbf{w}_{-j}}$ denote the joint distribution of $\{w_l\}_{l\neq j}$. We have that

$$\mathrm{E}\left[\mu_{ij} \cdot s_j^d\right] = \int\int_0^1\int_{b(c;\theta_{ij})}^1 (\beta - c)\cdot\sigma_j(w_1,\dots,\beta,\dots,w_n)dG(\beta|\tilde{K}_j)dG(c|k_i)dG_{\mathbf{w}_{-j}} \quad \text{(A32)}$$

where $dG(\beta|\tilde{K}_j)$ is the density of b_{-ij} and $dG(c|k_i)$ is the density of c_{ij}. From (A32), we see that θ_{ij} only impacts i's upstream profit to the extent that it impacts the probability that i wins the auctions for producer j and hence the probability that b_{-ij} is pivotal in determining w_j. Because each auction is independent and since θ_{ih} is chosen uniquely for each producer h, θ_{ij} has no impact on any other producers' input prices.

Differentiating (A32) with respect to θ_{ij}, we have

$$\frac{d\mathrm{E}\left[\mu_{ij}\cdot s_j^d\right]}{d\theta_{ij}} = -\frac{\partial b\left(c;\theta_{ij}\right)}{\partial\theta_{ij}} \times$$

$$\left[\int\int_0^1\left[b\left(c;\theta_{ij}\right)-c\right]\sigma\left(w_1,\dots,b\left(c;\theta_{ij}\right),\dots,w_m\right)g\left(b\left(c;\theta_{ij}\right)|\widetilde{K}_j\right)dG(c|k_i)dG_{\mathbf{w}_{-j}}\right], \quad \text{(A33)}$$

where

$$g\left(b\left(c;\theta_{ij}\right)|\widetilde{K}_j\right) \equiv \frac{d}{d\beta}G\left(\beta|\widetilde{K}_j\right)\Big|_{\beta=b\left(c;\theta_{ij}\right)} = \widetilde{K}_j\left[1-b\left(c:\theta_{ij}\right)\right]^{\widetilde{K}_j-1} \quad \text{(A34)}$$

is the instantaneous rate of change in the distribution of b_{-ij} in the neighborhood of $b_{-ij} = b\left(c;\theta_{ij}\right)$. Finally, given that $\frac{\partial b\left(c;\theta_{ij}\right)}{\partial\theta_{ij}} > 0$ and $b\left(c;\theta_{ij}\right) \geq c$ for all $\theta_{ij} \geq 1$, it follows that $\frac{\partial\mathrm{E}\left[\mu_{ij}\cdot s_j^d\right]}{\partial\theta_{ij}} \leq 0$, strictly so for $\theta_{ij} > 1$. Further, given $b(c;1) = c$ expression (A33) is zero evaluated at $\theta_{ij} = 1$. ■