-----Original Message-----
From: jchavas@gmail.com [mailto:jchavas@gmail.com]
Sent: Saturday, October 17, 2009 11:46 AM
To: ATR-Agricultural Workshops
Subject: Re: workshop paper

As a follow-up to my previous Email, please find attached another PDF file of my paper "On Storage Behavior Under Imperfect Competition" (by Jean-Paul Chavas) with an application to the US cheese market. (I believe that the file I sent you in my previous Email was not complete).
I mailed you two hard copies of the paper earlier. I would like to have the paper considered for the USDA/DOJ workshop.
Let me know if you need additional information.
Sincerely,
Jean-Paul Chavas

On Mon, Sep 14, 2009 at 1:01 PM, Jean-Paul Chavas <jchavas@wisc.edu> wrote:
> Please find attached a PDF file of the paper entitled with Application
> to the American Cheese Market" by Jean-Paul Chavas.
> The paper examines both conceptually and empirically the use of stocks
> in the exercise of market power. It was recently published in the
> Review of Industrial Organization.
> I would like to have the paper considered for the USDA/DOJ workshop.
> I will also send you two hard copies of the paper.
> Sincerely,
> Jean-Paul Chavas
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ON STORAGE BEHAVIOR UNDER IMPERFECT COMPETITION

Jean-Paul Chavas
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This paper investigates the economic implications of storage behavior under imperfect competition. It evaluates the economic dynamics implied by a storage firm under alternative market structures. This includes perfect competition as well as imperfect competition under Cournot behavior. The conceptual analysis is used to specify and estimate a model of storage behavior, with an econometric application to the US American cheese market. The empirical results provide statistical evidence of non-competitive storage behavior. They show that the exercise of market power contributes to reduced stock fluctuations and increased price instability in the US cheese market.

Keywords: storage, dynamics, imperfect competition

JEL Code: D4, L1, Q11

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Jean-Paul Chavas

1. Introduction

Much research has been conducted on the economics of commodity storage. Storage provides a means of smoothing anticipated market changes over time (e.g., Kaldor; Working, 1948, 1949; Brennan; Williams and Wright). In general, competitive storage activities contribute to stabilizing markets: they tend to stimulate demand in periods of low prices (by increasing stocks) and increase supply in periods of high prices (by decreasing stocks). When positive, optimal competitive stocks imply that the marginal cost of storage equals the discounted expected price change, thus providing an arbitrage relationship for commodity prices over time (e.g., Williams and Wright). But what if markets are not competitive? What are the implications of the exercise of market power for storage activities?

The analysis of non-competitive storage has been investigated by Allaz, Arvan, Kirman and Sobel, Judd, Mitraille, Moellgaard et al., Newbery, Rotemberg and Saloner, Saloner, Vedenov and Miranda, and Williams and Wright. In general, non-competitive storage behavior can be complex. For example, in the context of a Cournot duopoly game, Arvan has shown that the Nash value functions can be ill-behaved, implying that equilibrium inventory strategies may not be symmetric and may not even exist. These complexities mean that stock-holding under imperfect competition remains poorly understood. First, to help deal with these complexities, researchers have often focused their analysis on simple two-period models (e.g., Allaz, Arvan, Mitraille, Moellgaard et al., Rotemberg and Saloner, and Saloner). This has helped provide insights into the strategic role of inventory in the exercise of market power (e.g., Rotemberg and Saloner, Mitraille). However, two-period models seem overly restrictive. Indeed, we show below that the effects of market power on storage activities depend in general on the anticipated path of future stocks. This means that having only two periods imposes strong restrictions on imperfectly competitive storage behavior. Second, to help deal with the complexities of economic dynamics, a number of papers have restricted their analysis to stationary decision rules. This has facilitated the investigation of storage decisions under a long planning horizon (e.g., Judd; Thille; Vedenov and Miranda; Williams and Wright). However, this neglects the role of inventory under changing market conditions (e.g., due to the business cycle or to seasonality). Third, a relevant issue is: how does the exercise of market power affect storage behavior? Previous research has not provided a clear answer to this question. The reason is that the findings are sensitive to market conditions and to the nature of strategic interactions. For example, Rotemberg and Saloner argue that duopolies have an incentive to hold higher inventories to help maintain collusion when demand is high (by allowing a stronger punishment of cheaters). But this contrasts with Mitraille who finds that higher inventories are more likely to be used strategically in downturns (in the context of asymmetric strategies). In addition, Wright and Williams (chapter 11) and Newbery have shown that a monopoly would store less than a competitive firm under some scenarios but more under others. These findings do not make it clear how imperfect competition affects stock-holding. These limitations of previous research mean the existence of a significant gap between storage models and our understanding of actual market dynamics. Finally, partly due the complexities of dynamic non-competitive models, the empirical investigation of imperfectly competitive stock-holding remains poorly developed. This suggests a need to refine our conceptual and empirical knowledge of storage activities under non-competitive behavior.

This paper develops a model of economic behavior for storage activities. The analysis is presented under a general T-period planning horizon. The model allows for uncertainty and non-stationarity. And it covers situations of perfect competition as well as imperfect competition under Cournot behavior. The analysis focuses on a structural model of decisions made by a firm, leading to the specification of behavioral rules for stock-holding. Note that this structural approach does not model explicitly the nature of strategic interactions in the market. But it has the advantage of remaining valid irrespective of these strategic interactions. This leads to the specification and estimation of a structural econometric model of storage decisions under alternative market structures. In this context, we propose a model specification that nests both competitive and Cournot storage behavior. This provides a convenient framework to investigate the empirical relevance of both competitive and Cournot decision rules for private stock-holding.

The approach is applied to the determination of commercial stocks for American cheese in the US since 1993. The choice of American cheese is motivated by previous evidence of market power in the American cheese market (e.g., Mueller et al.; Muller and Marion). First, the US American cheese market is highly concentrated. Second, Mueller et al. and Muller and Marion found evidence that a few large American cheese processors have been in a position to affect market prices and exercise market power. This raises the possibility that private stock-holding may have been used in a non-competitive way. If so, the decision rule used to choose private American cheese stocks would not be the competitive decision rule. Our empirical investigation provides statistical evidence of non-competitive storage behavior in the US American cheese market. It also shows that the exercise of market power contributes to reduced stock fluctuations and increased price instability.

The paper is organized as follows. A conceptual model of storage activities is presented in section 2. The implications of the model for storage decisions are examined under competition in section 3. Section 4 extends the analysis to imperfect competition under Cournot behavior. Section 5 makes use of our conceptual results to specify a structural econometric model of storage activities under alternative market structures. The approach is then applied to the empirical investigation of storage activities in the US American cheese market. After presenting the data in section 6, section 7 reports the econometric results. Economic implications are discussed in section 8. Finally, section 9 concludes.

2. THE MODEL

Consider a firm involved in storage activities for a given commodity. Let $S_t \in R_+$ the quantity stored by the firm at time t. Denote by $C_t(S_t)$ the cost of storage at time t. Throughout, we assume that the cost function $C_t(S_t)$ is convex in S_t and satisfies $C_t'' = \partial^2 C_t(S_t)/\partial S_t^2 > 0$.[1] The commodity depreciates at a rate $\delta_t \in [0, 1)$ from time t to time (t+1). It follows that the net quantity sold by the storage firm at time t is $[(1-\delta_{t-1}) S_{t-1} - S_t]$, with $[(1-\delta_{t-1}) S_{t-1} - S_t] > 0$ (< 0) corresponding to a sale (a purchase). Denoting by p_t the market price of the commodity at time t, the firm profit from storage activities at time t is

$$\pi_t = p_t \cdot [(1-\delta_{t-1}) S_{t-1} - S_t] - C_t(S_t). \tag{1a}$$

The firm is managed by an owner-manager with a T-period planning horizon, where t = 1 is the current time and $T \geq 2$. The owner-manager receives the stream of profits $\{\pi_t: t = 1, \ldots, T\}$ which can be either consumed or invested. The owner-manager's budget constraint at time t is

$$q_t\, x_t \leq \pi_t - I_t + (1 + i_{t-1})\, I_{t-1} + N_t, \tag{1b}$$

where x_t is consumption at time t, $q_t > 0$ is the price of x_t, I_t denotes investment made at time t into a riskless asset yielding $[(1 + i_t) I_t]$ at time (t+1), $i_t > 0$ is the one-period interest rate at time t, and N_t denotes "other income" at time t. Note that N_t represents the income from other activities undertaken by the firm. This is relevant in the case where the firm is involved both in production and storage activities (in which case N_t includes the firm profit made from production activities at time t). This is an important point: while we focus our attention on storage decisions, the analysis presented below would remain valid if the firm manager also makes production and other investment decisions.

Decisions are made under uncertainty. Future prices are not known for certainty and are treated as random variables with a given subjective probability distribution. Under the expected utility hypothesis, the owner-manager's preferences are given by $E_1U(\mathbf{x})$, where E_1 is the expectation operator based on the information available at time t = 1, $U(\mathbf{x})$ is a von Neumann-Morgenstern utility function representing risk preferences, and $\mathbf{x} = (x_1, \ldots, x_T)$ denotes the consumption path over the planning horizon. Throughout, we assume that $U_t' = \partial U(\mathbf{x})/\partial x_t > 0$, and $\partial U^2(\mathbf{x})/\partial \mathbf{x}^2$ is a

[1] Note that we do not impose a priori restrictions on the sign of the marginal cost of storage $C_t' = \partial C_t(S_t)/\partial S_t$. This means that, following Kaldor and Working (1948, 1949), we allow for a "convenience yield" where the C_t' can become negative for low inventory levels.

negative semi-definite matrix. This allows for risk neutrality (when $\partial U^2(\mathbf{x})/\partial \mathbf{x}^2 = 0$), as well as risk aversion (when $\partial U^2(\mathbf{x})/\partial \mathbf{x}^2$ is a negative definite matrix, with $U_t'' = \partial U^2(\mathbf{x})/\partial x_t^2 < 0$).

We assume that prices p_t, q_t and i_t become observable at time t. However, the future is not known for sure. Uncertainty is represented by a general (subjective) distribution of future prices. Under Bayesian learning, the distribution of future prices evolves over time following the observation each time period of (p_t, q_t, i_t) and possibly other random variables correlated with future prices. Based on the information available at time t, future uncertainty is represented by a joint probability distribution of (p_{t+1}, …, p_T), (q_{t+1}, …, q_T) and (i_{t+1}, …, i_T). Below, we will assume that the random variables p_{t+1} and q_{t+1} are independently distributed. This assumption will simplify our analysis. It seems appropriate in situations where the commodity being stored involves only a small sector of the economy.

For given the initial conditions (I_0, S_0) and using backward induction, optimal decisions are given by the optimization problem

$$\text{Max}_{S_1,I_1,x_1} E_1\{\text{Max}_{S_2,I_2,x_2} E_2\{\ldots\text{Max}_{S_T,I_T,x_T} \{E_T U(x_1, \ldots, x_T)\text{: equations (1a)-(1b)}\}$$

$$= \text{Max}_{S_1,I_1} E_1\{\text{Max}_{S_2,I_2} E_2\{\ldots\text{Max}_{S_T,I_T} \{E_T U([\pi_1 - I_1 + (1 + i_0) I_0 + N_1]/q_1,$$

$$\ldots, [\pi_T - I_T + (1 + i_{T-1}) I_{T-1} + N_T]/q_T)\text{: equation (1a)}\}, \tag{2}$$

where E_t denotes the expectation operator based on the information available at time t. Since there is no incentive to store/invest beyond the end of the planning horizon, we set $S_T = 0$ and $I_T = 0$ as terminal conditions. From (2), the optimal decisions made at time t are $S_t^*(p_t, S_{t-1}, I_{t-1}, \cdot)$ and $I_t^*(p_t, S_{t-1}, I_{t-1}, \cdot)$. The properties of these decisions rules are explored next.

3. COMPETITIVE STORAGE

First, we consider the case of a competitive storage firm that takes market prices as given. It means that, from the view point of the firm, neither current prices nor the probability distribution of future prices are affected by the firm's decisions. In this context, using the envelope theorem, the first-order necessary conditions for S_t and I_t in (2) are[2]

$$E_t[U_{t+1}' \cdot (1-\delta_t)\, p_{t+1}/q_{t+1} - U_t' \cdot (p_t + C_t')/q_t] \leq 0, \tag{3a}$$

$$E_t[U_{t+1}' \cdot (1-\delta_t)\, p_{t+1}/q_{t+1} - U_t' \cdot (p_t + C_t')/q_t]\, S_t = 0, \tag{3b}$$

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[2] If the firm is involved in both storage and production activities, then in addition to equations (3)-(4), the first-order conditions associated with the maximization of expected utility with respect to production

$S_t \geq 0,$ (3c)

and

$$E_t[U_{t+1}' \cdot (1+i_t)/q_{t+1} - U_t'/q_t] = 0, \quad (4)$$

$t = 1, 2, \ldots, T-1$, where $C_t' = \partial C_t(S_t)/\partial S_t$ denotes the marginal cost of storage at time t, and $U_t' = \partial U(\mathbf{x})/\partial x_t$ evaluated at $\{S_{t+1}^*, I_{t+1}^*; S_{t-2}^*, I_{t+2}^*, \ldots\}$. Assume that p_t, q_t and i_t are observed at time t. After substituting equation (4) into (3), we obtain

$$E_t[(U_{t+1}'/q_{t+1}) \cdot (\beta_t\, p_{t+1} - p_t - C_t')] \leq 0, \quad (5a)$$

$$E_t[(U_{t+1}'/q_{t+1}) \cdot (\beta_t\, p_{t+1} - p_t - C_t')]\, S_t = 0, \quad (5b)$$

$$S_t \geq 0, \quad (5c)$$

where $\beta_t \equiv (1 - \delta_t)/(1 + i_t) \in (0, 1)$ is a discount factor reflecting the effects of both depreciation (through δ_t) and the opportunity cost of money (through the interest rate i_t). Equation (5) is an Euler equation characterizing the implications of optimal competitive storage for price dynamics. When $S_t > 0$, it can be alternatively written as

$$\beta_t\, E_t(p_{t+1}) - p_t = C_t' + R_t', \quad (6)$$

where $R_t' \equiv -\beta_t\, Cov_t(U_{t+1}'/q_{t+1}, p_{t+1})/E_t(U_{t+1}'/q_{t+1})$, and $Cov_t(a, b)$ is the covariance between "a" and "b" based on the information available at time t. Note that the covariance term is zero under two scenarios: 1/ when p_{t+1} is known at time t; and 2/ under risk neutrality, where $\partial^2 U(\mathbf{x})/\partial \mathbf{x}^2 = 0$. In this latter case, $U_{t+1}' = \partial U/\partial x_{t+1}$ is a constant and $Cov_t(U_{t+1}'/q_{t+1}, p_{t+1}) = 0$ under the independence of p_{t+1} and q_{t+1}. It follows that the marginal risk premium vanishes ($R_t' = 0$) either in the absence of price risk or under risk neutrality. Alternatively, under risk aversion, $\partial^2 U(\mathbf{x})/\partial \mathbf{x}^2$ is a negative definite matrix, implying that R_t' will in general be non-zero under price risk. As such, we interpret R_t' in (6) as a "marginal risk premium" measuring the private cost of risk bearing and reflecting the role of risk and risk aversion. This gives the following result.

Proposition 1: For a competitive firm, optimal storage at time t corresponds to equation (6), where R_t' is the marginal risk premium.

Equation (6) shows that, for a competitive firm, optimal storage corresponds to the standard result: marginal revenue associated with storing one more unit of S_t from time t to time t+1, β_t

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decisions would also apply (with the profit from production activities at time t being part of "other income" N_t in equation (2)).

$E_t(p_{t+1}) - p_t$, must equal marginal cost, $C_t' + R_t'$. The marginal cost includes the marginal cost of storage C_t' as well as the marginal risk premium R_t' reflecting the cost of private risk bearing. We have seen that risk neutrality implies that the marginal risk premium vanishes, $R_t' = 0$. Thus, under risk neutrality, the marginal cost reduces to C_t'. However, under risk and risk aversion, R_t' is in general non-zero.

Given $C_t'' > 0$, equation (6) implies that the optimal stock S_t is necessarily higher (lower) when $R_t' < 0$ (> 0). Noting that $R_t' = 0$ either under risk neutrality or in the absence of uncertainty about p_{t+1}, we obtain the following results.

Proposition 2: Consider a competitive firm facing $C_t'' > 0$. Then, *ceteris paribus* at time t:

- compared to risk neutrality, the firm would store more (less) under risk aversion when the marginal risk premium satisfies $R_t' < 0$ (> 0);
- compared to the riskless case, the risk averse firm facing uncertainty about p_{t+1} would store more (less) when the marginal risk premium satisfies $R_t' < 0$ (> 0).

Proposition 2 establishes the role of risk and risk aversion for a competitive storage firm. Under risk neutrality, the inventory choice for a competitive firm depends on the expected path of prices as well as marginal cost, but not on future price uncertainty. And under risk aversion, future price uncertainty matters as it affects the marginal risk premium R_t'. The marginal risk premium reflects the impact of price risk and risk aversion on optimal stocks. Intuitively, a positive marginal risk premium means that storage contributes to increasing the cost of risk, thus providing a disincentive to store. Alternatively, a negative marginal risk premium implies that storage tends to decrease the cost of risk, providing additional incentives to store.

Given $R_t' \equiv -\beta_t \, Cov_t(U_{t+1}'/q_{t+1}, p_{t+1})/E_t(U_{t+1}'/q_{t+1})$, $U_{t+1}' > 0$, and under future price uncertainty, note that the marginal risk premium R_t' is always of the sign of $[-Cov_t(U_{t+1}'/q_{t+1}, p_{t+1})]$. When the sign of $[\partial(U_{t+1}'/q_{t+1})/\partial p_{t+1}]$ can be determined *ex ante*, $Cov_t(U_{t+1}'/q_{t+1}, p_{t+1})$ is necessarily of the sign of $[\partial(U_{t+1}'/q_{t+1})/\partial p_{t+1}]$. With $U_{t+1}' = \partial U(\mathbf{x})/\partial x_{t+1}$, note that $(\partial U_{t+1}'/\partial p_{t+1})$ involves the term $[(\partial^2 U(\mathbf{x})/\partial x_{t+1}^2) \cdot ((1-\delta_t)\, S_t - S_{t+1}^*)]$. Under risk aversion (where $\partial^2 U(\mathbf{x})/\partial x_{t+1}^2 < 0$), this term is positive (negative) when $[(1-\delta_t)\, S_t - S_{t+1}^*] > 0$ (< 0). This shows that the sign of the marginal risk premium R_t' is in general ambiguous: R_t' can be positive or negative depending on the anticipated change in stock between time t and time t+1.

To provide an intuitive interpretation of this result, note that the variance of firm profit π_{t+1} is $Var_t[p_{t+1} \cdot ((1-\delta_t)\, S_t - S_{t+1}^*)]$, where $((1-\delta_t)\, S_t - S_{t+1})$ is the net sale at time (t+1). Under price risk, this means that the variance of profit (and thus risk exposure) tends to increase (decrease) with S_t when $((1-\delta_t)\, S_t - S_{t+1}^*)$ is positive and large (negative and large). For a risk averse agent who tries to reduce his/her risk exposure, this means an incentive to store less at time t when he/she anticipates a large sale next period (meaning a large decrease in next-period firm inventory if δ is small). This corresponds to situations where $R_t' > 0$. Alternatively, a risk averse agent would have incentive to store more at time t when he/she anticipates a large purchase next period (meaning a large increase in next-period firm inventory if δ is small). This corresponds to situations where $R_t' < 0$. Intuitively, by attempting to reduce its risk exposure, a risk averse agent has an extra incentive to reduce optimal stock fluctuations over time. This suggests that risk and risk aversion tend to diminish the capacity of a competitive storage firm to smooth market and price fluctuations over time.

The above analysis shows the importance of the length of the planning horizon T in the investigation of the effects of risk and risk aversion on storage. As noted above, the sign of $Cov_t(U_{t+1}'/q_{t+1}, p_{t+1})$ depends on the term $[(\partial^2 U(\mathbf{x})/\partial x_{t+1}^2) \cdot ((1-\delta_t) S_t - S_{t+1}^*)]$. When T = 2, then $S_2^* = 0$ and $Cov_1(U_2'/q_2, p_2) = \text{sign}\{(\partial^2 U(\mathbf{x})/\partial x_2^2) \cdot (1-\delta_1) S_1\}$. Under risk aversion (where $(\partial^2 U(\mathbf{x})/\partial x_2^2) < 0$), it follows that $Cov_1(U_2'/q_2, p_2) \leq 0$ and $R_1' \geq 0$. Thus, when the planning horizon has only two periods (T = 2), price risk <u>always</u> provides an incentive for a risk-averse firm to store less at the current time (t = 1). However this specific result is implied by the terminal condition $S_2^* = 0$ and does <u>not</u> hold in general for $T \geq 3$. Indeed, when $T \geq 3$, $((1-\delta_1) S_1 - S_2^*))$ can be either positive or negative, implying that the covariance term $Cov_1(U_2'/q_2, p_2)$ can also be either positive or negative under risk aversion. In this context, the complexity of the effects of price risk on storage behavior arises only when $T \geq 3$. When storage firms are characterized by risk aversion and a relatively long planning horizon (with $T \geq 3$), this stresses that the effects of price risk on stocks depend on the anticipated path of future stocks.

Finally, note that equation (6) and Propositions 1 and 2 provide a structural characterization of dynamic behavior for a competitive firm involved in storage activities. It is of interest to establish linkages between these firm-level results and market equilibrium. For the commodity of interest, denote the price dependent demand function at time t by $p_t(D_t, \cdot)$, where D_t is the aggregate quantity demanded and $\partial p_t(D_t, \cdot)/\partial D_t < 0$. Here, "$\cdot$" denotes other factors affecting demand (which can include demand shifters as well as past quantities reflecting demand dynamics). Under market equilibrium, the aggregate quantity demanded D_t equals aggregate supply, $(1-\delta_{t-1}) S_{t-1} - S_t + z_t$, where $[(1-\delta_{t-1}) S_{t-1} - S_t]$ is the quantity supplied by our storage firm, and z_t is the net quantity supplied from other sources at time t. Let $(S_1^c, \ldots, S_T^c)$ denote the optimal competitive storage given by equation (6). And let z_t^c denote the net supply obtained from all other sources under competitive conditions. The associated market equilibrium conditions are

$$p_t = p_t((1-\delta_{t-1}) S_{t-1}^c - S_t^c + z_t^c, \cdot), \tag{7}$$

t = 1, …, T. It follows that equations (6) and (7) provide the dynamics of market equilibrium under competitive storage. When p_t measures the marginal willingness-to-pay for consumers of the commodity, then equations (6) and (7) represent the dynamics of a competitive market under storage. Note that this dynamics always depends on the dynamics of information supporting the stock decisions. If information is free, then "rational expectations" would apply, where the subjective probability distribution of future prices is consistent with equation (7). However, if information is costly, it is possible for storage firms to evaluate future prices in a way that differs from the "market fundamentals" given in equation (7). In this context, note that equation (6) would remain a valid representation of competitive storage. Equations (6) and (7) would then reflect how the processing of information by storage firms influences price and market dynamics.

4. STORAGE UNDER IMPERFECT COMPETITION

Our analysis so far has focused on a competitive firm that cannot affect prices. This appears appropriate when the market has a competitive structure and includes a large number of firms. However, this may not apply if there are few firms and/or if some of the firms are relatively large compared to the market. In this case, one can expect the decisions of the firm to affect market prices. In this section, we explore the implications of storage under imperfect competition. As discussed in the literature, there are many ways to introduce imperfect competition in the analysis of storage activities (e.g., Kirman and Sobel; Arvan; Rotemberg and Saloner; Judd; Wright and Williams, chapter 11). As shown by Arvan, the strategic use of inventory can make the characterization of equilibrium difficult. Below, we focus our attention on a simple Cournot game, where each firm makes its own decisions taking as given the quantities chosen by other agents (e.g., Tirole).

Again, consider the price dependent demand function $p_t(D_t, \cdot)$ where D_t is the aggregate quantity demanded and $p_t' = \partial p_t(D_t, \cdot)/\partial D_t < 0$. Under market equilibrium, this can be written as $p_t((1-\delta_{t-1}) S_{t-1} - S_t + z_t, \cdot)$, where $[(1-\delta_{t-1}) S_{t-1} - S_t]$ is the net quantity supplied by our storage firm and z_t is the net quantity supplied from other sources. Under imperfect competition, changes in S_t affect the market price p_t. And under a Cournot game, the storage decision S_t is made conditional on the quantity z_t of "other net supply." Thus, in the analysis presented below, we investigate storage behavior for a Cournot firm which treats the z_t's as given.

As noted above, N_t in (1b) represents income from other activities undertaken by the firm. This is relevant in the case where the firm is involved both in production and storage activities. It means that the analysis presented below would remain valid if the firm manager also makes production decisions (the handling of strategic behavior will be discussed below).

In a Cournot game, given $p_t[(1-\delta_{t-1}) S_{t-1} - S_t + z_t]$, the first-order necessary condition for optimal stock S_t in (2) is

$$E_t[U_{t+1}' \cdot (1-\delta_t) [p_{t+1} + p_{t+1}' \cdot ((1-\delta_t) S_t - S_{t+1}^*)]/q_{t+1}$$

$$- U_t' \cdot (p_t + C_t' + p_t' \cdot ((1-\delta_{t-1}) S_{t-1} - S_t))/q_t] \leq 0, \quad (8a)$$

$$E_t[U_{t+1}' \cdot (1-\delta_t) [p_{t+1} + p_{t+1}' \cdot ((1-\delta_t) S_t - S_{t+1}^*)]/q_{t+1}$$

$$– U_t' \cdot (p_t + C_t' + p_t' \cdot ((1\text{-}\delta_{t-1}) S_{t-1} – S_t))/q_t] S_t = 0, \quad (8b)$$

$$S_t \geq 0, \quad (8c)$$

where $p_t' = \partial p_t/\partial D_t < 0$ at time t, t = 1, …, T-1. The first-order condition with respect to investment I_t remains the same as before and is given in (4).[3] Substituting equation (4) into (8) yields

$$E_t[(U_{t+1}'/q_{t+1}) \cdot [\beta_t \cdot [p_{t+1} + p_{t+1}' \cdot ((1\text{-}\delta_t) S_t – S_{t+1}^*))]$$

$$– p_t – C_t' – p_t' \cdot ((1\text{-}\delta_{t-1}) S_{t-1} – S_t)]] \leq 0, \quad (9a)$$

$$E_t[(U_{t+1}'/q_{t+1}) \cdot [\beta_t \cdot [p_{t+1} + p_{t+1}' \cdot ((1\text{-}\delta_t) S_t – S_{t+1}^*))]$$

$$– p_t – C_t' – p_t' \cdot ((1\text{-}\delta_{t-1}) S_{t-1} – S_t)]] S_t = 0, \quad (9b)$$

$$S_t \geq 0, \quad (9c)$$

where $\beta_t = (1 - \delta_t)/(1 + i_t)$. Equation (9) is an Euler equation characterizing the implications of optimal stocks for a Cournot firm. Define

$$M_t' \equiv E_t[(U_{t+1}'/q_{t+1}) \cdot [\beta_t \cdot p_{t+1}' \cdot ((1\text{-}\delta_t) S_t – S_{t+1}^*) – p_t' \cdot ((1\text{-}\delta_{t-1}) S_{t-1} – S_t)]/E_t(U_{t+1}'/q_{t+1}). \quad (10)$$

Then, when $S_t > 0$, equation (9) can be written as

$$\beta_t E_t(p_{t+1}) – p_t + M_t' = C_t' + R_t'. \quad (11)$$

Note that equation (11) reduces to (6) when $M_t' = 0$. Since (6) identifies the storage behavior of a competitive firm, it follows that M_t' reflects the exercise of market power by the storage firm. Equation (11) states that, at the optimum, the storage firm equates marginal revenue, $\beta_t E_t(p_{t+1}) – p_t + M_t'$, with marginal cost, $C_t' + R_t'$. It introduces the term M_t' as an additional part of marginal revenue. This term can be interpreted as the part of marginal revenue due to the exercise of market power by our storage firm. These results are summarized next.

Proposition 3: At time t and conditional on z_t, a Cournot firm chooses storage S_t that satisfies equation (11), where the marginal revenue due to the exercise of market power M_t' is given by equation (10).

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[3] Again, if the firm is involved in both storage and production activities, then in addition to equations (4) and (8), the first-order conditions associated with the maximization of expected utility with respect to production decisions would also apply (with the profit from production activities at time t being part of "other income" N_t in equation (2)).

Note that M_t' in equation (10) can be either positive or negative depending on the anticipated change in future firm stocks. Proposition 3 shows that, in general, the inventory choice for a Cournot firm depends on the expected path of prices, on marginal cost, on price risk (under risk aversion) and on the anticipated path of future firm inventory.

From Proposition 3, in a way similar to the Lerner index, a convenient measure of the exercise of market power is

$$L_t = -M_t'/p_t,$$

$$= [\beta_t\, E_t(p_{t+1}) - p_t - C_t' - R_t']/p_t, \qquad (12a)$$

using (11). L_t in equation (12a) is the excess marginal revenue due to the exercise of market power at time t, measured as a proportion of the market price p_t. Relative to p_t, it involves the discounted expected price change, $\beta_t\, E_t(p_{t+1}) - p_t$, minus the marginal storage cost C_t', minus the marginal risk premium R_t'. From equation (6), $L_t = 0$ under competition. However, in general, $L_t \neq 0$ in the presence of market power.

Given the definition of M_t', note that equation (12a) can be alternatively written as

$$L_t = -(1/p_t) \cdot E_t[(U_{t+1}'/q_{t+1}) \cdot \beta_t \cdot p_{t+1}' \cdot ((1-\delta_t)\, S_t - S_{t+1}^{*})$$

$$- p_t' \cdot ((1-\delta_{t-1})\, S_{t-1} - S_t)]/E_t(U_{t+1}'/q_{t+1}),$$

$$= \beta_t \cdot E_t[(U_{t+1}'/q_{t+1}) \cdot \varepsilon_{t+1}^{-1} \cdot (p_{t+1}/p_t) \cdot W_{t+1}]/E_t(U_{t+1}'/q_{t+1}) - \varepsilon_t^{-1} \cdot W_t, \qquad (12b)$$

where $\varepsilon_t = -[\partial \ln(p_t)/\partial \ln(D_t)]^{-1}$ is the elasticity of demand at time t, and $W_t = [(1-\delta_{t-1})\, S_{t-1} - S_t]/D_t$ is the market share of our storage firm at time t. Equation (12b) illustrates how the market share W affects the exercise of market power. When the market share W becomes small ($W \rightarrow 0$), then $L_t \rightarrow 0$ in (12b) and storage behavior can be described as competitive. Alternatively, when the storage firm has a significant market share W, then equation (12b) shows that L_t can differ from zero when the storage firm exhibits market power. This gives the intuitive result that a significant market share is a necessary condition for the storage firm to exercise market power. In addition, equation (12b) shows that, when non-zero, L_t can be either positive or negative depending on the anticipated future price movements. Finally, equation (12b) illustrates the role of the price elasticity of demand $\varepsilon = -[\partial \ln(p)/\partial \ln(D)]^{-1}$. In particular, a very high price elasticity ($\varepsilon \rightarrow \infty$) means that $L_t \rightarrow 0$. This gives the intuitive result that storage behavior becomes approximately competitive when market demand becomes highly price-elastic. Alternatively, equation (12b) shows that the potential for exercising market power rises when demand becomes more price-inelastic.

The implications of the exercise of market power for storage are explored next. (See the proof in the Appendix)

<u>Proposition 4</u>: *Ceteris paribus*, at time t and conditional on z_t, a Cournot firm stores more (less) than a competitive firm when $M_t' > (<) 0$, where M_t' is given in (10).

Proposition 4 shows how the exercise of market power affects storage behavior for a Cournot firm at time t. The stated results apply under risk neutrality as well as under risk aversion. First, consider the case of risk neutrality (where $\partial U^2(\mathbf{x})/\partial \mathbf{x}^2 = 0$ and $R_t' = 0$). Under risk neutrality, Proposition 4 implies that, ceteris paribus, a Cournot firm stores more (less) at time t than a competitive firm when $\beta_t \cdot E_t[(1/q_{t+1}) \cdot p_{t+1}' \cdot ((1-\delta_t) S_t - S_{t+1}^*)]/E_t(1/q_{t+1}) > (<) p_t' \cdot ((1-\delta_{t-1}) S_{t-1} - S_t)$. In the case where $p_t' = p_{t+1}' = p' < 0$ and q_{t+1} is known with certainty, this condition reduces to $\beta_t \cdot E_t[((1-\delta_t) S_t - S_{t+1}^*)] < (>) ((1-\delta_{t-1}) S_{t-1} - S_t)$. The term $\{\beta_t \cdot E_t[((1-\delta_t) S_t - S_{t+1}^*)]\}$ can be interpreted as the "discounted firm sale" at time t+1. This shows that a Cournot firm has an incentive to store more (less) than a competitive firm at time t when its discounted sale is expected to increase (decrease) over the next time period. Since increasing (decreasing) sale means reducing (increasing) stocks, this suggests that the exercise of market power tends to stimulate (reduce) S_t in periods when significant increases (decreases) in stocks are anticipated for the next period. To the extent that stock variations over time help smooth anticipated market shocks, this indicates that the exercise of market power reduces the ability of storage to absorb such shocks. These effects will be further evaluated in the empirical analysis presented below.

The role of anticipated future stocks stresses the importance of the length of the planning horizon. This is particularly relevant given that a number of previous papers have investigated non-competitive storage in the context of two-period models (e.g., Allaz; Arvan; Mitraille; Moellgaard et al.; Rotemberg and Saloner; Saloner). Noting that T = 2 implies that $S_2^* = 0$, it is clear that two-period models impose strong restrictions on the prospects for increasing future stocks. In particular, $S_2^* = 0$ means that "discounted sale" is less likely to decrease between t = 1 and t = 2. This implies that such models fail to capture some of the adverse effects of imperfect competition on storage. By showing that two-period models impose strong restrictions on Cournot behavior, this stresses the need to consider storage models with longer planning horizons (with $T \geq 3$).

Second, consider the case of risk averse firm, where $\partial^2 U(\mathbf{x})/\partial \mathbf{x}^2$ is a negative definite matrix. Under risk aversion, Proposition 4 implies that a Cournot firm tends to store more (less) at time t than a competitive firm when $\beta_t \cdot E_t[p_{t+1}' \cdot ((1-\delta_t) S_t - S_{t+1}^*)] + \beta_t \cdot Cov_t[U_{t+1}'/q_{t+1}, p_{t+1}' \cdot (S_t - S_{t+1}^*)]/E_t(U_{t+1}/q_{t+1}) > (<) p_t' \cdot ((1-\delta_{t-1}) S_{t-1} - S_t)$. This condition introduces a role for the covariance term $Cov_t[U_{t+1}'/q_{t+1}, p_{t+1}' \cdot ((1-\delta_{t-1}) S_t - S_{t+1}^*)]$. Under risk aversion, note that this covariance term can be either positive or negative depending on the anticipated path of future sales, i.e., on the anticipated path of future firm stocks. This indicates the presence of interaction effects between risk management and the exercise of market power. For example, under risk aversion, a positive covariance term would strengthen the incentive of a firm playing a Cournot game to store more than a competitive firm. Alternatively, a negative covariance term would strengthen the incentive for a Cournot firm to store less than a competitive firm.

Proposition 4 identifies the role of M_t'. This provides useful insights on how imperfect competition affects storage incentives. However, note that the results presented in Proposition 4 hold under rather strong *ceteris paribus* conditions. These conditions require than nothing else change besides the decision rule used for choosing stock S_t at time t. In a dynamic context, the exercise of market power will typically affect the storage decisions for all relevant periods. This suggests the need to explore the effects of imperfect competition in this broader context. To address this issue, define

$$W(\mathbf{S}, \mathbf{I}) = E_1U[p_1[((1-\delta_0) S_0 - S_1) + z_1] \cdot ((1-\delta_0) S_0 - S_1) - C_1(S_1) - I_1 + (1 + r_0) I_0 + N_1]/q_1, \ldots, p_T[((1-\delta_{T-1}) S_{T-1} - S_T) + z_T] \cdot ((1-\delta_{T-1}) S_{T-1} - S_T) - C_T(S_T) - I_T + (1 + i_{T-1}) I_{T-1} + N_T]/q_T], \quad (13)$$

where $\mathbf{S} = (S_1, \ldots, S_T)$, and $\mathbf{I} = (I_1, \ldots, I_T)$. Denote by $(\mathbf{S}^c, \mathbf{I}^c)$ the optimal decision rules obtained under perfect competition (corresponding to (3) and (4)). And denote by $(\mathbf{S}^m, \mathbf{I}^m)$ the optimal decision rules obtained under Cournot competition (corresponding to (8) and (4)), taking the z_t's as given. Note that, subject to information constraints, $(\mathbf{S}^m, \mathbf{I}^m) \in \text{argmax}_{\mathbf{S},\mathbf{I}} \{W(\mathbf{S}, \mathbf{I})\}$, and $W(\mathbf{S}^m, \mathbf{I}^m)$ is the *ex ante* utility function obtained under Cournot behavior. It follows that $W(\mathbf{S}^m, \mathbf{I}^m) \geq W(\mathbf{S}, \mathbf{I})$ for any feasible $(\mathbf{S}, \mathbf{I})$. Choosing $(\mathbf{S}, \mathbf{I}) = (\mathbf{S}^c, \mathbf{I}^c)$ generates the following result.

<u>Proposition 5</u>: Under Cournot behavior and conditional on $(z_1, \ldots, z_T)$, the exercise of market power tends to make the storage firm better off in the sense that

$$W(\mathbf{S}^m, \mathbf{I}^m) \geq W(\mathbf{S}^c, \mathbf{I}^c). \quad (14)$$

Proposition 5 gives the standard result that, conditional on $(z_1, \ldots, z_T)$, a Cournot firm benefits from the exercise of market power. These private benefits give an incentive for the firm to implement non-competitive behavior. Of course, to the extent that competitive behavior implements a Pareto efficient allocation (from the first welfare theorem), this means that the exercise of market power by storage firms also generates an inefficient allocation and creates a social cost. In the absence of other distortions, this social cost means that the private gains obtained by the storage firm are associated with losses suffered by other agents in the economy (i.e., producers and consumers), and that these losses are larger than the storage firm's gains.

The inequality $W(\mathbf{S}^m, \mathbf{I}^m) \geq W(\mathbf{S}, \mathbf{I})$ provides a basis for investigating the implications of imperfect competition. Note that these implications are typically complex since they involve dynamic behavior over a T-period planning horizon. The difficulties arise because the effects of imperfect competition can vary from one period to the next. To investigate these issues, consider the case where storage behavior S_t is associated with the following conditions

$$E_t[U_{t+1}' \cdot (1-\delta_t) [p_{t+1} + p_{t+1}' \cdot \alpha \cdot ((1-\delta_t) S_t - S_{t+1}^*)]/q_{t+1}$$

$$- U_t' \cdot (p_t + C_t' + p_t' \cdot \alpha \cdot ((1-\delta_{t-1}) S_{t-1} - S_t))/q_t] = 0, \quad (15a)$$

$$E_t[U_{t+1}' \cdot (1-\delta_t) [p_{t+1} + p_{t+1}' \cdot \alpha \cdot ((1-\delta_t) S_t - S_{t+1}^*)]/q_{t+1}$$

$$- U_t' \cdot (p_t + C_t' + p_t' \cdot \alpha \cdot ((1-\delta_{t-1}) S_{t-1} - S_t))/q_t] S_t = 0, \quad (15b)$$

$$S_t \geq 0, \quad (15c)$$

$t = 1, \ldots, T$. Conditional on α, define $(\mathbf{S}^a(\alpha)), \mathbf{I}^a(\alpha))$ as the optimal decisions satisfying (4) and (15). The decision rules $(\mathbf{S}^a(\alpha)), \mathbf{I}^a(\alpha))$ can represent competitive behavior as well as Cournot behavior as special cases. Indeed, competitive behavior is obtained when $\alpha = 0$, with $(\mathbf{S}^c, \mathbf{I}^c) = (\mathbf{S}^a(0)), \mathbf{I}^a(0))$. This follows from the fact that equation (15) reduces to equation (3) when $\alpha = 0$. Alternatively, Cournot behavior is obtained when $\alpha = 1$, with $(\mathbf{S}^m, \mathbf{I}^m) = (\mathbf{S}^a(1)), \mathbf{I}^a(1))$. This follows since equation (15) reduces to equation (8) when $\alpha = 1$.

Equation (14) implies the following result. (See the proof in the Appendix).

Lemma 1:

$$\int_0^1 (1-\alpha) \cdot \sum_{t=1}^{T} E_1 [M_t' \cdot \partial S_t^a(\alpha)/\partial\alpha] \cdot d\alpha \geq 0. \quad (16)$$

With $\mathbf{S}^a(1)$ and $\mathbf{S}^a(0))$ representing optimal stocks under Cournot behavior and competitive behavior, respectively, equation (16) generates the following result.

Proposition 6: Conditional on $(z_1, \ldots, z_T)$, comparing Cournot stock behavior $\mathbf{S}^a(1)$ with competitive stock behavior $\mathbf{S}^a(0)$, we have

- $\mathbf{S}^a(1) \geq \mathbf{S}^a(0)$ if $M_t' > 0$ for all $t = 1, \ldots, T-1$,
- $\mathbf{S}^a(1) \leq \mathbf{S}^a(0)$ if $M_t' < 0$ for all $t = 1, \ldots, T-1$.

These results are consistent with the results obtained in Proposition 4. However, Proposition 6 applies in the broader context of a T-period planning horizon. It indicates that the sign of M_t is the key determinant of how market power affects storage. It shows that, under Cournot behavior, the exercise of market power tends to have a positive (negative) effect on storage when $M_t' > 0$ (< 0). And as discussed above, the sign of M_t' depends on the anticipated path of future stocks. It means that, in general, the exercise of market power tends to stimulate (reduce) stockholding when stocks are anticipated to increase (decrease) over time. Note that this result is consistent with previous literature (e.g., Williams and Wright, p. 325) showing that market power tends to increase (decrease) stocks when initial stocks are low (high). However previous analysis has typically been presented under stationary conditions. Our analysis is developed under more general conditions. For example, by not imposing stationarity, our results apply to markets exhibiting business cycles, seasonality, or trends. They suggest that the exercise of market power reduces the ability of storage to smooth anticipated market shocks. This particular finding will be further discussed below. Finally, note that Proposition 6 does not give clear results when the sign of M_t changes over the planning horizon. In this case, the effects of market power on stocks can be either positive or negative. As discussed above, such effects in general depend on the anticipated path of future stocks.

Capturing strategic effects

Propositions 3-6 provide a structural characterization of storage behavior for a imperfectly competitive Cournot firm. By being conditional on $(z_1, \ldots, z_T)$, note that these results fall short of capturing the strategic effects of stocks. To investigate such strategic effects, consider that the "other supply" z_t is produced from (J+1) sources, with $z_t = \sum_{j=0}^{J} z_{jt}$, z_{jt} being the part of "other supply" from the j-th source at time t. Assume that the decision about z_{0t} is made independent of S_t, but that there are strategic interdependences among S_t and z_{jt}, j = 1, …, J. Denote by $S_t^m(\mathbf{z}_t, \cdot)$ the optimal storage given by equation (8) or (9), conditional on "other supplies" $\mathbf{z}_t = (z_{1t}, \ldots, z_{Jt})$. In a non-cooperative game with strategic effects among S_t and $\mathbf{z}_t$, denote by $z_{jt}^m(S_t, \mathbf{z}_{-j,t}, \cdot)$ the decision rule used in choosing z_{jt}, where $\mathbf{z}_{-j,t} = (\ldots, z_{j-1,t}, z_{j+1,t}, \ldots)$ and "·" represents other factors affecting z_{jt}, j = 1, …, J. The associated Nash equilibrium at time t involves a fixed point $(S_t^e, \mathbf{z}_t^e)$ satisfying $S_t^e = S_t^m(\mathbf{z}_t^e, \cdot)$ and $z_{jt}^e = z_{jt}^m(S_t^e, \mathbf{z}_{-j,t}^e, \cdot)$, j = 1, …, J. In general, it is well-known that such equilibrium may not exist, and if it exists it may not be unique (e.g., Tirole; Arvan). This creates significant challenges for the analysis of economic dynamics.

To simplify the discussion, assume that a Nash equilibrium exists satisfying $S_t^e = S_t^m(\mathbf{z}_t^e, \cdot)$ and $z_{jt}^e = z_{jt}^m(S_t^e, \mathbf{z}_{-j,t}^e, \cdot)$, j = 1, …, J. The associated market equilibrium conditions under imperfect competition are

$$p_t = p_t((1-\delta_{t-1})\, S_{t-1}^e - S_t^e + \sum_{j=0}^{J} z_{jt}^e, \cdot), \qquad (17)$$

t = 1, …, T. Equations (9) and (17) provide the dynamics of market equilibrium under Cournot-imperfect competition. As shown by Tirole, Arvan, and others, market equilibrium and its dynamics depend in general on the nature of strategic interactions between S_t and $\mathbf{z}_t$ (as characterized by the properties of the reaction functions $S_t^m(\mathbf{z}_t^e, \cdot)$ and $z_{jt}^m(S_t^e, \mathbf{z}_{-j,t}^e, \cdot)$, j = 1, …, J). This can represent various market conditions. To see that, consider first the case where the storage firm specializes in storage and is not involved in production activities. Then, the storage firm would be a monopoly if J = 0. It would be part of a duopoly when J = 1, with $z_{1t} = z_{1t}^m(S_t, \mathbf{z}_{-j,t}, \cdot)$) being the supply provided by the "other duopolist." And it would be part of an oligopoly when J ≥ 2. Second, consider the case where the storage firm is also involved in production activities. In this context, let z_{1t} represent the quantity supplied by this firm at time t, with $z_{1t}^m(S_t, \mathbf{z}_{-1,t}, \cdot)$ being the firm output produced at time t that maximizes expected utility (2). Then, the storing/producing firm would be a monopoly if J = 1. It would be part of a duopoly if J = 2 (with z_{2t} being the supply provided by the "other" duopolistic firm). And it would be part of an oligopoly when J ≥ 3. This illustrates how our approach can accommodate various market structures. Of course, each market structure would have different implications for market dynamics (depending on the nature of strategic interactions between storage S_t and "other supplies" $\mathbf{z}_t$). However, we want to stress that irrespective of these strategic interactions, equation (9) remains a valid representation of storage decisions under Cournot competition. As such, equation (9) provides a convenient structural representation of the effects of imperfect competition on storage decisions.

5. ECONOMETRIC SPECIFICATION

We start from the fact that equation (9) is a structural model for optimal storage S_t under Cournot competition. We propose an econometric specification for (9) that can support an empirical analysis of storage behavior.

Note that equation (9) suffers from an identification problem with respect to risk preferences. To see that, consider the case where the decision maker exhibits constant absolute risk aversion where $U(\mathbf{x}) = \sum_{t=1}^{T} -(K_t/r) \cdot \exp(-r \cdot x_t)$, $K_t > 0$ being a discount factor at time t, and r being the Arrow-Pratt risk aversion parameter. Risk neutral preferences are obtained when r = 0, while risk aversion corresponds to r > 0. Then, $U_t' = \partial U(x)/\partial x_t = K_t \cdot \exp(-r \cdot x_t)$. With $x_t > 0$, note that $U_t' \to 0$ as $r \to \infty$. It follows that equation (9) would always hold when $r \to \infty$. It means that the risk aversion parameter r is not identified in equation (9). In other words, equation (9) alone cannot provide a basis to investigate the nature of risk preferences.[4] On that basis, we proceed our empirical investigation assuming risk neutrality, with $\partial^2 U(\mathbf{x})/\partial \mathbf{x}^2 = 0$.

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[4] Note that risk preferences could be investigated empirically if the analysis were based instead on the joint estimation of equations (4) and (8). Subject to a normalization rule restricting the marginal utility of income to be positive, risk preferences would then be identified. The problem for us is that, under risk aversion, U_t' in equations (4) and (8) depends in general on "other income" N_t. In our case, we do not

Consider the storage cost specification $C_t(S_t) = c_1 S_t + ½ c_2 S_t^2$. It follows that $C_t' = \partial C_t(S_t)/\partial S_t = c_1 + c_2 S_t$. Finally, let $p_t(D_t) = \gamma_{0t} + \gamma_1 D_t$, with $p_t' = \partial p_t(D_t)/\partial D_t = \gamma_1 < 0$. Then, consider the following specification:

$$\beta_t \cdot [p_{t+1} + \phi \cdot ((1-\delta_t) S_t - S_{t+1}^*))] - [p_t + c_1 + c_2 S_t + \phi \cdot ((1-\delta_{t-1}) S_{t-1} - S_t))] = e_t, \qquad (18)$$

where $\beta_t = (1 - \delta_t)/(1 + i_t)$, and e_t is an error term distributed with mean zero and finite variance. Under risk neutrality (where U_{t+1}' is a positive constant) and given $S_t > 0$, compare equation (18) with equations (5) and (9). The comparison with equation (5) shows that equation (18) represents the competitive storage decision rule when $\phi = 0$. And given $p_{t+1}' = \gamma_1$, the comparison with equation (9) shows that equation (18) represents the Cournot decision rule when $\phi = \gamma_1$. Thus, equation (18) nests two important storage decision rules as special cases: $\phi = 0$ under competition, and $\phi = \gamma_1 < 0$ under Cournot behavior. In other words, in the context of equation (18), testing for competitive storage corresponds to the null hypothesis H_0: $\phi = 0$. And testing for Cournot behavior corresponds to the null hypothesis H_0: $\phi = \gamma_1 < 0$.

Could ϕ in equation (18) also have an economic interpretation n when $\gamma_1 < \phi < 0$? Letting $\phi = (1 + \eta) \gamma_1$, equation (18) can also be interpreted as the first-order condition associated with the maximization problem (2) with $p_t(D_t) = p_t((1-\delta_{t-1}) S_{t-1} - S_t + \sum_{j=0}^{J} z_{jt}^e, \cdot)$ and $\eta = \partial(\sum_{j=0}^{J} z_{jt}^e)/\partial S_t$. Following Bresnahan, Perry, and Dixit, η can be interpreted as the "conjectural variation" of the storage firm anticipating how "other supplies" $\sum_{j=0}^{J} z_{jt}^e$ would react to the choice of S_t. Again, this includes Cournot behavior and competitive behavior as special cases. Cournot behavior is obtained when $\eta = 0$, i.e. when the storage firm expects no quantity response from other firms. And competitive-Bertrand behavior is obtained when $\eta = -1$, i.e. when the choice of S_t has no effect on the market price p_t. But it also includes intermediate cases when $-1 < \eta < 0$, corresponding to partial departures from competitive conditions. The firm storage decisions then affect the market price ($-1 < \eta$), but this effect is less than under Cournot behavior due to the anticipated response of other firms ($\eta < 0$). With $\phi = (1 + \eta) \gamma_1$, it follows that $\gamma_1 < \phi < 0$ can be interpreted as reflecting the strength of departure from competitive conditions. Note that this interpretation is not solidly grounded in a game-theoretic model of strategic interactions in the market place. However, it provides a simple measure of departure from competition (e.g., Genesove and Mullin).

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have data on N_t. In other words, an empirical investigation of the risk preferences of the decision maker would require data on all sources of his/her income.

Under risk neutrality, equation (18) provides the specification used in our empirical investigation. Equation (18) is a structural equation representing optimal storage S_t. As a structural equation, note that it involves optimal storage at time t and (t+1): S_t and S_{t+1}. These variables are clearly endogenous at time t. Equation (18) also involves prices p_t and p_{t+1}. In a market equilibrium framework, these variables are jointly determined (since they are influenced by storage decisions) and should also be treated as endogenous. This means that, in general, the estimation of the structural equation (18) is subject to endogeneity problems leading to potential simultaneous equation bias. To deal with the endogeneity problems, we propose to estimate (18) using the generalized method of moments (GMM). This requires the use of instruments that are orthogonal to the error term e_t in (18). When this orthogonality condition is satisfied, and under some regularity conditions, GMM provides parameter estimates that are consistent and asymptotically normal. And upon the choice of an appropriate weighting scheme, the GMM estimator is also asymptotically efficient (Hansen, 1982).

6. DATA

This section presents an econometric application to the determination of stocks for American cheese in the US. As discussed in the introduction, the choice of American cheese is motivated in part by the fact that there is empirical evidence of exercise of market power in the American cheese market (e.g., Mueller et al., 1997; Muller and Marion, 2000). In the absence of disaggregate firm-level data, the analysis is applied at the aggregate level. This means that our empirical analysis relies on aggregate stock data for American cheese in the US. Applying our model to aggregate data forces us to assume that aggregate storage behaves *as if* it was generated by a representative firm. This has one important limitation: it does not allow the identification of which firms may be exercising market power. However, it still provides a useful framework to investigate empirically the question of whether aggregate private stock-holding has been managed in a non-competitive way over the last decade.

The data on the American cheese market were obtained from USDA. They consist of 157 observations on monthly prices and stocks for the period 1993 to March 2006. During this study period, government stocks of American cheese have been negligible.[5] American cheese stocks have been held almost exclusively by private firms since 1993. Thus, investigating the competitive nature of private storage behavior in the American cheese market over the last 13

[5] Before 1993, the US government price support program (which is part of US government dairy policy) was such that government purchases took place when the market price was falling below the support price for American cheese. As a result, significant government stocks of American cheese did accumulate in earlier periods. However, since 1993, the price support program for American cheese has been basically inactive: the market price has stayed consistently above the support price and government stocks have never exceeded 3 percent of total American cheese stocks.

years appears appropriate. The price data (p_t) used in our analysis involve monthly prices for 40 lb block cheddar cheese in Chicago.[6] The stock data (S_t) are for US commercial stock of American cheese at end of each month. The stock data vary from 297 million lbs to 628 million lbs over the sample period. This means that censoring issues do not arise as the data never get close to a "zero stock" situation. The consumer price (q_t) is measured by the Consumer Price Index for urban households reported by the US Bureau of Labor Statistics. Finally, the monthly riskless interest rate (i_t) is obtained from 6-month US Treasury bill.

7. ESTIMATION

Using the data just discussed, equation (18) was estimated for the US American cheese sector using a GMM estimator. GMM is an instrumental variable method. The following instruments were used: an intercept, a time trend, monthly seasonal dummies, and American cheese price and commercial stock lagged one and two periods. The time trend and seasonal dummies are supply/demand shifters in the US American cheese market. And the lagged variables capture market dynamics. The GMM parameter estimates of equation (18) are reported in Table 1.[7] The standard errors are White-corrected robust standard errors allowing for possible heteroscedasticity.

The GMM estimation of equation (18) provides a good explanatory power for prices: it explains 89 percent of the price variation over the sample period. To check for the appropriateness of the instruments, we tested the orthogonality conditions associated with overidentifying restrictions (Hansen). The test statistic is 0.09. Under the null hypothesis of orthogonality, the test has a χ^2 distribution with 14 degrees of freedom. Thus, we fail to reject the null hypothesis of orthogonality. This provides evidence that the instruments appear appropriate and provide consistent estimate of the parameters.

The estimates reported in Table 1 show that $C'' = c_2$ is positive. Also, while c_1 is estimated to be negative, note that the marginal cost of storage, $C' = c_1 + c_2 S_t$, remains positive within the range of the sample data. Thus, it appears that sample stock levels do not decline enough to uncover evidence supporting the presence of a "convenience yield." Finally Table 1 reports an estimate of

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[6] Note that our analysis assumes a standard storable commodity. It allows for depreciation of the inventory over time. However, it does not allow for the depreciation patterns to vary with the "vintage" of the stocks (e.g., the case where the depreciation rate varies with length of storage). Addressing this issue would require to have data on the "vintage" of inventories and their prices. Since such data are not available for American cheese, we neglect these issues in our analysis.

[7] The econometric estimates reported in Table 1 are obtained assuming that $\delta_t = 0$. Some sensitivity analysis was performed on the depreciation rate δ_t. We obtained the same qualitative conclusions (as reported below) under small positive δ.

$\phi = -1.097$. With a standard error of 0.451, this means that ϕ is found to be negative and significantly different from zero. As discussed above, $\phi = 0$ would correspond to competitive storage. Thus, our econometric analysis strongly rejects the null hypothesis of competitive storage.

We have seen above that Cournot behavior corresponds to the null hypothesis H_0: $\phi = \gamma_1$, where $\gamma_1 = \partial p_t(D_t)/\partial D_t < 0$. The elasticity of U.S. demand for cheese has been estimated in previous literature. The elasticity estimates vary from -0.33 (Huang), -0.44 for processed cheese (Gould and Lin), and -0.52 (Pagoulatos and Sorensen). Using the -0.44 elasticity estimate evaluated at sample means, it follows that $\gamma_1 = -13.82$. Then, testing the hypothesis that $\phi = \gamma_1$, we reject this hypothesis. Thus, we also find strong statistical evidence against Cournot behavior.

With $\gamma_1 = -13.82 < \phi = -1.097 < 0$, our empirical analysis suggests that neither competition nor Cournot behavior provides a satisfactory representation of storage behavior in the US American cheese market. However, noting that $\phi = -1.097$ is closer to 0 than it is to $\gamma_1 = -13.82$, our results suggest that the departure from competitive conditions, while significant, appears "moderate." The implications of our econometric estimates are discussed next.

8. Implications

Given the statistical evidence of non-competitive storage in the US American cheese market, we want to examine the nature and extent of the distortions created by the exercise of market power. This is a difficult task since storage activities are only parts of the economic activities affecting market dynamics. First, this requires information on the temporal evolution of production and consumption decisions. Second and perhaps more importantly, this requires information about how market participants anticipate the future. This involves a large number of possibilities. Under some scenarios, a monopoly producing a storable good may not able to exercise its market power (as exemplified by the "Coase conjecture"). Alternatively, the strategic use of inventory can be used to enforce collusion (e.g., Rotemberg and Saloner). Finally, the amount of information available to each market participant is always relevant. If information is costless, then a rational expectation equilibrium can be justified (e.g., Williams and Wright). However, if information is costly, then the information obtained by each market participant would depend on its cost. Assessing the amount of information used by various market participants remains a difficult task. Yet this information is expected to affect both storage and market dynamics. Given these difficulties, we focus our attention on the rather restrictive case of perfect foresight. While this is not a particularly realistic assumption, it will greatly simplify simulation exercises used to evaluate our econometric results.

Under perfect foresight, the future is correctly anticipated and all future variables can be treated as if they were known. This means that our econometric results can be evaluated using observed data. In this context, we evaluate the market power component M_t' given in equation (10) and the associated relative index $L_t = -M_t'/p_t$ given in equation (12a). Recall that L_t provides a relative measure of the excess marginal revenue due to the exercise of market power. The results for $L_t = -M_t'/p_t$ are reported in Figure 1. Figure 1 shows that L_t varies between -0.047 to +0.064 over the sample period. It illustrates that L_t (or M_t') can be positive or negative depending on the patterns of change in stocks. The largest value for L_t (+0.064) occurred in May 1999. It shows that the largest relative excess marginal revenue due to the exercise of market power amounted to 6.4 percent of the market price. In other words, under competitive storage, the market price could have been 6.4 percent lower in May 1999, *ceteris paribus*. As discussed above, this percentage is significantly different from zero, reflecting the price distortion generated under imperfectly competitive storage. However, the exercise of market power in storage activities appears to be moderate. This seems the case when we consider that 6.4 percent is the largest relative simulated price distortion obtained within the sample period.

To evaluate further the implications of the non-competitive price distortions, we simulated the implications of the exercise of market power for storage activities. Again, under perfect foresight (where all future variables are correctly anticipated), we simulated the effects of the excess marginal revenue M_t' on optimal stocks. The simulated change in stocks were calculated as $M_t'/(c_2 - \partial p_t/\partial D_t)$, given an elasticity of demand (evaluated at sample means) of -0.44. Under perfect foresight, actual stocks were taken to be the stocks obtained under imperfect competition. Competitive stocks were then simulated by adding to actual stocks the simulated change in stocks. The results are reported in Figure 2. As expected, Figure 2 shows that actual stocks and competitive stocks are similar when $|M_t'|$ or $|L_t|$ is small. However, compared to competition, $L_t < 0$ (> 0) means that the exercise of market power provides an added incentive (less incentive) to store. As shown in Figure 2, imperfectly competitive storage differs most from competitive storage in periods when stocks reach either a maximum or a minimum. This is expected: these are situations where the path of stock changes varies most. In a way consistent with Propositions 3-6, imperfect competition tends to reduce stocks when stocks get close to a maximum (because future stocks are then anticipated to decline). And imperfect competition tends to increase stocks when stocks are close to a minimum (because future stocks are then anticipated to increase). This illustrates how the effects of market power on stock-holding vary with market conditions. Figure 2 shows that imperfect competition reduces the magnitude of stock fluctuations over time. It indicates that the ability of stocks to buffer anticipated fluctuations in supply/demand conditions declines when market power affects storage decisions. This finding suggests that imperfectly competitive storage has contributed to increased price instability in the US cheese market.

9. Concluding Remarks

We have developed a model of storage behavior under competition as well as Cournot behavior. The model provides a structural representation of storage decisions under alternative market structures. This representation is used to specify and estimate a structural model of storage decisions. The analysis is applied to the US American cheese market. Several important results were obtained. First, the econometric analysis provides evidence of imperfect competition in storage activities for American cheese since 1993. In particular, we reject the null hypothesis of perfect competition. Second, the Cournot representation of imperfect competition in storage decisions is also rejected. Third, the empirical estimate suggests that the exercise of market power is somewhat moderate: the largest estimated impact of imperfect competition amounts to 6.4 percent of the market price. Fourth, simulations from the econometric model show that imperfect competition tends to reduce the ability of stocks to buffer anticipated fluctuations in supply/demand conditions. This suggests that imperfectly competitive storage contributes to increased price instability.

Our analysis has focused on a structural representation of dynamic storage decisions. This provides a convenient framework to investigate conceptually and econometrically the economics of storage. The approach has the advantage of being valid under broad conditions. For example, it applies under general supply/demand conditions, and various information scenarios. However, our structural analysis has its limitations. For example, we have not explored the strategic use of stocks in conjunction with joint production activities under imperfect competition. There is a need for further research to explore such issues. Also, our econometric analysis has focused on storage behavior in the US American cheese market. There is a need for additional research exploring the nature of storage competition in other markets.

APPENDIX

Proof of Proposition 4: Assume that the second-order conditions for the maximization problem (2) are satisfied in a Cournot game. Note that the first-order condition with respect to I_t in (4) remains the same with or without the exercise of market power. With (C_t'+ R_t') being increasing in S_t, equation (11) then implies that the optimal stock S_t is necessarily higher (lower) when $M_t' > (<) 0$. But $M_t' = 0$ for a competitive firm. This generates the desired result.

Proof of Lemma 1: Note that equation (14) implies that $W(\mathbf{S}^a(1), \mathbf{I}^a(1)) \geq W(\mathbf{S}^a(0), \mathbf{I}^a(0))$. It follows that

$$0 \leq W(\mathbf{S}^a(1), \mathbf{I}^a(1)) - W(\mathbf{S}^a(0), \mathbf{I}^a(0))$$

$$= \int_0^1 \{[\partial W(\mathbf{S}^a(\alpha), \mathbf{I}^a(\alpha))/\partial \mathbf{S}]\ (\partial \mathbf{S}^a(\alpha)/\partial\alpha) + [\partial W(\mathbf{S}^a(\alpha), \mathbf{I}^a(\alpha))/\partial \mathbf{I}]\ (\partial \mathbf{I}^a(\alpha)/\partial\alpha)\}\ d\alpha.$$

Using equation (13), we have $[\partial W(\mathbf{S}^a(\alpha), \mathbf{I}^a(\alpha))/\partial \mathbf{I}] = \mathbf{0}$ from (4). When $S_t > 0$, it follows that

$$\partial W(\mathbf{S}, \mathbf{I})/\partial S_t = E_1[U_{t+1}' \cdot (1-\delta_t)\ [p_{t+1} + p_{t+1}' \cdot ((1-\delta_t)\ S_t - S_{t+1}^{*})]/q_{t+1}$$

$$- U_t' \cdot (p_t + C_t' + p_t' \cdot ((1-\delta_{t-1})\ S_{t-1} - S_t))/q_t]$$

$$= (1-\alpha)\ M_t',$$

using equations (15), (3) and (10). Combining these expressions gives equation (16).

Table 1: GMM estimation

Parameters	Estimate	Standard error
c_1	-0.0102	0.0380
c_2	0.0409	0.0880
ϕ	-1.0972	0.4509

Number of observations = 157
R-square = 0.895
Minimum distance = 0.0984

Figure 1: Effect of market power relative to price: $L_t = -M_t'/p_t$

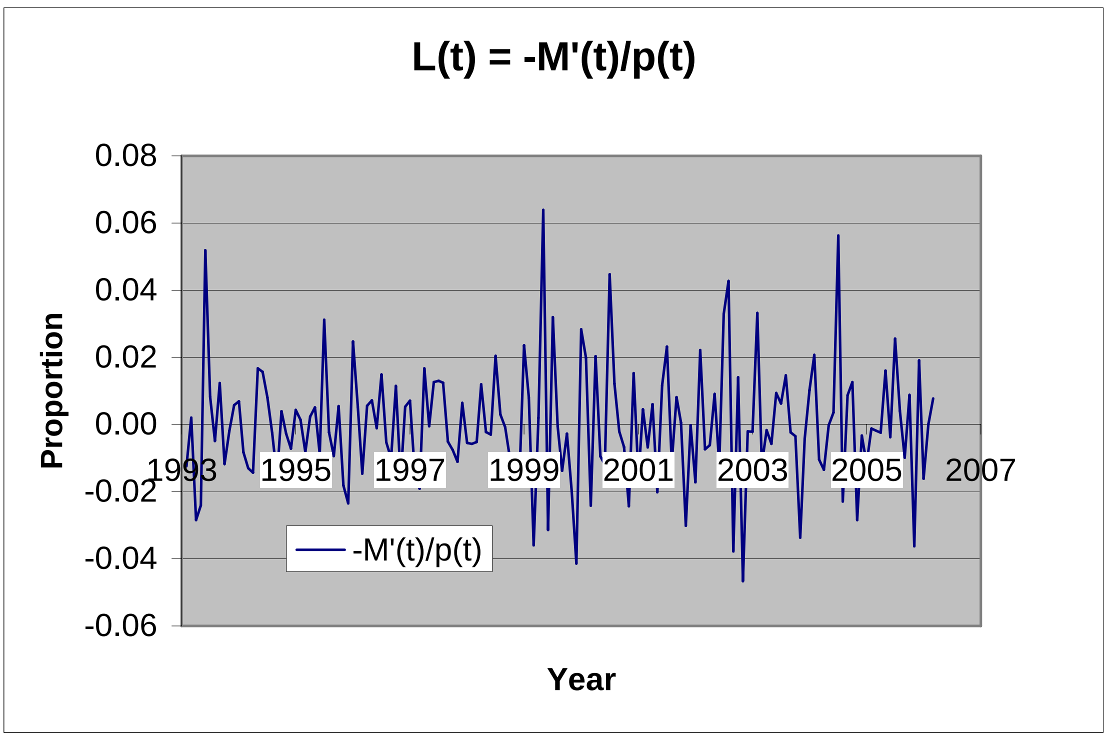
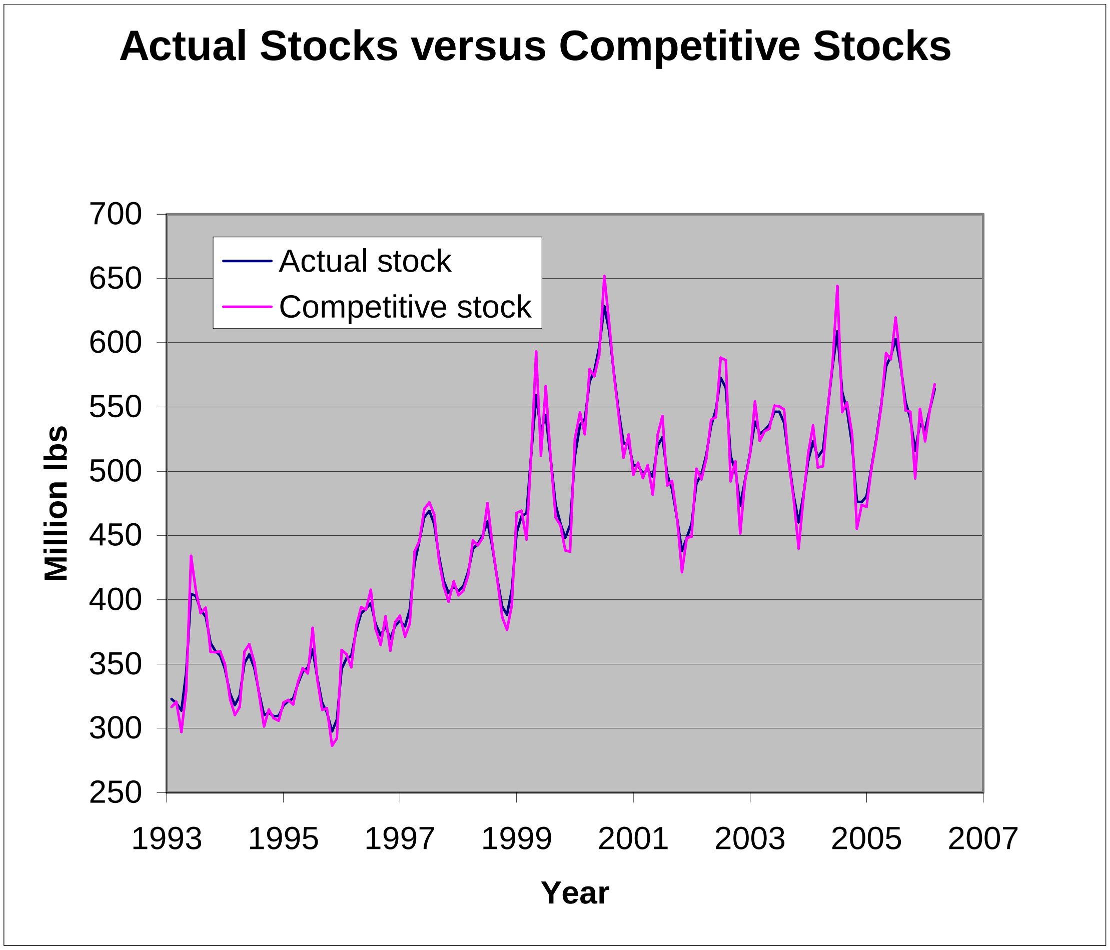


Figure 2: Simulated Effects of market power on stocks



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