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DISCUSSION PAPER

Cumulative Innovation and
Competition Policy

by

Alexander Raskovich
and Nathan H. Miller *

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Abstract

We model a “new economy” industry where innovation is sequential and monopoly is persistent but the incumbent turns over periodically. In this setting we analyze the effects of “extraction” (e.g., price discrimination that captures greater surplus) and “extension” (conduct that simply delays entry of the next incumbent) on steady-state equilibrium innovation, welfare and growth. We find that extraction invariably increases innovation and welfare growth rates, but extension causes harm under plausible conditions. This provides a rationale for the divergent treatment of single-firm conduct under U.S. law. Our analysis also suggests a rule-of-thumb, consistent with antitrust practice, that innovation proxies welfare.

*JEL codes:* D42, K21, L40, O30, O38

*Key words:* innovation, antitrust, monopoly, single-firm conduct
1 Introduction

It is well understood that competition policy has first-order implications for innovation and economic growth.\(^1\) Accordingly, antitrust authorities are very much concerned with the potential adverse effects on innovation of certain mergers and business practices. Yet despite the broad scope of competition policy, the economics literature has focused on the question of how market structure affects innovation incentives, with Schumpeter (1942) and Arrow (1962) standing on opposite sides of the classic divide.\(^2\) Comparatively little attention has been paid to how unilateral (i.e., “single-firm”) conduct affects innovation and economic growth.

In this paper, we explore single-firm conduct within a theoretical model of a “new economy” industry in which innovation is sequential and leads to a succession of incumbent monopolists. Each monopolist selects the quality of its innovation at the outset of its incumbency and is eventually displaced by a firm (the next monopolist) drawn from a pool of symmetric potential entrants. Higher quality innovations increase surplus flows and also lengthen the monopolist’s tenure. Two intertemporal externalities emerge immediately. First, firms stand on the shoulders of giants: innovation raises surplus flows not only in the current incumbency but in all future incumbencies as well. Second, firms beggar their successors: innovation, by lengthening the current incumbent’s tenure, delays the realization of all future innovations. The first of these externalities has been central to the sequential innovation literature and to discussions of patent policy.\(^3\) We highlight the second externality, which has largely been ignored in previous literature. The net intertemporal externality of innovation is positive when shoulders of giants outweighs beggar thy successor, and negative in the opposite case. The sign of the intertemporal externality plays a critical role in our analysis.

\(^1\) This is true for patentable as well as other types of innovation. O’Donoghue, Scotchmer and Thisse (1998) cite empirical evidence that the effective lives of patents are commonly shorter than their statutory lives, because of displacement in the market by non-infringing innovations. This suggests that competition policy and patent rules can play complementary roles in influencing the pace of innovation and growth.

\(^2\) See Gilbert (2006) and Katz and Shelanski (2005) for surveys of this impressive literature.

\(^3\) See e.g. Scotchmer (1991).
Within this framework, we examine the effects of monopoly extraction (e.g., price discrimination that captures greater surplus) and monopoly extension (e.g., conduct that lengthens incumbent tenure) on steady-state innovation and welfare. Our main findings are that:

1. monopoly extraction promotes innovation and welfare growth rates, but
2. monopoly extension inhibits innovation and welfare growth rates if, and only if, the net intertemporal externality of innovation is negative.

The latter finding of course begs the question of how the sign of the net intertemporal externality might be ascertained in practice. Further complicating matters, it is often difficult to cleanly categorize single-firm conduct as either extraction or extension. The model nevertheless provides a simple rule-of-thumb, consistent with antitrust practice, that may help in resolving such issues. Given that innovation and welfare growth rates tend to move together, the effect of single-firm conduct on innovation (which is potentially observable) can be taken as a proxy for its effect on welfare growth rates (more difficult to observe). If the single-firm conduct in question is associated with decreased innovation, this suggests both that the intertemporal externality is negative and that the conduct is a form of monopoly extension and thus harmful.

Our results offer a theoretical underpinning for the basic structure of U.S. antitrust law on single-firm conduct. In particular, Section 2 of the Sherman Act prohibits monopolization, but not monopoly. There is broad consensus among legal scholars that single-firm conduct is condemned only when it is exclusionary. Absent exclusion, conduct that merely extracts rents accruing to monopoly power, legitimately obtained, is

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5 As Areeda, Kaplow and Edlin (2004) observe, in the U.S. “courts and commentators are in uncommon accord that monopolization entails something more than monopoly. Monopolization requires exclusionary conduct.” There has been contentious debate, however, on how to identify exclusionary conduct. In 2006, the U.S. Department of Justice (DOJ) and Federal Trade Commission (FTC) jointly initiated a series of public hearings on single-firm conduct under Section 2 of the Sherman Act. At the conclusion of these hearings, the DOJ issued its findings in 2008, in a report in which the FTC pointedly did not join. With the change in administrations in 2009, the DOJ withdrew the report.
legal. Interestingly, Carlton and Heyer (2008) also argue that extraction and extension should be accorded very different legal treatment. They favor a permissive attitude toward monopoly extraction but an outright ban on the extension of monopoly. Their discussion is intuitive not formal, but their policy prescription is broadly consistent with the basic principle that exclusion is a necessary element of single-firm conduct that violates Section 2 of the Sherman Act.

Although the literature examining the relationship between single-firm conduct and cumulative innovation is sparse, some important work exists. The paper closest to ours is that of Segal and Whinston (2007). These authors develop a quality-ladders growth model in which (in the basic setting) two firms take turns as incumbent monopolist. The non-monopolist (i.e, the “potential entrant”) invests in each period, and the probability that the potential entrant displaces the monopolist increases with the level of investment. When entry occurs, the entrant’s quality increases by a fixed increment over that of the displaced incumbent. Segal and Whinston (2007) derive conditions under which competition policy that facilitates entry also stimulates innovation.

We depart from the modeling framework of Segal and Whinston (2007) in two important respects. First, we model the incumbent monopolist as actively investing in innovation, while potential entrants are passive. This assumption arguably fits the evolution of a number of “new economy” industries fairly well. Bill Gates (1995) has described Microsoft’s inception in this way:

“We were in the right place at the right time. We got there first, and our early success gave us the chance to hire more and more smart people.”

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6 Cf. Judge Learned Hand, who opined that “the successful competitor, having been urged to compete, must not be turned upon when he wins.” (United States v. Alcoa, 148 F.2d 416, 430 [2d Cir. 1945].)

7 The opposite assumption is commonly adopted in the innovation literature, namely that potential entrants are motivated to invest by the prospect of gaining monopoly stature but thereafter refrain from innovating to avoid self-cannibalization, reflecting Arrow’s (1962) “replacement effect.” Many innovation models, however, incorporate the feature that incumbents are motivated to invest to keep competition at bay, following the seminal contribution of Gilbert and Newbery (1982).
Other new-economy firms have likewise gained leadership positions through serendipity, undertaking substantial investment in innovation only after establishing themselves. Chad Hurley, a founder of YouTube, has said:

“Whether it’s [Google founders] Larry [Page] or Sergey [Brin] or other people like [Facebook co-founder] Mark Zuckerberg ... We’re all coming from these simple ideas. We were all really lucky to be in the right place at the right time.” (Owen, 2008)

We believe our modeling approach can potentially be applied to many industries where the lion’s share of R&D is undertaken by the leading firm after the firm has attained its leadership position.

The second point of divergence from the quality-ladders approach of Segal and Whinston (2007), and of many others in the endogenous growth literature, is that we treat R&D as affecting both the frequency and quality of innovation. Quality-ladders models focus on innovation frequency to the exclusion of innovation quality. Akcigit (2009) calls attention to the importance of redressing this imbalance with empirical evidence that variation in the quality of innovations is substantial. Our modeling approach can be interpreted as an initial foray into generating results when both the quality and frequency of innovation vary endogenously.8

The remainder of the paper is organized as follows. We lay out the economic setting in Section 2, derive the private optimum in Section 3 and relate this to the social optimum in Section 4. We discuss the implications for competition policy in Section 5 and conclude in Section 6.

2 Economic Setting

We analyze an industry characterized by persistent monopoly but with periodic turnover in the incumbent monopolist. A period in our model represents the lifespan of a

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8 The quality and frequency of innovation are not independent choice variables in our framework, however. Both are jointly determined by the level of R&D.
given incumbent. At the outset of each period $t$, a new incumbent is drawn without replacement from a pool of symmetric potential entrants; we refer to the firm operating in period $t$ as firm $t$. Firm 1 remains the incumbent for some span $\tau_1$. Period 2 commences at time $\tau_1$, at which point firm 2 becomes the incumbent and remains so for some span $\tau_2$. Firm 3 becomes the incumbent at time $\tau_1 + \tau_2$, and so on. Figure 1 depicts the game timeline for arbitrary values of $\tau_t$.

![Figure 1. Incumbency Timeline](image)

The length of period $t$, which is the lifespan of firm $t$, is given by

$$\tau_t = \phi(y_t),$$

where $y_t \geq 0$ is the level of innovation selected by firm $t$ (and undertaken at the outset of period $t$), $\phi(0) > 0$ and $\phi' > 0$. The incumbent tenure function $\phi(\cdot)$ captures the notion that a firm’s technological leadership position will be more durable the greater the firm’s investment in innovation. Thus firms in our setting are motivated to innovate in part to extend their tenure as monopolists.\(^9\)

Innovation also increases the flow rate of total gross surplus $S_t$, which is surplus gross of both monopoly deadweight losses and R&D costs, as discussed more fully below. This flow rate evolves according to $S_t = (1 + y_t) S_{t-1}$. In period 1, $S_0$ is an exogenously given base flow rate. We assume monopoly deadweight losses are a fixed

\(^9\) This is in the spirit of Gilbert and Newbery (1982), who show that a monopolist has an incentive to innovate to preempt competition from a potential entrant, as well as Aghion et al. (2001), who model firms as innovating in part to escape from competition by distancing themselves from rivals. For concreteness, one might think of potential entrants in our setting as receiving a stream of private ideas over time, as in O’Donoghue et al. (1998). Upon receiving a sufficiently good idea, an entrant displaces the incumbent, but this takes longer the greater the incumbent’s innovation.
proportion \( \delta < 1 \) of \( S_t \),\(^{10}\) so that the flow of surplus net of these losses is simply \( s_t = (1 - \delta) S_t \), and we adopt the normalization \( S_0 = 1/(1 - \delta) \) so that \( s_0 = 1 \). The flow rate \( s_t \) can thus be written as

\[
s_t = \prod_{i=1}^{t} (1 + y_t).
\]

(2)

Taking equation (1) into account, the present value of the stream \( s_t \) realized within period \( t \), evaluated as of the beginning of the period, can be written as

\[
\phi(y_t) \int_0^{s_t} e^{-rx} \, dx = \frac{1}{r} \left( 1 - e^{-r \phi(y_t)} \right) (1 + y_t) s_{t-1},
\]

(3)

where \( r > 0 \) is a discount rate common to all firms / periods. This is the gross social benefit (gross of R&D costs) of \( y_t \) that is realized within period \( t \). The gross social benefit in equation (3) can be usefully decomposed as the product of two factors \( b(y_t) s_{t-1} \), where \( s_{t-1} \) is the flow of surplus firm \( t \) inherits from her immediate predecessor and

\[
b(y_t) \equiv \frac{1}{r} (1 + y_t) \left( 1 - e^{-r \phi(y_t)} \right)
\]

(4)

is firm \( t \)'s contribution to the gross social benefit.

In turn, \( b(y_t) \) can be further decomposed as the product of the two factors \( \frac{1}{r} (1 + y_t) \) and \( 1 - e^{-r \phi(y_t)} \) on the right-hand side of (4). The innovation \( y_t \) increases surplus flows not only within period \( t \) but in all future periods as well. The overall present value of these social benefits is proportional to \( \frac{1}{r} (1 + y_t) \), however only the fraction \( 1 - e^{-r \phi(y_t)} \) of the benefit stream is realized within period \( t \) and so is potentially appropriable by firm \( t \).

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10 To explore what this implies about the underlying market, begin by supposing that demand and marginal cost are both linear in output and trace out a triangle whose area is \( S_t \). If the monopolist prices uniformly, deadweight loss is a triangle of area \( \delta S_t \). If period \( t \) innovation induces parallel shifts in demand and cost relative to period \( t - 1 \), then \( \delta \) is invariant to innovation. Relaxing the linearity assumption, \( \delta \) is fixed so long as innovation changes the demand and cost curves in a way that scales up the relevant areas similarly. Further relaxing the uniform pricing assumption, \( \delta \) is fixed so long as product and process innovations leave the efficiency of the monopolist’s price discrimination or extraction technology unchanged.
The monopolist captures a portion $\mu \in [0, 1]$ of the surplus flow $s_t$, the remainder being retained by consumers. In keeping with the terminology of Carlton and Heyer (2008), we refer to $\mu$ as the rate of monopoly “extraction,” which we assume is common to all incumbents / periods. We do not model the extraction process, but rather treat $\mu$ as a parameter, one that is potentially influenced by competition policy.\footnote{In particular, we assume $\mu$ is invariant to innovation. Though greater innovation might allow higher extraction by softening competition with rival (outside) goods, we set this possibility aside in our monopoly setting to highlight the intertemporal effects of innovation through its impact on monopoly tenure, $\phi(\cdot)$.} We interpret changes in $\mu$ as reflecting changes in the monopolist’s ability to extract surplus through more flexible price discrimination.\footnote{Given the typically ambiguous welfare effects of price discrimination, we treat $\mu$ as independent of the deadweight loss parameter $\delta$.}

The R&D cost of innovation $y_t$, incurred at the beginning of period $t$, is

$$k_t(y_t) = c'(y_t) s_{t-1},$$

where $c(0) = c'(0) = 0$ and for $y_t > 0$, $c', c'' > 0$. Thus innovation by earlier incumbents makes later innovation more difficult, consistent with empirical evidence that in many countries patents per researcher have declined substantially over time (Kortum, 1997).

Although the R&D costs of innovation grow with time, so too do the associated rewards. The assumed balancing of these effects (compare equations (2) and (5)) simplifies the analysis considerably by yielding a steady-state equilibrium rate of innovation and average growth. It is also consistent with the empirical evidence on “growth without scale effects” (Jones, 1995a,b), as well as modeling approaches taken in the “second wave” of endogenous growth theories (e.g., Jones, 1995b; Kortum, 1997; Segerstrom, 1998; Howitt, 1999; Aghion et al., 2001).

3 The Private Optimum

Firm $t$’s profit is the difference between the present value of innovation benefits appropriated by the firm and the associated R&D costs:

$$\Pi_t = \left( \mu b(y_t) - c(y_t) \right) s_{t-1} \equiv \pi(y_t) s_{t-1}. \quad (6)$$

The private optimum $y^*$ is implicitly defined by the first-order condition
\[ \mu b'(y) - c'(y) = 0. \]  
(7)

We further assume that profit is concave,
\[ \mu b''(y) - c''(y) < 0, \]  
(8)
to ensure the optimum is unique.

**Lemma 1.** *The privately optimal innovation choice is stationary.*

**Proof:** By inspection of equation (6), firm \( t \)'s optimum does not depend on \( s_{t-1} \). \( \square \)

Given stationarity, we drop subscripts \( t \) hereafter.

### 3.1 Intertemporal Externalities of Innovation

Differentiating equation (4) yields
\[ b'(y) = \frac{1}{r} \{ 1 - e^{-r \phi(y)} [1 - (1 + y) r \phi'(y)] \} > 0. \]  
(9)

Equation (9) gives the within-period marginal social benefit of innovation.¹⁴ Of particular interest is the expression within square brackets,
\[ 1 - (1 + y) r \phi'(y), \]  
(10)
which represents the intertemporal externality of innovation. Expression (10) is central to our main results on innovation and welfare.

**Definition.** *Innovation has a positive (negative) intertemporal externality at innovation level \( y \) if and only if \( 1 - (1 + y) r \phi'(y) > 0 \) (< 0).*

Observe that if innovation were to have no effect on incumbent tenure, i.e., \( \phi'(\cdot) = 0 \), expression (10) would simplify to 1 and equation (9) would become \( b'(y) = \frac{1}{r} \{ 1 - e^{-r \phi(y)} \}. \) In this case, current-period innovation would have a *positive* intertemporal externality: only the fraction \( 1 - e^{-r \phi(y)} \) of the marginal social benefit of innovation would be realized in the current period. Insofar as innovation tends to lengthen tenure, \( \phi' (\cdot) > 0 \), expression (10) falls below one and this incumbency effect of innovation helps to internalize the positive intertemporal externality (so long as expression (10) remains nonnegative).

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¹³ One can more readily see that \( b'(y) > 0 \) by rewriting equation (9) as \( b'(y) = \frac{1}{r} \{ 1 - e^{-r \phi(y)} \} + (1 + y) \phi'(y) e^{-r \phi(y)}. \)

¹⁴ When multiplied by \( s_{t-1} \) for firm \( t \).
When expression (10) falls zero, equation (9) becomes $b'(y) = \frac{1}{r}$. The intertemporal externality then disappears: the full marginal social benefit of innovation is captured within the current period. The reason is that in this case two countervailing intertemporal effects of innovation exactly balance. A positive spillover from innovation is that later innovators stand on the shoulders of giants: current-period innovation boosts the flow of gross surplus in all future periods. The shoulders-of-giants effect pushes innovators toward underinvestment. A negative spillover is that innovators beggar their successors: current-period innovation, by lengthening the incumbent’s tenure, delays the realization of all future innovations. This pushes innovators toward overinvestment. When expression (10) is zero these effects cancel and each incumbent fully internalizes the intertemporal externalities of innovation.

Finally, when expression (10) turns negative, $b'(y) > \frac{1}{r}$. In this case, the beggar-thy-successor effect dominates the shoulders-of-giants effect, giving incumbents an excessive incentive to innovate from a social, intertemporal perspective.

3.2 Monopoly Extraction and Innovation

A second, within-period externality arises when firms fail to fully capture the social benefits of innovation that flow during their tenure in the market, i.e., when $\mu < 1$.

**Proposition 1.** Steady-state equilibrium innovation increases with monopoly extraction.

**Proof:** Differentiating the first-order condition (7) with respect to $\mu$ yields

$$\frac{\partial y^*}{\partial \mu} = \frac{-b'}{\mu b'' - c''},$$

which is positive by conditions (8) and (9). □

Hereafter, let $y^*(\mu)$ denote steady-state equilibrium innovation for a given monopoly extraction rate $\mu$. Clearly $y^*(0) = 0$, and we assume $y^*(1)$ is finite.

3.3 Monopoly Extension and Innovation

We now decompose incumbent tenure as

$$\phi(y) = \lambda(y) + \epsilon,$$

where $\epsilon$ is an additive “monopoly extension” parameter.
Proposition 2. Steady-state equilibrium innovation increases (decreases) with monopoly extension if the intertemporal externality is positive (negative).

Proof: Differentiating the first-order condition (7) with respect to the monopoly extension parameter \( \varepsilon \) yields

\[
\frac{\partial y^*}{\partial \varepsilon} = \left( \frac{\mu}{\mu b' - c} \right) \frac{\partial b'}{\partial \varepsilon}
\]

by the implicit function theorem. The expression in parentheses is positive by condition (8) and thus \( \partial y^*/\partial \varepsilon \) has the same sign as \( \partial b'/\partial \varepsilon \). Differentiating equation (9) with respect to \( \varepsilon \), taking equation (12) into account, then yields

\[
\frac{\partial b'}{\partial \varepsilon} = \left[ 1 - (1 + y) r \phi'(y) \right] e^{-r\phi(y)}.
\]

(14)

Given that the bracketed expression in equation (14) is expression (10), \( \partial y^*/\partial \varepsilon \) has the same sign as the intertemporal externality of innovation. □

Intuitively, monopoly extension helps to internalize the intertemporal externality, spurring innovation if the externality is positive and paring it back if the externality is negative. This does not necessarily imply, however, that monopoly extension is welfare improving, as we show in the next section.

4 The Social Optimum

For a given steady-state innovation \( y \), the present value of the resulting stream of total surplus (net of periodic R&D costs), evaluated at the start of the game \( t = 0 \), can be written as

\[
W(y) = w(y) X(y),
\]

(15)

where \( w(y) \) is the component of periodic welfare common across periods,

\[
w(y) \equiv b(y) - c(y),
\]

(16)

and

\[
X(y) \equiv \sum_{t=1}^{\infty} \left[ (1 + y) e^{-r \phi(y)} \right]^{t-1}
\]

(17)

is a scaling factor. We refer to \( W(\cdot) \) as total welfare. We initially focus on the case in which

\[
(1 + y^*(\mu)) e^{-r \phi(y^*(\mu))} < 1
\]

(18)
for all $\mu \in [0, 1]$. When condition (18) holds, the series in (17) converges for any feasible $\mu$, and thus both the scaling factor $X(y^*(\mu))$ and total welfare $W(y^*(\mu))$ are finite. We treat the case in which condition (18) does not hold in Section 4.3, where we consider welfare growth rates as an alternative welfare metric.

Differentiating (17) yields

$$\frac{dx}{dy}(y) = \left[1 - (1 + y) r \phi'(y)\right] e^{-r \phi(y)} \sum_{t=2}^{\infty} (t-1) [(1 + y) e^{-r \phi(y)}]^{t-2}. \quad (19)$$

The intertemporal externality of innovation is embodied in $X'(y)$:

**Lemma 2.** $X'(y)$ has the same sign as the intertemporal externality.

**Proof:** By inspection of equation (19), $X'(y)$ and expression (10) have the same sign. □

From (15), the derivative of total welfare with respect to innovation is

$$W'(y) = w'(y)X(y) + w(y)X'(y). \quad (20)$$

The term $w'(y)X(y)$ in equation (20) reflects within-period effects of innovation, whereas the term $w(y)X'(y)$ reflects intertemporal effects. In the absence of an intertemporal externality (i.e., for $X'(y) = 0$), innovation has only within-period effects, in which case $W'(y)$ necessarily has the same sign as $w'(y)$. Otherwise, innovation also affects welfare through the intertemporal effect $w(y)X'(y)$, tending to raise welfare when the externality is positive and lower welfare when the externality is negative.

### 4.1 Monopoly Extraction and Welfare

**Lemma 3.** Assume condition (18) holds and let $\mathcal{M} \equiv \arg\max_{\mu} W(y^*(\mu))$, i.e., $\mathcal{M}$ is the set of monopoly extraction rates $\mu$ that, given the resulting steady-state equilibrium innovation $y^*(\mu)$, maximize total welfare.

(a) If the intertemporal externality is nonnegative for all $y \in [y^*(0), y^*(1)]$, then $\mathcal{M} = \{1\}$.

(b) If the intertemporal externality is strictly negative at $y^*(1)$, then every member of $\mathcal{M}$ is strictly less than one.

**Proof:** Note that $\mathcal{M}$ is nonempty by the extreme value theorem, given that $W(y^*(\mu))$ is continuous on $[y^*(0), y^*(1)]$. Recall equation (20) and Proposition 1. For proof of part (a), first observe that for $y \in [y^*(0), y^*(1)]$ we have $w'(y) > 0$. Thus $X'(y) \geq 0$ (by
hypothesis and Lemma 2) implies \( W'(y) > 0 \) for each such \( y \), and therefore total welfare is highest for \( \mu = 1 \). For proof of part (b), note that \( w'(y^*(1)) = 0 \). By hypothesis \( X'(y^*(1)) < 0 \), which implies \( W'(y^*(1)) < 0 \) and therefore total welfare is higher for some \( \mu < 1 \). \( \square \)

According to Lemma 3(b), full monopoly extraction is socially suboptimal when the intertemporal externality of innovation is negative. This suggests a possible policy rationale for limits on price discrimination. Intuitively, a very high rate of monopoly extraction can harm welfare by slackening the pace of innovation. Although extraction increases the magnitude of periodic innovation (Proposition 1), it also decreases the frequency of innovation, through the incumbent tenure effect \( \phi(y) \). On net, further extraction retards the pace of innovation and harms welfare when extraction is already very high and the intertemporal externality of innovation is negative.

**Proposition 3.** Assume condition (18) holds and let \( \hat{\mu} < 1 \) be some initial rate of monopoly extraction.

(a) If the intertemporal externality is nonnegative at \( y^*(\hat{\mu}) \), then total welfare increases with monopoly extraction at this point.

(b) Assume \( W(y^*(\mu)) \) is strictly concave on \([y^*(0), y^*(1)]\) so that \( M = \{ \mu^* \} \) for some \( \mu^* \in [0, 1] \). If the intertemporal externality is strictly negative at \( y^*(\hat{\mu}) \), then total welfare increases (decreases) with monopoly extraction at this point if \( \hat{\mu} < (>) \mu^* \).

**Proof:** Part (a) follows from equation (20) and Proposition 1, given that \( w'(y^*(\hat{\mu})) > 0 \) for \( \hat{\mu} < 1 \) and \( X'(y^*(\hat{\mu})) \geq 0 \) by hypothesis and Lemma 2. For proof of part (b), note first that \( \mu^* < 1 \) by Lemma 3 and concavity. Part (b) then likewise follows from Proposition 1. \( \square \)

When the intertemporal externality is positive, total welfare unambiguously increases with monopoly extraction, because in this case the within-period externality and the intertemporal externality have the same sign. Total welfare then necessarily increases with innovation, which is spurred by greater monopoly extraction.

The case of a negative intertemporal externality is less straightforward, both because \( \mu^* < 1 \) in this case and because the welfare function may not be concave.
Imposing concavity puts only modest restrictions on the tenure function $\phi(\cdot)$. When the intertemporal externality is negative, it has the opposite sign of the within-period externality. On the margin, then, the effect of innovation on total welfare depends on which externality is larger in absolute value; the two externalities exactly balance when the reigning extraction rate is $\hat{\mu} = \mu^* < 1$.

4.2 Monopoly Extension and Welfare

The welfare effects of monopoly extension depend on whether innovation is initially above or below the social optimum as well as on the sign of the intertemporal externality, given that this sign is also the direction in which innovation changes as a result of monopoly extension (Proposition 2).

Proposition 4. Assume condition (18) holds and $W(\mu)$ is strictly concave on $[y^*(0), y^*(1)]$. Let $\hat{\mu} < 1$ be the prevailing rate of monopoly extraction.

(a) If the intertemporal externality is positive at $y^*(\hat{\mu})$, then total welfare increases (decreases) with monopoly extension at this point if $\hat{\mu} < (>)\mu^*$.

(b) If the intertemporal externality is negative at $y^*(\hat{\mu})$, then total welfare decreases (increases) with monopoly extension at this point if $\hat{\mu} < (>)\mu^*$.

Proof: Follows from Proposition 2 and Lemma 3.

Comparing Propositions 3 and 4, note that extraction and extension move welfare in the same direction when the intertemporal externality is positive, but move it in opposite directions when the externality is negative. We consider the potential policy implications of this observation in Section 5. Insofar as a primary goal of competition policy is to prohibit single-firm conduct that harms welfare, divergent treatment of extraction and extension may be appropriate when the intertemporal externality from innovation is negative.

4.3 Welfare Growth Rates

When condition (18) does not hold, the series in (17) diverges and the present value of the net surplus stream is infinite. This will be the case for the most innovative industries, whose growth greatly exceeds that of the broader economy, as reflected in

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15 In particular, $W'' < 0$ is consistent with each of $\phi''$ and $X''$ having either positive or negative sign.
market discount rates. Such rapidly innovating industries are of course of particular interest as regards competition policy.

As an alternative welfare metric, we consider the average rate of growth in the industry’s flow of net surplus. Given innovation \( y \) realized every period, the flow of net surplus in period \( t \) is \( w(y)(1 + y)^t \), expressed as a present value as of the beginning of period \( t \). The time elapsed from the beginning of the game through the end of period \( t \) is \( t \phi(y) \). We refer to the ratio of these as the welfare growth rate through period \( t \):

\[
R_t(y) = \frac{w(y)(1 + y)^t}{t \phi(y)}.
\]  
(21)

The first derivative of the welfare growth rate through period \( t \) can be written as

\[
R_t'(y) = \left( \frac{w(y)}{\tau(y)} \right) \left( \frac{(1 + y)^t}{ty} \right) \left( \eta_{w,y} - \eta_{\phi,y} + \left( \frac{y}{1 + y} \right) t \right),
\]  
(22)

where

\[
\eta_{w,y} \equiv \frac{yw'(y)}{w(y)}
\]  
(23)

is the elasticity of the welfare component \( w(y) \) with respect to \( y \) and

\[
\eta_{\phi,y} \equiv \frac{y\phi'(y)}{\phi(y)}
\]  
(24)

is the elasticity of incumbent tenure \( \phi(y) \) with respect to \( y \).

**Lemma 4.** Greater steady-state equilibrium innovation raises the welfare growth rate through every period if

\[
\eta_{\phi,y} < \eta_{w,y} + \frac{y}{1 + y}.
\]  
(25)

**Proof:** By inspection of equation (22). \( \square \)

Because the magnitude and frequency of innovation move in opposite directions (given the incumbent tenure function \( \phi(y) \)), the net effect of innovation on welfare growth rates is ambiguous. Lemma 4 states, however, that if condition (25) holds then innovation raises welfare growth rates on net. Condition (25) requires that incumbent tenure not be (much) more elastic with respect to innovation than is within-period net surplus. This is arguably consistent with a common understanding of what constitutes an
“innovation.”\textsuperscript{16} If condition (25) were sharply violated, the activity in question would begin to look more like naked monopoly extension than genuine innovation.

**Proposition 5.** Assume condition (25) holds.

(a) Greater monopoly extraction raises the welfare growth rate through every period.

(b) Greater monopoly extension raises (lowers) the welfare growth rate through every period if the intertemporal externality is positive (negative).

**Proof:** Given Lemma 4, part (a) follows from Proposition 1 and part (b) follows from Proposition 2. □

We consider the results in Proposition 5 on welfare growth rates to be crisper than those in Propositions 3 and 4 on (the present value of) total welfare. Total welfare depends on the discount rate, which puts greater weight on surplus flows in earlier periods, whereas welfare growth rates are independent of discounting. When the intertemporal externality is negative, innovation may reduce total welfare but not welfare growth rates (so long as condition (25) holds). In practice, competition policy commonly presumes that innovation is good, which seems more in line with an emphasis on promoting welfare growth rates rather than total welfare. Regardless, Proposition 5 suggests a qualitatively similar conclusion to that of Propositions 3 and 4: divergent treatment of extraction and extension may be justified when the intertemporal externality of innovation is negative.

### 5 Competition Policy Implications

As the foregoing analysis reveals, monopoly extraction and monopoly extension have distinct effects, and the distinctions turn on the sign of the intertemporal externality of innovation. Extraction and extension tend to have similar effects on innovation, welfare and growth when the intertemporal externality is positive, but their effects can diverge sharply when the externality is negative. Monopoly extension inhibits steady-state equilibrium innovation and welfare growth rates when the intertemporal externality

\textsuperscript{16} This point has a similar flavor to O’Donoghue’s (1998) finding that a patentability requirement (a minimum innovation size to obtain patent protection) can improve dynamic efficiency.
is negative, whereas monopoly extraction tends to promote both innovation and welfare growth rates regardless of the sign of the externality.

Insofar as an important consideration in competition policy is the promotion of innovation and growth, the analysis suggests that appropriate policy toward monopoly extension should turn on the sign of the intertemporal externality. The analysis suggests a theoretical rationale for broadly permitting monopoly extraction but prohibiting monopoly extension if the intertemporal externality of innovation is negative. But this begs the question of how the sign of the externality might be determined. A related and equally thorny question is how monopolization might be distinguished in practice from innovative activity that extends monopoly tenure through competition on the merits.

The analysis in Section 3 suggests an answer of sorts to both quandaries. Recall that monopoly extension induces the incumbent to either expand R&D and innovation, when the intertemporal externality is positive, or to pare them back when the externality is negative. In the former case, the activity in question typically improves welfare, but in the latter case the activity often harms welfare and typically slackens welfare growth rates. This suggests a (deceptively simple) rule-of-thumb regarding single-firm conduct that extends monopoly tenure: antitrust authorities should permit such conduct when it is associated with increased innovation, but prohibit it when it is associated with decreased innovation. Determining what constitutes “innovation” by a monopolist may pose hard challenges, but arguably the task is less complex than measuring welfare changes or evaluating the sign of the intertemporal externality directly.

Indeed, our proposed rule-of-thumb appears consistent with antitrust practice. We believe there is broad consensus that innovative activity which extends monopoly tenure by delivering higher quality or lower price to consumers is a legitimate form of competition on the merits, whereas conduct that extends monopoly tenure without generating the salutary effects of increased innovation constitutes illegal monopolization. Our analysis provides a theoretical rationale for this policy.

With respect to monopoly extraction, we show that increased extraction invariably increases innovation and typically raises welfare growth rates. This suggests that a permissive policy toward pure monopoly extraction (e.g., more extractive but non-exclusionary forms of price discrimination) is warranted. Such a policy also appears
consistent with antitrust practice, given that exclusion is a necessary element of single-firm conduct found to violate Section 2 of the Sherman Act.

6 Conclusions

We develop a model of sequential innovation to explore the potentially divergent effects of monopoly extraction and monopoly extension on innovation, welfare and long-term growth. We find that monopoly extension’s effects turn on the sign of the intertemporal externality of innovation. Our analysis provides a theoretical rationale for prohibiting monopoly extension if the externality is negative. The externality’s sign is revealed by the directional effect monopoly extension has on the incumbent monopolist’s choice of innovation intensity or quality. Changes in this intensity that accompany suspected acts of monopoly extension might be more visible to competition authorities and courts than are changes in welfare growth rates, and so might serve as useful proxies for changes in welfare growth rates.

In this paper, we have interpreted changes in the monopoly extraction rate as flowing from changes in the way the monopolist price discriminates. Our motivation has been that, in the U.S. at least, simple monopoly pricing above marginal cost is legal under Section 2 of the Sherman Act. The European Union, however, further prohibits “abuse of dominance” that is purely “exploitative.”17 Article 82(a) of the EC Treaty prohibits a dominant firm from “imposing unfair purchase or selling prices.”18 In future research, we could model the effects of such a policy on innovation and welfare. Restricting the monopolist to pricing below the monopoly optimum would clearly inhibit innovation (Proposition 1), but would also limit static efficiency losses.

18 In practice, differences between U.S. and European competition policy toward single-firm conduct may not be as stark as the foregoing suggests. In his survey of European case law, Wahl (2007), a judge at the Court of First Instance, concludes that “the Court has not yet condemned a particular pricing practice, in a free and unregulated market, as amounting to unfairly high and exploitative prices and thus constituting an infringement of Article 82.” Cf. Werden (2009) for further discussion. In the U.S., on the other hand, conduct that may represent pure price discrimination can run a risk of being condemned as exclusionary.
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Kathy Burt
U.S. Department of Justice
450 5th Street, NW, Suite 9452
Washington, DC 20530
202-307-5794

kathy.burt@usdoj.gov