Adverse Effects of Patent Pooling on Product Development and Commercialization*

by

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Abstract

The conventional wisdom is that the formation of patent pools is welfare enhancing when patents are complementary, since the pool avoids a double-marginalization problem associated with independent licensing. This conventional wisdom relies on the effects that pooling has on downstream prices. However, it does not account for the potentially significant role of the effect of pooling on innovation.

The focus of this paper is on (downstream) product development and commercialization on the basis of perfectly complementary patents. We consider development technologies that entail spillovers between rivals, and assume that final demand products are imperfect substitutes. When pool formation facilitates information sharing and either increases spillovers in development or decreases the degree of product differentiation, patent pools can adversely affect welfare by reducing the incentives towards product development and product market competition—even with perfectly complementary patents.

The analysis modifies and even negates the conventional wisdom for some settings and suggests why patent pools are uncommon in science-based industries such as biotech and pharmaceuticals that are characterized by tacit knowledge and incomplete patents.

Keywords: Patent Pools, Research and Development, Innovation, Tacit Knowledge, Biotechnology, Pharmaceuticals

JEL classifications: O3 (Technological Change; Research and Development; Intellectual Property Rights), L1 (Market Structure, Firm Strategy, and Market Performance), L2 (Firm Objectives, Organization, and Behavior), L4 (Antitrust Issues and Policies), L6 (Industry Studies: Manufacturing), K2 (Regulation and Business Law), D2 (Production and Organizations), D4 (Market Structure and Pricing)
1 Patent Pools and the Structure of Innovation

In many important industries, prominently so in electronics, computer software, telecommunications, pharmaceuticals and biotechnology, it has been suggested that innovation has been stifled by a so-called patent-thicket: “a dense web of overlapping intellectual property rights that a company must hack its way through in order to actually commercialize new technology” (Shapiro, 2001, p. 120). This has been an integral part of the debate among academics and policymakers concerning the reform of patents and patent law, with arguments ranging from the abolition of intellectual property (IP) rights altogether (e.g., Boldrin and Levine, 2009) or limiting patents in certain affected areas (e.g., Heller and Eisenberg, 1998), to cross-licensing or the deliberate bundling of related patents in so-called patent pools (e.g., Clark et al., 2000, Ménière, 2008). In this paper we consider the latter suggestion by exploring the interplay of IP-licensing and knowledge transfer in product development and commercialization.

A patent pool is an arrangement in which patent holders bundle distinct patents and then collectively license these. The first such combination in the United States was the formation of a patent pool covering patents related to sewing machines in 1856. After initially being viewed as fully protected under the doctrine of freedom of contract, in 1912 the U.S. Supreme Court ruled that patent pools were subject to antitrust scrutiny. Since then a nuanced view of patent pooling has emerged in which “blocking” (complementary) and “competitive” (substitutable) patents are distinguished.

Shapiro (2001) notes that when the patents that are included in the pool are perfect complements, a pool should be viewed benignly. The insight rests upon applying Cournot’s (1838) analysis of independent monopolies providing perfectly complementary inputs to a downstream producer, in which neither of the upstream suppliers incorporate the negative externality that their pricing decision has on the other. The implied (horizontal) double-marginalization then results in lower producer and consumer surplus. The analysis was

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1 See, e.g., E. Bement & Sons v. National Harrow Co., 186 U.S. 70 (1902).
3 Cournot illustrates his point by considering the pricing decisions of a monopolist for copper and a monopolist for zinc who are providing the necessary inputs to a downstream producer of brass.
further refined in a general model by Lerner and Tirole (2004), who also conclude that
the more complementary the patents in the pool are, the greater are the welfare benefits
associated with the formation of a pool.4 Quite in line with these theoretical findings,
contemporary antitrust recommendations and practice in Europe and the U.S. hold that
pooling complementary patents is generally not anti-competitive.5

These insights rely on an analysis of the implication of pool formation on downstream
prices. What is not addressed, however, is the degree to which pool formation may affect
subsequent innovation—which generally varies between industries and which may account
for why pools are not a universal feature across industries.

Examples of successful pools are found abundantly in consumer electronics (e.g., the
DVD6C patent pool that was formed by nine leading home entertainment companies to
facilitate technology related to digital versatile discs); or in the software industry (e.g., the
several MPEG patent pools that govern video and audio compression and transmission).
These types of pools contain patents that are “standard-essential” and are therefore deemed
to be complementary.

In contrast to the many pools in electronics and software, patent pools in pharmaceuticals
and biotechnology are actually rare, despite complementary IP and active advocacy for pool
formation.6 And where pools have been initiated these have been driven by the demand
side of the market, i.e., by downstream users, rather than by upstream IP-holders. In fact,
almost all attempted patent pools in the biotech and pharma industries have been initiated

4It should be noted that they recognize that, in the context discussed, the notions of complementarity
and substitutability are not actually as clear-cut as it might seem, but a meaningful distinction is nonetheless
possible on the basis of changes in patentees’ willingness to pay for additional patents.

5Cf. the Guideline on the Application of Art. 81 of the European Commission Treaty to Technology
Transfer Agreements (2004/C 101/02), and Chapter 3 of USDOJ/FTC (2007).

6For example, the several entities that had sequenced parts or the whole of the severe acute respiratory
syndrome associated coronavirus (SARS-CoV) proved unable to form a pool to facilitate the development
of an effective vaccine. Similarly, the development of a DNA Microarray to arrange 300 cancer-associated
genes would greatly facilitate the diagnosis and possible treatment of many cancers; yet such a DNA chip
would require the pooling of widely dispersed patents, which has not happened. In similar vein, patents on
receptors are useful for screening potential pharmaceutical products. To learn as much as possible about
the therapeutic effects and side effects of potential products at the pre-clinical stage, firms want to screen
products against all known members of relevant receptor families. But when these receptors are patented
and controlled by different owners, gathering the necessary licenses may be difficult. See, e.g., USTPO’s
white paper on the subject, Clark et al. (2000), Gaulé (2006), Ménier (2008), Verbeure (2009), van Zimmeren
et al. (2011), or van Overwalle (2012).
or sponsored by governments, NGOs, or other types of non-profits.\footnote{A notable exception is the current attempt by MPEG-LA's Librassay, a genetic diagnostic testing patent pool to support molecular diagnostics testing.}

We believe that differences in pool-formation across industries are specifically tied to differences in the development and commercialization processes. In industries such as consumer electronics and computing, obtaining IP-rights is frequently the necessary and sufficient condition to allow for product commercialization. As a result, it is often the case that securing the requisite IP-licenses is the final step before commercialization begins, after all development is complete.

In biotech and pharmaceuticals, in contrast, securing IP-rights is a necessary, but not a sufficient condition; and obtaining IP-rights is necessarily done early in the development process. With many discoveries the pioneer inventions owned by patent holders are ‘incomplete’ and need further innovation before being embodied in a marketable product. Powell, \textit{et al.} (1996) discuss how inter-organizational collaboration contributes to organizational learning and the emergence of the biotech industry. They argue that when the knowledge base of an industry is both complex and expanding and the sources of expertise are widely dispersed, the locus of innovation is found in networks of learning, rather than in individual firms. The large-scale reliance on interorganizational collaborations in the biotech industry reflects a fundamental and pervasive concern with access to knowledge.

The point is illustrated in the following example. Patent applications can be filed on newly identified DNA sequences, including gene fragments, before identifying a corresponding gene, protein, biological function, or potential commercial product. But the characterization of nucleic acid sequence information is only the first step in the utilization of genetic information. Significant and intensive research efforts are required to glean the information from the nucleic acid sequences for use in the development of pharmaceutical agents for disease treatment.

In addition to patents being incomplete, resulting in a large divide between foundations knowledge and commercial viability, many relevant discoveries are further characterized by ‘tacit knowledge.’ In particular, “[t]o the extent that the knowledge is both scarce and tacit, it constitutes intellectual human capital retained by the discovering scientist,” (Zucker \textit{et al.} 1996).
al., 2001, p. 153); so product innovation based on those scientific discoveries must start with the transformation from tacit to codified knowledge, which requires the interaction of the patent holders with the developing firms.

Due to patents covering incomplete technologies and tacit knowledge, discovering scientists are closely involved in the subsequent development of products. When this is the case, the formation of a pool not only facilitates cooperative marketing among patent holders; it also determines how knowledge is transferred from IP-holders (acting in unison) to downstream developers and retailers, and pools become information-sharing institutions. Thus, “a patent pool leads to the institutionalized exchange of patented knowledge, as well as technical information not covered by patents through a mechanism for sharing technical information relating to the patented technology, which would otherwise be kept secret. This is reflected by an exchange of know-how brought along by the set-up of a patent pool ...”8 Such information sharing may affect both the downstream development process and the resulting product market competition.

Our aim is to examine how development and commercialization may be impacted by pool formation; and how this helps explain why pools are infrequent in the bio-technology and pharmaceutical industries. There are two principal channels through which information-sharing within the pool affects development and final product competition.

The first is spillover effects among competitors in the process of developing a marketable product. Spillover effects have been recognized to affect R&D efforts within industries for several reasons. Reverse engineering and strategic hiring of rivals’ personnel are common occurrences, but it has also been noted that employees within a given industry often share information concerning the development and design of products across rival firms (see, e.g., Severinov, 2001). When patent-holders who are involved in further development of products form a pool, then the pool leads to direct communication between the individual patent holders and thus increases spillovers in the development.

The second impact affects horizontal product differentiation in the final product market. The more closely aligned are the research paths that are pursued, the smaller is the degree

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8See Van Overwalle (2012), in refer to Verbeure (2009). The aforementioned USPTO white paper on patent pools in biotechnology (see footnote 6) also cites information sharing specifically as an advantage of pool formation (id. p. 10).
of horizontal product differentiation that result from research efforts that are undertaken to develop and commercialize final products. Hence, pooling leads to less-differentiated products, due to the congruence inherent in research trajectories that are closely interrelated.

Both of these effects impact firms’ anticipated market profits and, hence, their incentives to apply effort at the development stage. As a result, patent holders may prefer to remain independent, despite the otherwise recognized advantages of pool-formation. It is even possible that overall welfare is reduced with pooling, calling into question the unqualified policy recommendations made concerning pool formation of complementary patents.

There is a growing literature on the affects of patent pooling, although the explicit consideration of downstream development and commercialization has largely not been addressed. Thus, Kim (2004) finds that vertical integration can alleviate double-marginalization problems, but Schmidt (2009) shows that this can also lead to incentives to raise rivals’ costs. Brenner (2009), building upon Lerner and Tirole’s (2004) framework, looks at rules that govern the formation of pools to discern how welfare-enhancing pools can be made stable and welfare-reducing pools can be destabilized. Gallini (2011) considers additional potential anti-competitive concerns tied to pooling, and Choi’s (2010) focus is altogether different, as he considers the welfare implications of patent uncertainty and possible litigation as reasons for pool formation.

There is also an emerging literature examining the relationship between patent-pooling and R&D. Thus, Gilbert and Katz (2011) study how reward schemes affect R&D incentives leading up to the development of perfectly complementary inputs. Dequiedt and Versaevel (2012) show that the anticipation of the formation of a pool of essential patents increases R&D activities prior to pool formation and slows these afterwards, and Llanes and Trento (2012) show in the context of sequential innovation that pools can become unstable over time. Taking a look at R&D after the formation of a pool, Lampe and Moser (2010, 2012) and Joshi and Nerkar (2011) find empirical evidence to suggest that pool formation reduces further patenting activities. It has been suggested that this is tied to a reduction in competition or free-riding among pool members. This is similar to a result in our model, although our model is tied to downstream development and commercialization of final products given the existing IP, rather than upstream IP generation, or the subsequent development of derivative
IP, *viz.*, the further development of additional patents.

In order to formalize the effect of patent pooling on subsequent development and commercialization, we depart from the previous literature on patent pooling and expressly consider a product development stage, in which development efforts entail spillovers across firms and the degree of product differentiation in the final demand market is also determined by information sharing during the development process.

The importance of the innovation structure on product development and downstream competition has been studied in the context of research joint ventures (RJVs). Since the seminal papers by Katz (1986) and d’Aspremont and Jacquemin (1988) implications of spillovers in product development have been studied extensively.\(^9\) However, the focus is generally on cooperation between rivals, designed to internalize spillovers, avoid cost-duplications and generally coordinate development efforts. This is in contrast to the effect of pooling on development with spillovers. The decision to pool is made by IP holders, rather than the developing firms; and the existence of a pool does not induce any cooperation or coordination among the competing downstream developing firms.

An exception to the majority of the literature on coordination and spillovers in RJVs is the notion of research sharing joint ventures (RSJVs) in which firms agree to share the results of their research, but do not coordinate their efforts (Greenlee, 2005). Kamien *et al.* (1992) consider an extreme version of this where industry-wide joint ventures yield complete spillovers. Going beyond this, Greenlee (2005) considers endogenous joint venture sizes and allows the degree of spillovers to vary. Citing many such joint ventures over the past decades, Greenlee suggests that RSJVs (rather than cooperative RJVs) are actually the prevalent form of joint ventures in the development of products.\(^10\) From a modeling standpoint, Greenlee’s variations in the degree of spillovers is akin to our notion of spillovers tied to patent pool formations, suggesting that the analysis of our model may—to a degree—carry over to RSJVs. Nevertheless, as mentioned above, an important distinction between our model of patent pooling and RSJVs is that the pooling decision does not lie in the hands


\(^10\)See also Erkal and Minehard (2010), who present a dynamic model of research exchange among rivals and consider the endogenous timing of information sharing.
of the firms that undertake the commercialization and then compete in the product market, but rather, it depends on the incentives and interests of the upstream patent holders.\footnote{Of course, if the pooling incentives of patent holders and firms are perfectly aligned \textit{(e.g., if the licensors are also licensees, \textit{i.e., the case of cross-licensing)} then our model can also be interpreted as a version RSJVs similar to Greenlee (2005), and our insights carry over to such a setting.}

The effect of the degree of product differentiation on development efforts has also been examined elsewhere, with some models specifically examining endogenous product differentiation. A precursor to this literature is Choi (1993) who examines the private and social incentives of research collaboration in anticipation of its effect on product market profits. However, he considers generic profits, rather than derived profits in a closed form model. Similarly, Amir \textit{et al.} (2003) also use generic profit functions and consider differences between cooperative and non-cooperative R&D. As for the interplay of effort and spillovers in development, Moltó \textit{et al.} (2005) have a closed-form model with a result that is similar to one of ours (albeit in a very different set-up) in that the social planner may wish to limit the extent of spillovers in development, as these lead to under-performance due to free-riding. Bourreau and Doğan (2010) allow for cost sharing in development and study how increased collaboration in development leads to diminished product differentiation. However, effort is not part of the development process. Ghosh and Morita (2008) also study possible trade-offs concerning development collaboration and product differentiation, using a circular city model with a focus on how insiders differ from outsiders.\footnote{In contrast to these approaches that postulate a positive relationship between collaboration and product similarity, Lin and Saggi (2002) consider the case where firms coordinate to increase product differentiation.}

\section{The Model and the Downstream Equilibrium}

Our model of product development and commercialization consists of three stages. Prior to the first stage two fundamental discoveries have been made that resulted in two patents being awarded to two distinct patent holders. In Stage I of the model these patent holders decide whether they form a patent pool or remain independent. In Stage II downstream firms acquire access to the relevant IP and undertake costly efforts to each develop a differentiated product. Finally, in Stage III, the developing firms engage in Bertrand price competition against each other in the downstream market.
In this section the three stages of the model are first characterized in greater detail and the equilibrium actions of the downstream firms are derived. Throughout we refer to the upstream providers of the perfectly complementary IP inputs as ‘patent holders,’ and the downstream developers and competitors of imperfectly substitutable goods are the ‘firms.’13 To avoid potential confusion, it bears repeating and emphasizing that the patents involved are perfect complements (in production); whereas the final goods produced are imperfect substitutes (in consumption).

2.1 The Basic Framework

Stage I: Pool Formation Stage I begins after foundation research has already been completed and two patents have been awarded to two distinct patent holders, \( k \) and \( l \). The two patents are both deemed essential in the further development and commercialization of a final product. That is, the patents constitute perfectly complementary inputs. Patent holders can either license their patents independently to downstream developing/retailing firms, or they can form a pool and license both patents jointly.

There are two possible types of licensing contracts between patent holders and the developing/retailing firms that we consider. The first are per-unit-of-output royalty rates, denoted by \( R \). This is the prevalent type of contract found in pools (Serafino, 2007; Gilbert, 2010), because it avoids the need to estimate the commercial viability of the developed products and preserves the IP-holders interest in the subsequent commercial success. Moreover, compared to a flat upfront licensing fee, the per unit royalty rate reduces downstream output and increases prices and profits. Absent a pool, each patent holder independently (non-cooperatively) sets a royalty rate for each of the developing/retailing firms, whereas a uniform royalty rate for the downstream firms is agreed upon between the patent holders when they have formed a pool.

As the double marginalization caused by independently set royalty payments provides a central rationale for pool formation of perfectly complementary inputs, we also consider

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13Thus, we assume that the upstream IP-holders are non-practicing entities. In biotech and pharmaceuticals many IP-holders are indeed small research laboratories or universities that do not themselves commercialize. Nevertheless, the findings of the model carry over to more complex settings in which patent holders are also developers and manufacturers.
non-distortionary licensing arrangements for comparison purposes. Thus, the second form of contract is an upfront fixed fee $F$ that firms pay to access the patent rights. Because the fee constitutes a fixed cost for the firms, it does not distort downstream actions. In particular, it does not affect the firms’ marginal costs of production in Stage III and, because the firm is the residual claimant of all market profit, it also does not distort efforts applied in product development in Stage II.

Since our focus is on the welfare implications of pool formation in light of its effects on development and product market competition, we preclude the possibility of strategic foreclosure (e.g., the deliberate creation of monopoly in the final-demand market by excluding all but one downstream firm from access to the patents). Indeed, foreclosure would be the subject of independent antitrust concerns, and in both European and U.S. jurisprudence patent pools are subject to non-discrimination rules.

When patents cover incomplete technology or tacit knowledge is relevant for the subsequent development process, then the pooling decision also affects the joint transfer of knowledge tied to the patents. Spillovers between the downstream firms in the development stage (Stage II) are augmented by the pool, as the pool is a conduit for knowledge transfer. And, because research trajectories become more similar, the products that are sold in the final demand market (Stage III) are more similar to one another. That is, the degree of horizontal product differentiation is diminished and product homogeneity increases.

**Stages II and III: Product Development and Commercialization** Much of the literature on IP-licensing and patent pools assumes either a monopoly or a perfectly competitive downstream market. Both of these polar extremes obscure important aspects of downstream activities. The former fails to recognize important trade-offs that exist in the degree of product differentiation among rival downstream firms, while the latter fails to appreciate how downstream market interactions affect development efforts of the firms. We capture both of these important aspects by assuming that there are two imperfectly competitive downstream firms $i$ and $j$ poised to develop and market a product based on the two patents.

Following the modeling framework of Singh and Vives (1984), the firms engage in (non-cooperative) differentiated goods price competition in the final demand market in Stage III.
Inverse demand for each firm’s product is linear and is given by:

\[ P_i = A_i - Q_i - \gamma Q_j, \quad i, j = 1, 2; \quad i \neq j, \quad (1) \]

where \( \gamma \in \{\gamma_p, \gamma_n\} \) is the degree of product differentiation, with \( 1 > \gamma_p \geq \gamma_n > 0 \) where \( p \) denotes the case that a pool has been formed, and \( n \) that patent holders operate independently.

Departing from Singh and Vives (1984), \( A_i \) is the base demand, or market size, for firm \( i \)'s product. Its size depends on efforts \( e \) expended in development (Stage II) prior to product market competition. In principle this effort may be jointly applied by the patent-holder/scientist and the researchers/developers in the firm. However, since the outside patent holders in such instances are generally fully compensated for their efforts, the cost of effort only enters the firms’ objective function.\(^{14}\) In particular,

\[ A_i = a + e_i + \beta e_j, \quad i, j = 1, 2; \quad i \neq j, \quad (2) \]

where \( \beta \in \{\beta_p, \beta_n\} \), with \( 1 \geq \beta_p \geq \beta_n \geq 0 \), denotes the degree of spillovers in development, measuring how much of firm \( j \)'s effort is captured and appropriated by firm \( i \) augmenting firm \( i \)'s base demand.

Firms face a quadratic cost of effort in development and for simplicity we assume that the only production costs are associated with acquiring IP. Thus, the marginal cost is given by any royalty rates the firms pay, \( R \), and any upfront license fees, \( F \), constitute the firms’ (sole) fixed costs.

The sequence of events characterizing the structure of innovation and competition is the basis for Figure 1.

2.2 The Continuation Equilibrium

We seek the subgame perfect Nash equilibrium and solve the model through backward induction. We first consider the product market competition for a generic degree of product homogeneity \( \gamma \in \{\gamma_n, \gamma_p\} \), arbitrary demand intercepts, \( A_i \) and \( A_j \), and an arbitrary licensing (royalty/fee) structure. Thereafter we analyze the optimal development efforts for

Figure 1: The Structure of Innovation and Competition

generic spillovers \( \beta \in \{ \beta_n, \beta_p \} \). The analysis is conducted from firm \( i \)'s point of view, which is without loss of generality as firms are symmetric.

The firms’ inverse demand functions, given in (1), are solved for the firms’ demands as functions of the strategic variables, namely the prices \( P_i \) and \( P_j \):

\[
Q_i = \frac{(A_i - P_i) - \gamma(A_j - P_j)}{1 - \gamma^2}.
\]

While all production costs apart from licensing expenses are normalized to zero, firms may face (per unit) royalty rates \( R \). Moreover, for the case of fixed fees, firms make an upfront payment to patent holders of \( F \). Letting \( 1 \in \{0, 1\} \) be an indicator denoting the type of the licensing arrangement, with 1 designating the case of royalties and 0 the case of fixed fees, firm \( i \)'s objective is to choose a price to maximize

\[
\pi_i = (P_i - 11R)Q_i - (1 - 11)F = (P_i - 11R)\frac{(A_i - P_i) - \gamma(A_j - P_j)}{1 - \gamma^2} - (1 - 11)F.
\]

Detailed derivations of the model are found in Appendix A, where it is shown that the Bertrand-Nash equilibrium of this game yields,

\[
Q_i^* = \frac{(A_i - P_i^*) - \gamma(A_j - P_j^*)}{(1 - \gamma)(1 + \gamma)} = \frac{(2 - \gamma^2)A_i - \gamma A_j}{(2 - \gamma)(1 + \gamma)} - 11R.
\]
with
\[ \pi^*_i(A_i, A_j) = \frac{(1 - \gamma) \left( \frac{(2 - \gamma^2) A_i - \gamma A_j}{2 - \gamma^2 - \gamma} - 1 R \right)^2}{(2 - \gamma)^2 (1 + \gamma)} - (1 - 1) F. \] (6)

Equation (6) gives equilibrium market profits as a function of the demand intercepts \( A_i \) and \( A_j \). In accordance with (2), these depend on the firms’ effort levels, \( \text{viz} \). \( A_i = a + e_i + \beta e_j \).

Thus, given quadratic effort costs of \( e_i^2 \), the firm’s objective is given by
\[ \max_{\{e_i\}} \Pi_i(e_i, e_j) = \frac{(1 - \gamma) \left( a - 1 R + \frac{(2 - \gamma^2 - \gamma \beta) e_j + (2 \beta - \gamma^2 - \gamma \beta) e_j}{2 - \gamma^2 - \gamma} \right)^2}{(2 - \gamma)^2 (1 + \gamma)} - (1 - 1) F - e_i^2. \] (7)

The first-order condition\(^{15}\) yields a best response function of
\[ e^*_i(e_j) = \left( a - 1 R + \frac{(2 \beta - \gamma^2 - \gamma \beta) e_j}{2 - \gamma^2 - \gamma} \right) \frac{(2 - \gamma^2 - \gamma \beta)(2 - \gamma^2 - \gamma)}{(2 - \gamma)^2 (1 - \gamma^2)(2 + \gamma)^2 - (2 - \gamma^2 - \gamma \beta)^2}. \] (8)

Given symmetry, the equilibrium effort choices are
\[ e^* = \frac{2 - \gamma^2 - \gamma \beta}{(2 - \gamma)^2 (1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma \beta)}. \] (9)

Thus far the firms’ equilibrium behaviors for the general set-up of the development process and the downstream market competition. We now consider the implications of patent pooling for this general setting, proceeding first with the conventional analysis that abstracts from any possible effects that pooling may have on the subsequent development and commercialization. This is followed by a discussion of the impact of marginal changes in spillovers or product differentiation on welfare independent of the pooling structure. On the basis of this we then examine the welfare implication and potential pitfalls of patent pooling in Section 5, where we differentiate between license fees and royalties.

3 Benchmark Analysis

Given the equilibrium effort and pricing decisions of the firms, we now consider the patent holders’ incentives concerning the formation of a pool and analyze how welfare is affected by the pooling structure.

\(^{15}\)The first order conditions are sufficient and yield an interior solution (\textit{i.e.}, positive equilibrium effort) provided that \( \gamma \lesssim 0.9325 \) — an assumption that we henceforth maintain.
While we extend the existing literature on patent pooling by explicitly modeling the costly development of differentiated products in an imperfectly competitive market, in our benchmark analysis we remain in line with the received literature by initially supposing that the formation of a pool has no effect on the parameters governing the interaction between the downstream firms. That is, we assume that possible spillovers in the development process are unaffected by the pooling decision so that $\beta_n = \beta_p = \beta$; and pooling also does not affect the degree of horizontal product differentiation so that $\gamma_n = \gamma_p = \gamma$.

### 3.1 Licensing

Let $R$ denote the per-unit-of-output royalty rate that firms are charged. The patent holder’s objective is to maximize the revenue obtained from the firms. We first consider the case of a pool in which the patent holders jointly set a royalty rate $R_p$ for the firms.

The patent holders’ objective is to choose a royalty rate $R_p$ that maximizes $V := R_p \times 2Q^*$, while recognizing that the firm’s equilibrium output is a function of the royalty rate, i.e., $Q^* = Q^*(R_p)$. The firm’s output is given in (5) and the equilibrium value is derived in Appendix A and given by (45), from whence it follows that the patent holders’ objective is

$$\max_{\{R_p\}} R_p^2(a - R_p) \frac{4 - \gamma^2}{(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)},$$

with solution

$$R_p^* = \frac{a}{2}.$$  \hspace{1cm} (11)

Absent a pool, both patent holders independently choose a royalty rate ($r_k$ and $r_l$, respectively) that they will charge to each firm for each unit of output sold. Hence, patent holder $k$’s objective is for given $r_l$ to choose a royalty rate $r_k$ that maximizes the revenues obtained from the two downstream firms, $v_k := r_k \left( Q^*_i(r_k, r_l) + Q^*_j(r_k, r_l) \right)$.

Since the patents are essential (i.e., perfectly complementary) each of the downstream firms contract with and pay royalties to both patent holders so that their unit costs are given by $R_n := r_k + r_l$.

Using (45) once again, the patent holder’s objective is

$$\max_{\{r_k\}} r_k^2(a - r_k - r_l) \frac{4 - \gamma^2}{(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)},$$

with solution

$$R_n^* = \frac{a}{2}.$$  \hspace{1cm} (12)
yielding the best response function of \( r^*_k(r_l) = \frac{a-r_l}{2} \). The symmetric equilibrium royalty rates are, thus, \( r^* = \frac{a}{3} \) so that

\[
R^*_n = 2r^* = \frac{2}{3}a. \tag{13}
\]

Consider now the case of upfront fixed fees. Because our primary focus is on overall welfare implications of patent pooling, we abstract from explicitly modeling how upfront fees are set and we have therefore treated them simply as a fixed cost from the firms’ perspectives. An implication of this is that the fee itself is welfare-neutral as it is merely a transfer from firms to patent holders.

Noting that a firm’s willingness to pay for access to the required IP is increasing in the profits that it obtains by using the IP, we assume that the patent holder’s ability to extract rents is also increasing in the firms’ market profit. Therefore it is in the interest of the patent holders to base their pooling decision on whichever format (pool or no-pool) generates the greater profit for the firms they are dealing with.

### 3.2 Welfare

We now derive welfare in the benchmark and revisit the conventional wisdom concerning the pooling of perfectly complementary patents for the case in which subsequent product development is explicitly modeled and the resulting products are differentiated. We let \( x \) denote the given pooling structure (pool or no pool) with \( x \in \{p,n\} \), but continue to maintain for this section that \( \gamma_p = \gamma_n (= \gamma_x) \) and \( \beta_p = \beta_n (= \beta_x) \). Moreover, we remind the reader that \( 1 \in \{0,1\} \) is an indicator with \( 1 = 1 \) indexing royalties and \( 1 = 0 \) fixed fees. With this indexation, market profit \( \Pi_x \) and consumer surplus \( CS_x \), which are derived in Appendix A as a result of equilibrium effort given in (9) are

\[
\Pi_x = (a - 1R_x)^2 \left\{ \frac{(2-\gamma_x)^2(1-\gamma_x^2)(2+\gamma_x)^2 - (2-\gamma_x^2-\gamma_x\beta_x)^2}{[(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)]^2} - (1 - 1)F_x \right\}. \tag{14}
\]

and

\[
CS_x = (a - 1R_x)^2 \left\{ \frac{(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x)^2}{[(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)]^2} \right\}. \tag{15}
\]

Note that for the case of royalty payments the patent holder payoffs are given by (10) for the case of a pool, whereas for the case without a pool they are given by twice (12). With fixed
fees, total patent holders’ payoffs are generically $2F_x$ for a given pooling structure. Thus, patent holders’ total payoffs are

$$V_x = 2R_x(a - R_x) \frac{(2 - \gamma_x)(2 + \gamma_x)}{(2 - \gamma_x)(1 + \gamma_x)(2 + \gamma_x) - (1 + \beta_x)(2 - \gamma_x^2 - \gamma_x \beta_x)} + 2(1 - 1)F_x.$$

Because we do not establish an algebraic value of $F_x$, profit and patent holders’ payoffs are not further detailed for the case of upfront fees. However, since $F_x$ is merely a transfer payment between firms and patent holders, the magnitude of $F_x$ has no total welfare implication, so total welfare is given by

$$TW_x := CS_x + 2\Pi_x + V_x$$

$$= (a - 1R_x)^2 \frac{3 - 2\gamma_x(2 - \gamma_x)(2 + \gamma_x)^2(1 + \gamma_x)(2 + \gamma_x) - 2(2 - \gamma_x^2 - \gamma_x \beta_x)^2}{[(2 - \gamma_x^2)(1 + \gamma_x)(2 + \gamma_x) - (1 + \beta_x)(2 - \gamma_x^2 - \gamma_x \beta_x)]^2} + (a - 1R_x)2\Pi_x \frac{(2 - \gamma_x)(2 + \gamma_x)}{(2 - \gamma_x)^2(1 + \gamma_x)(2 + \gamma_x) - (1 + \beta_x)(2 - \gamma_x^2 - \gamma_x \beta_x)}.$$

We can now reiterate the conventional wisdom concerning the pooling of perfectly complementary patents for the case in which subsequent product development is explicitly modeled and the resulting products are differentiated.

**Theorem 1 (Generalized Conventional Wisdom)** Pooling increases all measures of welfare when there are royalty contracts, even when products are differentiated and there are spillovers in development;

$$W_p > W_n, \quad \forall \gamma, \beta \text{ and } W \in \{CS, \Pi, V, TW\} \text{ and } 1 = 1 \text{ (i.e., royalties).}$$

Thus, regardless of the size of spillovers in subsequent product development and independent of the degree of the resulting product differentiation, consumer surplus, market profit, licensing revenue, and hence total welfare are all strictly greater with pooling when there are royalties. The insight follows readily: royalty revenue, given by (16), is maximized at $R_p^*$, so patent holder payoffs are lower when there is no pool compared to the case where there is a pool. Notice from Equations (14) and (15) that firm profit and consumer surplus are decreasing in $R$. Hence, since $R_n^* = \frac{2}{3}a > \frac{1}{2}a = R_p^*$, firm profit and consumer surplus, in addition to patent holder payoffs, are all higher under pool-formation when compared to the absence of a pool. This is the double-marginalization, or ‘royalty stacking,’ argument.
that reflects the negative externalities that are not accounted for when royalty rates are set independently across perfectly complementary inputs.

For the case of fees, in contrast to Theorem 1, note that since we have assumed that pooling does not affect either the level of spillovers, or the degree of product differentiation, it is clear that the pooling decision also does not affect equilibrium effort levels of the firms, and ultimately also has no effect on profits or consumer welfare. We conclude for the case of license fees that the formation of a patent pool has no overall welfare effects.

\textbf{Remark 1} Absent any distortions total welfare is greater with fees than under royalties. However, this does not imply that patent holders would generally prefer the non-distortionary fee arrangement over royalties. Indeed, if products are sufficiently homogenous so that downstream firms have little market power, then royalties serve as a means for reducing output and extracting consumer surplus. In contrast, if products are more differentiated, then firms’ market power generates enough profit that can be extracted by means of a fee whereas royalties would introduce a vertical double marginalization.

The more general point here is that pool formation of complementary IP in itself should not raise antitrust concerns, even when one considers more general frameworks of competition with differentiated products that first require further development. Indeed, for the case of royalties pool formation is strictly preferred over independent licensing. Furthermore, if transactions costs of contractual agreements between licensees and licensors are lower in the pool structure (an argument that is sometimes made, but goes beyond our stylized model), then pooling is also strictly preferred to a situation without a pool in the case of upfront fixed fees.

We now consider how marginal changes in spillovers and the degree of product differentiation impact the analysis.

\section{Spillover- and Differentiation-Effects}

To lay the groundwork for a discussion of how the interactions between pooling, development efforts, and product differentiation play out, this section deals with how marginal changes in spillovers and product differentiation affect payoffs assuming a given pooling structure.
Where the academic literature on patent pools addresses efficiency, total welfare is generally used as the standard for assessing the best structure for licensing patents. In the benchmark case in Theorem 1 any further differentiation between welfare measures leads to the same insights as an exclusive focus on total welfare, so any separate evaluation of payoffs to producers or patent holders or consumers does not lead to any additional insight regarding the desirability of pooling. However, in the presence of spillover and differentiation effects this is no longer necessarily the case and it needs to be determined when disparate measures of welfare are in congruence and when they are in conflict when it comes to evaluating the formation of patent pools.

A direct consideration of patent holder payoffs indicates when the formation of pools might be initiated by patent holders. Industry profit is relevant in this context as this will indicate in which circumstances the industry would lobby for or against policies that facilitate the formation of pools. Consumer surplus is also pertinent for our analysis, since, in contrast with much academic literature, antitrust practice often views consumer welfare as the guiding criterion that is to be considered when evaluating a given policy.\textsuperscript{16}

\subsection{Spillover Effects}

We first consider the impact of changes in the amount of spillovers in development. Specifically, assuming a given licensing contract (either royalties or fees), we determine the marginal payoff implications of changes in spillovers for arbitrary constellations of initial spillovers and fixed levels of product differentiation.

\textit{Ceteris paribus}, increasing the spillover effect increases welfare by generating a greater demand base $A$. Hence, all else equal, patent holders view increased spillovers favorably. However, \textit{ceteris non paribus}: When considering the impact that spillovers in the development process have on optimal effort choices, the degree of product differentiation plays a critical role.

\textbf{Lemma 1} \textit{Equilibrium effort at the development stage is increasing in the amount of spillovers if products are strongly differentiated, but decreasing if products are similar. Specifically, there}

\textsuperscript{16}See, \textit{e.g.}, Farrell and Katz (2006), or the contrasting positions in Heyer (2006) and Pittman (2007); but also Lyons (2002).
exists a function \( S_e \) such that

\[
\frac{d e^*}{d \beta} \geq 0 \iff \gamma \leq S_e. \tag{19}
\]

The intuition here is straightforward. Strong spillovers bestow a positive externality on a firm’s rival. If the rival is a close competitor, i.e., \( \gamma > S_e \), then firms recognize a larger negative impact on their profits from the rival’s strength, that is, \( \pi_i^* \), given in (6), is decreasing in \( \gamma A_j \). As a result, for \( \gamma \) sufficiently large, firms reduce the amount of effort applied to the development process if spillovers increase.

This negative effect can be sufficiently strong so that increases in the spillovers in development actually have negative effects on equilibrium base demands as measured by the demand intercept \( A^* \).

**Lemma 2** When products are sufficiently homogenous, increased spillovers reduce the market size. Specifically, there exists a function \( S_A \) with \( S_A > S_e \) such that

\[
\frac{d A^*}{d \beta} \geq 0 \iff \gamma \leq S_A. \tag{20}
\]

The two critical thresholds for the degree of product differentiation are depicted in Figure 2, with positive relations between the variables and changes in spillovers occurring to the left of the lines.

![Figure 2: Impact of the Spillover Effect on Effort and Market Size](image)

For the case that there is a license fee arrangement in place, we postulate that the incentives of the patent holders are aligned with those of the firms. As a result of Lemma 2, patent holders therefore prefer increased spillovers, for a given licensing contract, only if the degree of product homogeneity is sufficiently small.
Proposition 1  Unless inherent spillovers are very large and products are close substitutes, increased spillovers are beneficial for fee-charging patent holders and firms. That is, there exists a function \( S_{V,F,\Pi} \) such that

\[
\frac{dV^*_F}{d\beta} \cdot \frac{d\Pi^*_F}{d\beta} \geq 0 \quad \iff \quad \beta \lesssim S_{V,F,\Pi},
\]

with \( S_{V,F,\Pi} > 0.85 \).

Figure 3 illustrates the combinations of product differentiation and inherent spillovers referenced in the proposition, with increased spillovers being beneficial below the line identified as \( S_{V,F,\Pi} \).

If instead of a fee arrangement there are per-unit royalty payments then this divorces the patent holders’ incentives from those of the firms. In particular, while the firms’ objectives are profits, the patent holders’ interests are tied to the level of output.

In this case the constellations for which increased spillovers are beneficial to the patent holders is smaller—that is, firms benefit more on the margin from increased spillovers compared to patent holders under royalties. This is due to the fact that firms benefit from the reduced effort costs associated with lower efforts when goods are more substitutable (Lemma 1); whereas the patent holders benefit from increased output associated with increases in the base market size; that is, \( Q \), given in (5), is increasing in \( A \).

The same intuition for preferring increased spillovers also applies to consumers. In fact, when a given royalty contract is in place, patent holders’ interests in terms of increased spillovers are perfectly aligned with those of consumers and are characterized by Lemma 2.
Proposition 2 Unless products are close substitutes an increase in spillovers is beneficial for a given royalty rate for patent holders and also for consumers. In particular, there exists a function $S_{V_r,C_S}$, with $S_{V_r,C_S} = S_A$ such that

$$\frac{dV^*_r}{d\beta}, \frac{dC^*_S}{d\beta} \geq 0 \quad \iff \quad \gamma \leq S_{V_r,C_S}. \tag{22}$$

The critical dividing line is again depicted in Figure 3, with increased spillovers being beneficial below and to the left of the line $S_{V_r,C_S}$.

Given the discrepancy between firms’ and consumers’ interests, with the patent holders’ interests aligning with those of the former for the case of fees and with the latter for the case of royalties, it is instructive which interests weigh more when looking at total welfare. As one would expect, the effect of changed spillovers on total welfare lies (necessarily) between those of firms and consumers, being closer to consumers in the case of royalties.

Proposition 3 Unless products are close substitutes, the spillover effect makes pooling more attractive from a total welfare perspective. That is, there exists functions $S_{T W_r}$ and $S_{T W_F}$ with $S_{V_r,C_S} < S_{T W_r} < S_{T W_F} < S_{V_F,\Pi}$, such that

$$\frac{dT W^*_x}{d\beta} \geq 0 \quad \iff \quad \gamma \leq S_{T W_x}, \quad x \in \{F, R\}. \tag{23}$$

The overall conclusion from this discussion is that in isolation, that is, absent differentiation effects and for a given licensing contract, spillover effects tend to be beneficial provided that products are sufficiently differentiated.

4.2 Differentiation Effects

We now consider the impact of marginal decreases in product differentiation for given licensing contracts and given degrees of spillovers in development. Again, a critical feature in understanding distinct welfare effects of changes in product differentiation is to understand firms’ incentives to provide effort at the development stage.

In contrast to changes in spillovers, the effect of marginal changes in the degree of product differentiation on equilibrium development effort is unambiguous, and therefore also results in an unambiguous effect on the products’ base market size $A$. 

20
Lemma 3  Equilibrium effort, and hence equilibrium base market size, is decreasing in the degree of product homogeneity, i.e.,
\[
\frac{de^*}{d\gamma} < 0 \implies \frac{dA^*}{d\gamma} < 0, \quad \forall \beta, \gamma.
\] (24)

The intuition is straightforward. As \(\gamma\) increases, products become more similar and product market competition becomes more fierce, which decreases the returns on investment in development efforts. This, in turn, reduces the firm’s market size, which adversely affects the firm’s profit and, for the case of licensing fees, also directly affects the patent holders’ interests. Formally,

Proposition 4  Increases in the degree of product homogeneity adversely affect fee-charging patent-holders’ and firms’ interests. That is,
\[
\frac{dV^*_F}{d\gamma}, \frac{d\Pi^*_F}{d\gamma} \leq 0, \quad \forall \beta, \gamma.
\] (25)

As discussed, Proposition 4 reflects that increases in \(\gamma\) translate into fiercer product market competition. However, while firms and fee-charging patent holders eschew fiercer competition, if this translates into increased output, then per-unit royalty-charging patent holders may actually benefit from decreases in product differentiation. Similarly, consumers also might benefit from increased competition. Indeed, this may, but need not be the case. Figure 4 depicts the regions in which decreased differentiation is beneficial, which is formalized in the following proposition.

Proposition 5  When spillovers are sufficiently large, a decrease in differentiation is undesirable from both the royalty-charging patent holders’ and the consumers’ viewpoints. Specifically, there exist functions \(D_{VR}\) and \(D_{CS}\) with \(D_{VR} < D_{CS}\) such that
\[
\frac{dV^*_R}{d\gamma} \geq 0 \iff \beta \geq D_{VR},
\] (26)
\[
\frac{dCS^*}{d\gamma} \geq 0 \iff \beta \geq D_{CS}.
\] (27)

The reason that royalty-charging patent holders and consumers may not find the differentiation effect desirable is because of the equilibrium incentives to exert effort in the development process. Due to Lemma 3, if spillovers in the development process are large.
then the adverse effect of diminished effort results in a reduction in equilibrium output $Q^*$, which negatively impacts consumers’ and patent holders’ interests. Otherwise, if spillovers are sufficiently small (provided $\gamma$ is not too small), royalty-charging patent holders and consumers benefit from the differentiation effect.

This raises the question of what the overall welfare implications of the differentiation effect is, which, it turns out, is unambiguous for the case of fees, but depends on product differentiation not being too large and spillovers not being too small for the case of royalties.

**Proposition 6** A decrease in the degree of differentiation decreases total welfare unambiguously under fees and does so for royalties if spillovers are sufficiently small whenever goods are fairly homogenous to begin with. Thus, there exists $D_{TW,R} < D_{V_R}$ with

$$\frac{dTW^*_x}{d\gamma} \begin{cases} < 0 & \forall \beta, x = F, \\ \leq 0 & \iff \beta \geq D_{TW,R}, x = R. \end{cases}$$

Thus, despite the fact that consumers may benefit from the increased competition brought about by reduced differentiation, this is more than offset by reductions in profits. That is, once one accounts for the effort incentives in development, total welfare is unambiguously increasing in product differentiation for the case of fees and also so for the case of royalties provided intrinsic differentiation is not too large and spillovers not too small.

We now turn to how spillover and differentiation effects affect the incentives to form patent pools and determine what the implications of patent pooling are for welfare.
5 Welfare Effects of Patent Pools

Having derived the marginal impact of spillover and differentiation effects for a given contract structure, we now evaluate the overall incentives to pool and derive the welfare implications of patent pooling. We first consider the case of upfront licensing fees, since for this case some insights can directly be gleaned from the analysis of the previous section. In contrast, when it comes to pool formation with (per-unit) royalties, the avoidance of double-marginalization and royalty-stacking adds another distinct element to consider when contemplating pools.

5.1 Fees

In the case of upfront fixed fees, the incentives implied by the spillover and differentiation effects carry over and can directly be applied to the analysis of pool formation, provided that the pool formation does not alter the innovation and competitive structures too dramatically. Even so, because spillover effects and differentiation effects do not paint a consistent picture across interests and generally depend on the size of spillovers and the degree of product differentiation, there are few immediate and straightforward results. Nevertheless some patterns emerge and some noteworthy constellations exist.

Of the three market participants—patent holders, firms, and consumers—the direction of marginal welfare effects are most sensitive to spillovers and product differentiation when it comes to consumers, and least so when it comes to firms, with patent holders in between. That is, whether consumers benefit or suffer on the margin from either of the effects generally depends on the degree of spillovers and the degree of product differentiation, whereas for firms most constellations of parameters have the same implications concerning the marginal impact of the effects. In particular, firms and fee-charging patent holders largely benefit from increases in spillovers (Proposition 1) and decreases in product homogeneity (Proposition 4).

However, while it may generally be easy to evaluate the marginal effects for firms and hence also for fee-charging patent holders, this does not mean that the incentive to form a pool is straightforward. Notice, thus, from Propositions 1 and 4 and the accompanying Figures 3 and 4 that from the fee-charging patent holders’ perspective the two effects almost always operate in opposite directions so that any definitive evaluation of the desire to pool
must account for the magnitude of the two effects. In general, whenever the differentiation
effect increases, to keep the incentives for pooling the same, there must also be an increase
in the spillover effects.

The only exception to the fee-charging patent holders’ two incentives moving in opposite
directions is the case characterized in Proposition 1. Indeed, since here the patent holders’
critical threshold on the parameter values is entirely encased by that of consumers, this also
yields the only unambiguous prediction concerning the desirability of pooling that can be
drawn on the basis of the previous section.

**Theorem 2** If the degree of product differentiation is small and spillovers in development
are sufficiently high, then firms, consumers, and patent holders are all worse off by the
formation of a pool. Formally,

\[ W_p < W_n, \quad \forall \beta_n, \gamma_n \Rightarrow \beta_n > S_{V_F, \Pi} (\gamma_n) \quad \text{and} \quad W \in \{CS, \Pi, V_F, TW_F\}. \] (29)

Thus, quite remarkably, the only strong result to follow from the analysis of Section 4
is a sufficient condition in which the pooling of perfectly complementary patents actually
unambiguously lowers consumer surplus, profit and total welfare. A stark contrast to the
conventional wisdom concerning the benefits of pooling complementary patents.

Underlying the result is that when there are large degrees of spillovers in development
then for relatively homogenous products there is a lot of free-riding in the development
process. As a result, increases in spillovers and increases in product homogeneity due to
pooling actually reduce overall development efforts to the detriment of all involved.

Several important remarks concerning Theorem 2 are in order. First, the Theorem gives
sufficient conditions for a pool to be welfare reducing for all involved. The conditions are not
necessary and indeed there are many other constellations concerning spillovers and product
differentiation and how these are affected by pooling that yield the same implication. Second,
in all of these cases, because patent holders also are better off without a pool, an inefficient
pool would not emerge on its own.

In contrast to the theorem, however, there are also many constellations in which—in
congruence with the conventional wisdom—all parties involved strictly benefit from the for-
mation of a pool. The following example, depicted in Figure 5, illustrates some of these
Example 1. Let \( \beta_n = 0.7 \) and \( \gamma_n = 0.2 \), that is, products are strongly differentiated and there are strong spillovers in development. Now consider spillover and differentiation effects such that \( \beta_p \in [0.7, 1] \) and \( \gamma_p \in (0.2, 0.9) \), then there exist functions \( F_{CS} \) and \( F_{V, \Pi} \), with \( F_{CS} > F_{V, \Pi} \), such that

\[
CS_p < CS_n, \quad \forall \beta_p, \gamma_p \Rightarrow \beta_n/\beta_p > F_{CS} (\gamma_n/\gamma_p), \tag{30}
\]

\[
W_p < W_n, \quad \forall \beta_p, \gamma_p \Rightarrow \beta_n/\beta_p > F_{V, \Pi} (\gamma_n/\gamma_p) \quad \text{and} \quad W \in \{V_F, \Pi\}. \tag{31}
\]

Example 1 shows how pooling can be undesirable, even for initially very differentiated goods, provided that spillover effects are small (i.e., \( \beta_n/\beta_p \) large) and differentiation effects are large (i.e., \( \gamma_n/\gamma_p \) small). In contrast, if differentiation effects are small, then all parties prefer the pooling outcome.

Moreover, as Figure 5 illustrates, as the differentiation effect becomes smaller (i.e., \( \gamma_n/\gamma_p \) increases) or the spillover effect becomes larger (i.e., \( \beta_n/\beta_p \) decreases) it is first consumers and only later the fee-charging patent holders who prefer the pooling structure. For this example, this implies two things. First, a sufficient condition for pooling to be overall beneficial is that patent holders prefer to pool. And second, there are constellations for which consumers would prefer the pooling structure, while patent holders do not; and overall welfare would be higher without pooling. Indeed, \( F_{TW} \) in Figure 5 shows the threshold for which pooling becomes beneficial from a total welfare standpoint.

\[\text{Figure 5: Pooling and Non-Pooling with Fees}\]

The examples were calculated and the figures were generated using Mathematica®. The associated files are available from the authors upon request.
The tradeoffs described in Example 1 and illustrated in Figure 5 are somewhat typical for large areas of the parameter space. In particular, it can be shown that the incentives to pool are much stronger for consumers than for patent holders in most cases. However, a universal policy recommendation to the effect that pool-formation initiated by patent holders would necessarily benefit consumers is unfortunately not possible. This is illustrated in the following example.

**Example 2** Let \( \beta_n = 0.2 \) and \( \gamma_n = 0.8 \). Thus, products are fairly homogenous and spillovers in development are moderate when there is no pool. Now suppose that \( \beta_p = 0.8 \) so that there are large spillover effects from pooling. Consumers will surely not want a pool to form in this case. However, for a small differentiation effect, i.e., \( \gamma_p \lesssim 0.846 \), patent holders wish to pool; which is only overall desirable (from a total welfare perspective) if differentiation effects are truly minimal, i.e., if \( \gamma_p \lesssim 0.816 \).

Figure 6 depicts the parameter space in which \( \beta_n = 0.2, \gamma_n = 0.8, \beta_p \in (0.2, 1] \) and \( \gamma_p \in (.8, 0.9] \). The dotted horizontal line gives the case where \( \beta_p = 0.8 \) and the intersections of that line imply the values of \( \gamma_p \) that are given as cut-offs in Example 2.

![Figure 6: A Case of Profit-Maximizing Pooling that Reduces Total Welfare](image)

Note that to the degree that this type of example is not deemed pathological, in terms of spillovers and product differentiation and how these are affected by pooling, then this is cause for some policy concern: Despite patents being perfect complements, this reveals constellations in which patent pools would be expected to form, yet pool formation is against the consumers’ interests and also lowers total welfare.
5.2 Royalties

The preceding analysis stems directly from the applications of the spillover and differentiation effects tied to pooling. However, one of the central arguments in the discussion about pool formation is the avoidance of the distortions associated with double-marginalization and royalty-stacking that occur under independently set royalties. We now see if this outweighs other concerns tied to the effect of spillover and differentiation effects on product development. An immediate implication of double-marginalization with royalties is the following theorem.

**Theorem 3** A necessary condition for pooling to be welfare reducing under royalties, is that pooling is welfare reducing under a fee structure.

Of course an immediate corollary to Theorem 3 is that a sufficient condition for pool-formation to be welfare reducing under fees, is that pools are welfare reducing under royalties;

\[(W_p < W_n|R) \implies (W_p < W_n|F), \quad W \in \{CS, \Pi, V\}.\] (32)

We now consider when pooling may be of concern. An implication of Theorem 3 is that for wide areas of the parameter space pooling is the preferred structure of all market participants—from which, of course, it readily follows that total welfare is generally also greatest under a pooling structure. Indeed, it turns out that because of the strong distortions that independently-set royalty rates have on output, consumers unambiguously prefer the pool formation. This is so, independent of the degree or product differentiation and the amount of spillovers in development; and independent of the magnitude of spillover and differentiation effects. That is, consumer surplus is always strictly greater under a patent pool when licensing arrangements contain per-unit-of-output royalty rates. This can be viewed as a partial corroboration and extension of the conventional wisdom. Formally:

**Theorem 4** Given per-unit-of-output royalties, the pooling of perfectly complementary patents always generate an increase in consumer surplus, i.e.,

\[CS_p > CS_n, \quad \forall \beta_n, \beta_p, \gamma_n, \gamma_p.\] (33)
This is an important finding from an antitrust perspective, when consumer surplus is used as the deciding policy guide. However, the picture is more nuanced for firms and, more importantly, in terms of the patent holders’ interests as well. As was shown in the previous section, the differentiation effect makes pooling less attractive for firms (Proposition 4), and if spillovers are large then the spillover effect may also make pooling less profitable (Proposition 1). Analogous considerations exist for royalty-charging patent holders as well (see Propositions 5 and 2). Thus, it is typically the case that for firms or patent holders to refrain from pooling, differentiation effects must be very strong. When this is the case, the aversion to pooling can then even be independent of spillover effects; as the following typical example illustrates.

**Example 3** Let $\beta_n = 0.5$ and $\gamma_n = 0.5$, that is, products are moderately differentiated and there are moderate spillovers in development. Now consider spillover and differentiation effects such that $\beta_p \in [0.5, 1]$ and $\gamma_p \in (0.5, 0.9]$, then there exist functions $R_{\Pi}$ and $R_{\mathcal{V}_R}$, with $R_{\Pi} > R_{\mathcal{V}_R}$, such that

$$\Pi_p < \Pi_n, \quad \forall \beta_p, \gamma_p \Rightarrow \beta_n/\beta_p < R_{\Pi} (\gamma_n/\gamma_p),$$

$$V_p < V_n, \quad \forall \beta_p, \gamma_p \Rightarrow \beta_n/\beta_p < R_{\mathcal{V}_R} (\gamma_n/\gamma_p).$$

Figure 7: Unprofitable Pooling with Royalties

Despite the fact that patent holders, and even more so firms, may eschew a pool formation, total welfare is commonly larger with a pool, due to the increase in consumer surplus under a pool. However, the adverse effects of pooling on profits and royalty revenues may be large enough to overcome the advantages of pooling for consumers. This is only the
case when the products are highly differentiated, but there are very strong differentiation
effects that result in goods becoming close substitutes for one another, as is illustrated in
the following example.

**Example 4** Let $\beta_n = 0.8$ and $\gamma_n = 0.1$, that is, products are highly differentiated and there
are large spillovers in development. Now consider spillover and differentiation effects such
that $\beta_p \in [0.8, 1]$ and $\gamma_p \in (0.1, 1]$, then there exist $R_{TW}$, such that

$$TW_p < TW_n, \quad \forall \beta_p, \gamma_p \ni \beta_n/\beta_p < R_{TW}(\gamma_n/\gamma_p). \quad (36)$$

![Figure 8: Reduction of Total Welfare due to Pooling with Royalties](image)

The function $R_{TW}$ from Example 4 is depicted as the dashed line in Figure 8. Note that
the thresholds for desiring pooling from the firms’ and the patent holders’ perspectives are
also depicted there, using analogous notation. It is worth noting that in contrast to Example
3 and Figure 7 it is now patent holders who more readily reject the pool formation compared
to the firms.

An immediate and very important corollary to Theorem 4 and the examples is that the
industry’s desire to prefer pooling is a sufficient condition to guarantee that overall welfare
is increased if a pool is formed. In these instances, then, a good policy guide would be to
facilitate the industry’s desire. This is in contrast to the findings of Example 2, further
demonstrating that general implications concerning the welfare effects of pooling perfectly
complementary patents are hard to establish.
6 Conclusion

In the debate about overcoming the so-called ‘patent-thicket,’ patent pooling has been seen as a possible solution, provided that patents placed in the pool are complementary. This conventional wisdom—present in the academic literature, in policy circles, and antitrust practice—relies on static price effects tied to royalty stacking, but overlooks potential implications of pool formation for dynamic innovation in downstream product development and commercialization. Largely missing from the debate is the potential impact of patent pooling on the transfer of embodied and tacit knowledge on the subsequent development and commercialization process of goods.

In this paper we begin to fill this gap by considering a model in which perfectly complementary patents are incomplete and tacit knowledge needs to be transferred in the licensing process—features that are present in biotechnology and the pharmaceutical industry. Forming pools for scientific discoveries results in patent holding scientists becoming more connected, thus providing opportunities for information sharing. This has also been cited as a strong reason to favor patent pools—in particular in biotechnology and the pharmaceutical industry. Because the pool serves as an information-sharing device, the formation of a pool increases spillovers in subsequent product development and decreases the degree of product differentiation in the final product market. Once the development incentives of the downstream firms are accounted for in light of this, patent pools—even for perfectly complementary patents—may be welfare decreasing.

Nevertheless, there are many constellations for which patent pools are beneficial. If consumer surplus is viewed as the relevant criterion for antitrust sanctioning of pools and royalties are paid on a per-unit-of-output basis, the pooling structure is always preferred to the non-pooling structure, regardless of the degree of spillovers and product differentiation and how pooling affects these.

However, when IP is licensed on an up-front fee basis, consumer surplus may be reduced under pooling. This happens, for instance, if products are relatively close substitutes and there are large spillovers in development, because free-riding in the development process lowers development efforts. In these cases firms’ profits and patent holders’ revenues are
also diminished under pooling. Similarly, when evaluating total welfare, pooling is also
detrimental when products are not close substitutes, but there are large differentiation effects,
regardless of whether spillovers in development are affected by pooling.

A corollary of sorts to this observation also emerges from our analysis. In many instances,
a sufficient condition for total welfare to increase under pool formation is that patent holders
prefer the pooling structure and therefore would seek it of their own volition. However, there
are exceptions to this guide. When products are close substitutes and spillovers are initially
small, but become large due to pooling, then firms may benefit from reduced effort in the
development stage to the detriment of consumers.

In sum, we have found constellations in which even though the industry desires to pool,
consumer surplus (and even total welfare) is lower under a pool. For the case of royalties, total
welfare may decrease under pooling even without any spillover effects, provided that spillovers
are already large, products are relatively close substitutes and there are differentiation effects
from pooling. Finally, for the case of up-front fees, even minuscule spillover effects alone can
decrease welfare when products are relatively similar and spillovers are large.

The model demonstrates that the welfare implications of pooling complementary patents
is sensitive to industry specifics, and general policy recommendations based solely on the
complementarity of patents ought to be avoided. Although the conventional wisdom may
prevail in industries such as consumer electronics where spillovers and product differentiation
are not affected by pooling; it may fail in industries such as biotech and pharmaceuticals,
where knowledge transfer creates spillover and differentiation effects tied to pooling.

Indeed, Van Overwalle (2012), citing two surveys in the medical biotechnology sector,
reports that for most respondents an important reason not to form a pool is the fear of loss
of secrecy and control, consistent with the anticipation of free-riding due to spillovers and
reduced profits due to diminished product differentiation. These concerns suggest that in
order for pools to successfully form, patent-holders and licensees need to find mechanisms
to control spill-over and differentiation effects in the development and commercialization
process. Yet, any such mechanisms would be cause for added antitrust and regulatory
scrutiny. In the meantime there is evidence to suggest that innovators in biotechnology and
pharmaceuticals are able to navigate the patent thicket problem without pool structures.
Thus, Katznelson and Howells (2012) note that there are substantial clinical and economic benefits tied to design-around activities in pharmaceuticals.

Appendix A: Derivations

Market Profit

The Bertrand-Nash equilibrium of this game yields:

\[ P^*_i = \frac{(2-\gamma^2)A_i - \gamma A_j - 1 R}{2 - \gamma}, \quad i, j = 1, 2; \ i \neq j. \]  

(37)

Hence, (3) becomes

\[ Q^*_i = \frac{(A_i - P^*_i) - \gamma(A_j - P^*_j)}{(1 - \gamma)(1 + \gamma)} = \frac{(2-\gamma^2)A_i - \gamma A_j - 1 R}{2 - \gamma(1 + \gamma)}. \]  

(38)

Note also that

\[ P^*_i - 1 R = \frac{(1 - \gamma)\left(\frac{(2-\gamma^2)A_i - \gamma A_j - 1 R}{2 - \gamma - \gamma} - 1 R\right)}{2 - \gamma} = \left(1 - \gamma^2\right)Q^*_i; \]  

(39)

So, from (38) and (39) one obtains profit of

\[ \pi^*_i(A_i, A_j) = (P^*_i - 1 R)Q^*_i - (1 - 1)F = (1 - \gamma^2)(Q^*_i)^2 - (1 - 1)F \]
\[ = (1 - \gamma)\left(\frac{(2-\gamma^2)A_i - \gamma A_j - 1 R}{2 - \gamma - \gamma - \gamma - \gamma} - 1 R\right)^2 \]
\[ = (2 - \gamma^2)(1 + \gamma) - (1 - 1)F, \]  

(40)

which is (6).

Equilibrium Consumer and Producer Surplus

Substituting the equilibrium effort level (9) into the firm’s payoff (7) yields

\[ \Pi^*_i = (a - 1 R)^2 \frac{(2 - \gamma^2)(1 - \gamma^2)(1 + \gamma)^2 - (2 - \gamma^2 - \gamma^2)^2}{[(2 - \gamma^2)(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma^2)]^2} - (1 - 1)F. \]

(41)

To derive consumer surplus in the market, we use the representative consumer’s preferences that underlie the demand structure \[ U(Q_i, Q_j) = A_i Q_i + A_j Q_j - (Q_i^2 + 2\gamma Q_i Q_j + Q_j^2)/2 \]
(see Singh and Vives, 1984). For the symmetric equilibrium this reduces to

\[ U^* = 2A^* Q^* + (1 + \gamma)Q^*^2. \]

(42)
Substituting (9) into (2) gives

$$A^*(e^*) = \frac{a(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - R(1 + \beta)(2 - \gamma^2 - \gamma\beta)}{(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)}.$$ (43)

Further substitution into (37) yields

$$P^* = 1 - (a - \Pi)\frac{(1 + \gamma)(4 - \gamma^2)}{(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)},$$ (44)

and substituting (43) and (44) into (3), results in

$$Q^* = (a - \Pi)\frac{4 - \gamma^2}{(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)}.$$ (45)

Consumer surplus is gross utility minus expenditures, i.e., $CS^* := U^* - P^*2Q^*$, so, using (42)–(45),

$$CS^* = (a - \Pi)^2\frac{(2 - \gamma)^2(1 + \gamma)(2 + \gamma)^2}{[(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (1 + \beta)(2 - \gamma^2 - \gamma\beta)]^2}.$$ (46)

### Appendix B: Proofs

Proofs that are straightforward or are implied by the discussion in the main text are omitted.

**Proof of Lemma 1**

Equilibrium effort is given by (9). After taking the derivative, dropping the denominator and consolidating it follows that \(\frac{de^*}{d\beta}\) carries the same sign as

$$-\gamma(2 - \gamma)^2(1 + \gamma)(2 + \gamma) + (2 - \gamma^2 - \gamma\beta)^2.$$ (47)

Setting this equal to zero and solving for \(\beta\) yields

$$S_e = \frac{2 - \gamma^2 - \sqrt{\gamma(2 - \gamma)^2(1 + \gamma)(2 + \gamma)}}{\gamma}.\quad (48)$$

**Proof of Lemma 2**

Beginning with (43), the proof follows *mutatis mutandis* that of the previous Lemma with

$$S_A = \frac{2 - \gamma - \gamma^2}{2\gamma}.\quad (49)$$

**Proof of Proposition 1**

By assumption \(V_F(\cdot)\) is perfectly aligned with equilibrium market profit and hence \(\frac{dV^*_F}{d\beta}\) carries the same sign as \(\frac{d\Pi^*}{d\beta}\). Equilibrium profit \(\Pi^*\) is given by (14).
Applying the quotient rule in taking the derivative and dropping the denominator, it follows after some simplification that \( \frac{dT_W^F}{d\beta} \) carries the same sign as
\[
\left[ 6 + 4\gamma + \beta^2\gamma - 5\gamma^2 - \gamma^3 + \gamma^4 - \beta (2 - \gamma - \gamma^2) \right] \times \\
\left[ 24 - 44\gamma^2 + 4\gamma^3 + \beta^3\gamma^3 + 30\gamma^4 - \gamma^5 - 9\gamma^6 + \gamma^8 - 3\beta^2\gamma^2 (2 - \gamma^2) - \\
\beta\gamma (20 + 8\gamma - 32\gamma^2 - 6\gamma^3 + 14\gamma^4 + \gamma^5 - 2\gamma^6) \right].
\]

The first factor can be written as
\[
(2 - 2\beta) + (4 - 4\gamma^2) + (\gamma - \gamma^2) + (3\gamma - \gamma^3) + \gamma^4 + \beta (\gamma^2 + \gamma) + \beta^2\gamma,
\]
which is clearly positive. Setting the second factor equal to zero and solving for \( \beta \) yields \( S_{V,R,II} \) and the derivative properties follow. \( \square \)

**Proof of Proposition 2** Taking the derivatives of (16) and (15) with respect to \( \beta \) when \( 1 = 1 \) reveals that the sign is determined by the sign of \( - (2 - \gamma^2 - \gamma\beta) - (1 - \beta)\gamma \), hence \( S_{V,R,CS} = S_A \). \( \square \)

**Proof of Proposition 3** It can be shown that \( \frac{dT_W^F}{d\beta} \) carries the same sign as
\[
G := 80 - 8(-2 + 9\beta)\gamma - 4 (34 + 12\beta + 3\beta^2) \gamma^2 + 2 \left( -8 + 40\beta + 3\beta^3 \right) \gamma^3 + \\
(78 + 28\beta + 6\beta^2) \gamma^4 + (7 - 30\beta)\gamma^5 - 4(5 + \beta)\gamma^6 + (-1 + 4\beta)\gamma^7 + 2\gamma^8
\]
Setting \( G \equiv 0 \) gives an implicit equation that can be solved for \( \beta \) to yield
\[
S_{TW_F} = \frac{246\gamma^3(1-i\sqrt{3})(-2+\gamma)^2\gamma^4(-6-9\gamma-\gamma^2+3\gamma^3+\gamma^4)}{H^{1/3}} - 66^{1/3} (1+i\sqrt{3}) H^{1/3} - 72\gamma^2 (-2 + \gamma^2);
\]
where
\[
H := 864\gamma^6 + 720\gamma^7 - 1296\gamma^8 - 792\gamma^9 + 774\gamma^{10} + 261\gamma^{11} - 198\gamma^{12} - 27\gamma^{13} + 18\gamma^{14} + \\
\sqrt{3}\sqrt{(-2 + \gamma)^6\gamma^{12} (2 + 3\gamma + \gamma^2)^3 (-2970 + 81\gamma + 2943\gamma^2 - 1044\gamma^4 + 128\gamma^6)}.
\]
Note that \( \frac{dT_W^F}{d\beta} \geq 0 \) whenever \( \beta \leq \frac{1}{2} \) \( S_{TW_F} \).
\[
\frac{dT_W^F}{d\beta} \text{ carries the same sign as }
\]
\[
I := 176 + 32\gamma - 320\gamma^2 - 28\gamma^3 + 182\gamma^4 + 9\gamma^5 - 44\gamma^6 - \gamma^7 + 4\gamma^8 + \\
2\beta^3\gamma^2 (-8 + \gamma + 2\gamma^2) + 6\beta^2\gamma (8 - 6\gamma - 6\gamma^2 + 2\gamma^3 + 4\gamma^4) + \\
2\beta (-16 - 68\gamma - 40\gamma^2 + 80\gamma^3 + 23\gamma^4 - 31\gamma^5 - 3\gamma^6 + 4\gamma^7).
\]
Setting $I \equiv 0$ gives an implicit equation that can be solved for $\beta$ to yield

$$S_{TW_R} = \frac{J^{1/3}}{62^{1/3} \gamma^2 (-8 + \gamma + 2\gamma^2)} \left[ \frac{8 - 6\gamma - 6\gamma^2 + 2\gamma^3 + \gamma^4}{\gamma (-8 + \gamma + 2\gamma^2)} - \frac{-768 + 9792\gamma + 4800\gamma^2 - 13536\gamma^3 - 3216\gamma^4 + 6468\gamma^5 + 756\gamma^6 - 1308\gamma^7 - 60\gamma^8 + 96\gamma^9}{32^{2/3} J^{1/3} (-8 + \gamma + 2\gamma^2)} \right];$$

where

$$J := 27648\gamma^4 - 262656\gamma^5 - 286848\gamma^6 + 454464\gamma^7 + 342144\gamma^8 - 331776\gamma^9 -$$

$$150120\gamma^{10} + 117828\gamma^{11} + 28296\gamma^{12} - 19980\gamma^{13} - 1944\gamma^{14} + 1296\gamma^{15} +$$

$$\left[ 4 (-768\gamma^2 + 9792\gamma^3 + 4800\gamma^4 - 13536\gamma^5 - 3216\gamma^6 +
6468\gamma^7 + 756\gamma^8 - 1308\gamma^9 - 60\gamma^{10} + 96\gamma^{11} \right]^3 +$$

$$\left( 27648\gamma^4 - 262656\gamma^5 - 286848\gamma^6 + 454464\gamma^7 + 342144\gamma^8 - 331776\gamma^9 -$$

$$150120\gamma^{10} + 117828\gamma^{11} + 28296\gamma^{12} - 19980\gamma^{13} - 1944\gamma^{14} + 1296\gamma^{15} \right)^2]^{1/2}.$$

Note that $\frac{dTW_R}{d\beta} \geq 0$ whenever $\beta \leq S_{TW_R}$.}

**Proof of Lemma 3** Equilibrium effort is given by (9). After taking the derivative, dropping the denominator and consolidating it follows that $\frac{de^*}{d\gamma}$ carries the same sign as

$$-(2 - \gamma) \left[ 2(2 - \gamma - \gamma^2) + 3\gamma^3 + 2\gamma^4 + \beta \left(4 + 2\gamma + 4\gamma^2 + 3\gamma^3\right) \right]. \quad (52)$$

Both factors are obviously positive so that the negative of their product is negative; which is also sufficient to prove the second statement.

**Proof of Proposition 4** As remarked in the proof to Proposition 1, $\frac{dV^*}{d\gamma}$ carries the same sign as $\frac{d\Pi^*}{d\gamma}$. Applying the quotient rule in taking the derivative of (14) with respect to $\gamma$, it follows after some simplification that $\frac{d\Pi^*}{d\gamma}$ carries the same sign as

$$\left[ -2(\gamma - 2)^2(1 + \gamma) \right] \left[ 6 + 4\gamma + \beta^2\gamma - 5\gamma^2 - \gamma^3 + \gamma^4 - \beta(2 - \gamma - \gamma^2) \right] \times$$

$$\left[ 12 + 10\gamma^3 + 2\gamma^4 - 3\gamma^5 - \gamma^6 + \beta\gamma(\gamma - 2)(1 + \gamma)^2 + \beta^2(4 + 2\gamma + 3\gamma^2 + 2\gamma^3) \right] \quad (53)$$

Of the three factors it is straightforward to show that the first is negative and the third is positive. The middle factor is shown to be positive in the proof to Proposition 1, from which it follows that $\frac{d\Pi^*}{d\gamma} < 0$. 

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Proof of Proposition 5 We undertake the same steps as in the proof to Proposition 2, but now take derivatives with respect to $\gamma$. From this it follows that \( \frac{dV}{d\gamma} \) has the same sign as
\[
- \left[ 16 - 28\gamma - 8\gamma^2 + 16\gamma^3 + \gamma^4 - 2\gamma^5 + \beta(2 + \gamma)^2 + \beta^2(4 + \gamma^2) \right].
\] (54)
Setting this equal to 0 and solving for $\beta$ yields
\[
\mathcal{D}_{VR} = \frac{(2 + \gamma)^2 + (2 - \gamma) \sqrt{-60(1 - \gamma) + 97\gamma^2 + 28\gamma^4 + 8\gamma^5}}{4 + \gamma^2}.
\] (55)
Similarly one derives that \( \frac{dCS}{d\gamma} \) has the same sign as
\[
- \left[ 16 - 28\gamma - 8\gamma^2 + 16\gamma^3 + \gamma^4 - 2\gamma^5 + \beta(2 + \gamma)^2 + \beta^2(4 + \gamma^2) \right] \times
\left[ 8 - 40\gamma - 46\gamma^2 + 24\gamma^3 + 25\gamma^4 - 3\gamma^5 - 3\gamma^6 + \right.
\beta(2 + \gamma)^2 (4 - \gamma + \gamma^2) + \beta^2 (8 + 4\gamma + 2\gamma^2 + 3\gamma^3) \right].
\] (56)
Of the three factors it is straightforward to show that the first is negative and in the proof to Proposition 1 it is shown that the second is positive. Setting the third factor equal to zero and solving for $\beta$ yields
\[
\mathcal{D}_{CS} = \frac{(2 - \gamma) \sqrt{\gamma (384 + 964 (\gamma + \gamma^2) + 669\gamma^3 + 454\gamma^4 + 205\gamma^5 + 36\gamma^6)}}{2 (8 + 4\gamma + 2\gamma^2 + 3\gamma^3)}
+ (2 + \gamma)^2 (4 - \gamma - \gamma^2).
\] (57)
□

Proof of Proposition 6 It can be shown that \( \frac{dT^2}{d\gamma} \) carries the same sign as $-(112 - 24\gamma - 180\gamma^2 + 82\gamma^3 + 56\gamma^4)$, which is negative for all $\gamma \in [0, 1]$. Hence, \( \frac{dT^2}{d\gamma} < 0 \forall \beta $.
Now, \( \frac{dT^2}{d\gamma} \) carries the same sign as $K \left( \frac{K}{(96 + 4\gamma + \beta^2(4 + \gamma^2) - 5\gamma^2 - 5\gamma^3 + 4\gamma^4 + 4\gamma^5 + \gamma^6)} \right)^2$, where
\[
K := -608 + 576\gamma + 1296\gamma^2 - 1096\gamma^3 - 794\gamma^4 + 636\gamma^5 + 185\gamma^6 - 147\gamma^7 - 
15\gamma^8 + 12\gamma^9 - 4\beta^4\gamma (4 + \gamma^2) - 4\beta^3 (-8 + 8\gamma + 6\gamma^2 + 2\gamma^3 + \gamma^4) - 
\beta^2 (160 + 160\gamma - 144\gamma^2 + 20\gamma^3 + 34\gamma^4 - 15\gamma^5 + 4\gamma^6) - 
\beta (32 + 464\gamma - 24\gamma^2 - 328\gamma^3 + 48\gamma^4 + 81\gamma^5 - 13\gamma^6 - 4\gamma^7).
\]
Setting $K \equiv 0$ gives an implicit equation that can be solved for $\beta$ to yield

$$D_{TW_R} = \frac{-B}{4A} + \frac{1}{4\sqrt{6A}} \left( \frac{6B^2}{A} - 4C - \frac{2^{4/3}(C^2 + 12(4EA - BD))}{(F + G)^{1/3}} \right) + \frac{2^{2/3}(F + G)^{1/3}}{4} + \frac{\frac{B^2}{8A^2} - \frac{C}{12A} + \frac{C^2 + 12(4EA - BD)}{24 \cdot 2^{2/3}A(F + G)^{1/3}} + \frac{(F + G)^{1/3}}{48 \cdot 2^{1/3}A^3}}{\sqrt{\frac{3}{2}(BCA - B^3 - 2DA^2)}} \left( \frac{6B^2 - 4CA + \frac{2^{4/3}A(C^2 + 12(4EA - BD))}{(F + G)^{1/3}} + 2^{2/3}A(F + G)^{1/3}}{4A} \right)^{1/2},$$

where

$$A = \gamma(4 + \gamma^2), \quad B = -8 + 8\gamma + 6\gamma^2 + 2\gamma^3 + \gamma^4,$$

$$C = 160 + 160\gamma - 144\gamma^2 + 20\gamma^3 + 34\gamma^4 - 15\gamma^5 + 4\gamma^6,$$

$$D = 32 + 464\gamma - 24\gamma^2 - 328\gamma^3 + 48\gamma^4 + 81\gamma^5 - 13\gamma^6 - 4\gamma^7,$$

$$E = 608 - 576\gamma - 1296\gamma^2 + 1096\gamma^3 + 794\gamma^4 - 636\gamma^5 - 185\gamma^6 + 147\gamma^7 + 15\gamma^8 - 12\gamma^9,$$

$$F = 36BCD - 432EB^2 - 2C^3 + 288AEC - 108AD^2,$$

and

$$G = 2\sqrt{(C^3 - 18BCD + 54AD^2 + 72E(3B^2 - 2AC))^2 - (C^2 - 12BD + 48AE)^3}.$$

Then $\frac{dD_{TW_R}}{d\gamma} \geq 0$ whenever $\beta \leq D_{TW_R}$.

**Proof of Theorem 2** It can be shown that $S_{V_p,\Pi} > D_{CS}$, whereupon the assertion follows immediately as a corollary to Propositions 1, 2, 4 and 5.

**Proof of Theorem 4** Setting $\gamma_p = 0.85$ and $\beta_p = 1$ Mathematica’s FindInstance[$\{CS_p < CS_n, 0 < \gamma_n < 0.85, 0 < \beta_n < 1\}, \{\gamma_n, \beta_n\}$], shows that no such instance exists on the given domain. Since consumer surplus is concave, it then follows that the theorem holds for the entire domain.

**References**


